Abstract

The Derivatives Sourcebook is a citation study and classification system that organizes the many strands of the derivatives literature and assigns each citation to a category. Over 1800 research articles are collected and organized into a simple web-based searchable database. We have also included the 1997 Nobel lectures of Robert Merton and Myron Scholes as a backdrop to this literature.
Publisher’s Note

The Derivatives Sourcebook is a valuable bibliography of the literature for the derivatives research community – both academic and professional. We felt that the Foundations and Trends format, which allows for updating, would be ideal to keep this bibliographic effort alive and current. In addition, the classification provided by the authors and the links to the original articles gives the reader a tremendous reach into the research in this area and should make finding and accessing this research much easier.

We are grateful to the Nobel Foundation and Professors Robert C. Merton and Myron S. Scholes for allowing us the republish their Nobel lectures in this issue. While FnT Finance typically publishes review articles that are commissioned, written, and reviewed for the journal itself, we felt that there was no one better positioned to write such a survey and these lectures provide an historic perspective of this research topic.

We hope you find this bibliographic resource valuable, use it frequently, and help us keep it current by submitting any new references that appear in the literature. You can submit updates via our web site <www.nowpublishers.com> and look for “Update an FnT” on the homepage. Thank you.
One of the most important breakthroughs in modern finance is the pricing and hedging of derivative securities. By now, the fascinating history of the derivatives pricing literature is well known, having been chronicled by a number of authors including Bernstein (1992), MacKenzie (2006), Mehrling (2005), and Fischer Black himself (1989). Indeed, the remarkable twists and turns leading up to the publication of Black and Scholes (1973) and Merton (1973) is no less gripping than the story of the discovery of the structure of DNA as told by James Watson (1968) in *The Double Helix*.

But the intellectual lineage of the derivatives literature has received somewhat less attention, partly because bibliographies are simply not that exciting, but also because this literature has spread so widely and so fast. We conjecture that, in the modern history of all the social sciences, no other idea has had a bigger impact on theory and practice in such a short period of time. In academia, the impact of Black-Scholes and Merton has been profound – their papers have led to new insights into the dynamic structure of real and financial asset markets, the nature of intertemporal risks and hedging activity, macroeconomic risk exposures, and the value of flexibility in a variety of economic and,
in a few cases, non-economic contexts. And in industry, the impact of Black-Scholes and Merton has been equally profound, becoming the standard references and theoretical underpinnings for at least three distinct businesses – the listed options markets, the OTC structured products market, and the burgeoning credit derivatives market. This breakneck pace of research and development and the many corresponding industrial innovations – recall that the Black-Scholes and Merton papers are not yet 35 years old – have left little time for reflection on the breadth and reach of their original publications. This was the original motivation for the Derivatives Sourcebook Project (DSP).

The DSP began in 1997 a few weeks before the Nobel Prize Award Ceremony in Stockholm. Initially intended as a citation study of the Black-Scholes and Merton papers, the DSP took on a life of its own. Encouraged and supported by Bob Merton, we created a classification system to organize the many strands of the derivatives literature, and assigned each citation to a category. Even a cursory glance at the many and varied categories should generate a certain degree of intellectual vertigo in any academic – the comparison to Helen of Troy, the face that launched a thousand ships, springs to mind. We have also placed these citations – over 1,800 research articles – into a simple web-based searchable database (http://lfe.mit.edu/dsp/) where researchers can search the derivatives literature by category, author, title, and other characteristics. The website will eventually allow users to submit updates to the database which we hope to incorporate periodically, so as to allow the DSP to evolve organically as the literature continues to develop.

Because this endeavor was prompted by the 1997 Nobel Prize, it seems only appropriate to include the Nobel lectures of Bob Merton and Myron Scholes here, and they are reprinted in their entirety in Sections 2 and 3, respectively. While much has happened since those lectures, they continue to provide a remarkably current and timely framework for this literature. In Section 4, we provides the classification codes and categories of the citations listed in Section 5. With many of the citations linked to the original articles in the online version, and the ability to update the citations database, we hope this will become a useful tool for academics and practitioners alike.
1.1 References


2

Applications of Option-Pricing Theory: Twenty-Five Years Later

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2.1 Introduction

The news from Stockholm that the prize in economic sciences had been given for option-pricing theory provided unique and signal recognition to the rapidly advancing, but still relatively new discipline, within economics which relates mathematical finance theory and finance practice. The special sphere of finance within economics is the study of allocation and deployment of economic resources, both spatially and across time, in an uncertain environment. To capture the influence and interaction of time and uncertainty effectively requires sophisticated mathematical and computational tools. Indeed, mathematical models of modern finance contain some truly elegant applications of probability and optimization theory. These applications challenge the most powerful computational technologies. But, of course, all that is elegant and challenging in science need not also be practical; and surely, not all that is practical in science is elegant and challenging. Here we have both.

In the time since publication of our early work on the option-pricing

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1 This section draws on Merton (1995, 1997b).
model, the mathematically complex models of finance theory have had a direct and wide-ranging influence on finance practice. This conjoining of intrinsic intellectual interest with extrinsic application is central to research in modern finance.

It was not always thus. The origins of much of the mathematics in modern finance can be traced to Bachelier’s (1900) dissertation on the theory of speculation, framed as an option-pricing problem. This work marks the twin births of both the continuous-time mathematics of stochastic processes and the continuous-time economics of derivative-security pricing. Itô (1987) was greatly influenced by Bachelier’s work in his development in the 1940s and early 1950s of the stochastic calculus, later to become an essential mathematical tool in finance. Paul Samuelson’s theory of rational warrant pricing, published in 1965, was also motivated by the same piece. However, Bachelier’s important work was largely lost to financial economists for more than a half century. During most of that period, mathematically complex models with a strong influence on practice were not at all the hallmarks of finance theory. Before the pioneering work of Markowitz, Modigliani, Miller, Sharpe, Lintner, Fama, and Samuelson in the late 1950s and 1960s, finance theory was little more than a collection of anecdotes, rules of thumb, and shuffling of accounting data. It was not until the end of the 1960s and early 1970s that models of finance in academe become considerably more sophisticated, involving both the intertemporal and uncertainty dimensions of valuation and optimal decision-making. The new models of dynamic portfolio theory, intertemporal capital asset pricing, and derivative-security pricing employed stochastic differential and integral equations, stochastic dynamic programming, and partial differential equations. These mathematical tools were a quantum level more complex than had been used in finance before and they are still the core tools employed today.

The most influential development in terms of impact on finance practice was the Black-Scholes model for option pricing. Yet paradoxically, the mathematical model was developed entirely in theory, with essentially no reference to empirical option-pricing data as motivation for its formulation. Publication of the model brought the field to almost immediate closure on the fundamentals of option-pricing theory. At the
same time, it provided a launching pad for refinements of the theory, extensions to derivative-security pricing in general, and a wide range of other applications, some completely outside the realm of finance. The Chicago Board Options Exchange (CBOE), the first public options exchange, began trading in April 1973, and by 1975, traders on the CBOE were using the model to both price and hedge their option positions. It was so widely used that, in those pre-personal-computer days, Texas Instruments sold a handheld calculator specially programmed to produce Black-Scholes option prices and hedge ratios. That rapid adoption was all the more impressive, as the mathematics used in the model were not part of the standard mathematical training of either academic economists or practitioner traders.

Academic finance research of the 1960s including capital asset pricing, performance and risk measurement, and the creation of the first large-scale databases for security prices essential for serious empirical work have certainly influenced subsequent finance practice. Still the speed of adoption and the intensity of that influence was not comparable to the influence of the option model. There are surely several possible explanations for the different rates of adoption in the 1960s and the 1970s. My hypothesis is that manifest “need” determined that difference. In the 1960s, especially in the United States, financial markets exhibited unusually low volatility: the stock market rose steadily, interest rates were relatively stable, and exchange rates were fixed. Such a market environment provided investors and financial-service firms with little incentive to adopt new financial technology, especially technology designed to help manage risk. However, the 1970s experienced several events that caused both structural changes and large increases in volatility. Among the more important events were: the shift from fixed to floating exchange rates with the fall of Bretton Woods and the devaluation of the dollar; the world oil-price shock with the creation of OPEC; double-digit inflation and interest rates in the United States; and the extraordinary real-return decline in the U.S. stock market from a peak of around 1050 on the Dow Jones Industrial Average in the beginning of 1973 to about 580 at the end of 1974. As a result, the increased demand for managing risks in a volatile and structurally different economic environment contributed to the major
success of the derivative-security exchanges created in the 1970s to trade listed options on stocks, futures on major currencies, and futures on fixed-income instruments. This success in turn increased the speed of adoption for quantitative financial models to help value options and assess risk exposures.

The influence of option-pricing theory on finance practice has not been limited to financial options traded in markets or even to derivative securities generally. As we shall see, the underlying conceptual framework originally used to derive the option-pricing formula can be used to price and evaluate the risk in a wide array of applications, both financial and non-financial. Option-pricing technology has played a fundamental role in supporting the creation of new financial products and markets around the globe. In the present and in the impending future, that role will continue expanding to support the design of entirely new financial institutions, decision-making by senior management, and the formulation of public policy on the financial system. To underscore that point, I begin with a few remarks about financial innovation of the past, this adumbration to be followed in later sections with a detailed listing of applications of the options technology that include some observations on the directions of future changes in financial services.

New financial product and market designs, improved computer and telecommunications technology and advances in the theory of finance during the past quarter-century have led to dramatic and rapid changes in the structure of global financial markets and institutions. The scientific breakthroughs in financial modeling in this period both shaped and were shaped by the extraordinary flow of financial innovation which coincided with those changes. Thus, the publication of the option-pricing model in 1973 surely helped the development and growth of the listed options and over-the-counter (OTC) derivatives markets. But, the extraordinary growth and success of those markets just as surely stimulated further development and research focus on the derivative-security pricing models. To see this in perspective, consider some of the innovative changes in market structure and scale of the global financial system since 1973. There occurred the aforementioned fall of Bretton Woods leading to floating-exchange rates for currencies; the development of the national mortgage market in the United States which
in turn restructured that entire industry; passage of the Employee Retirement Income Security Act (ERISA) in 1974 with the subsequent development of the U.S. pension-fund industry; the first money-market fund with check writing that also took place in 1974; and the explosive growth in mutual fund assets from $48 billion 25 years ago to $4.3 trillion today (a ninety-fold increase), with one institution, Fidelity Investments, accounting for some $500 billion by itself. In this same period, average daily trading volume on the New York Stock Exchange grew from 12 million shares to more than 300 million. Even more dramatic were the changes in Europe and in Asia. The cumulative impact has significantly affected all of us – as users, producers, or overseers of the financial system.

Nowhere has this been more the case than in the development, refinement and broad-based implementation of contracting technology. Derivative securities such as futures, options, swaps and other contractual agreements – the underlying substantive instruments for which our model was developed – provide a prime example. Innovations in financial-contracting technology have improved efficiency by expanding opportunities for risk sharing, lowering transactions costs and reducing information and agency costs. The numbers reported for the global use of derivative securities are staggering (the figure of $70 trillion appeared more than once in the news stories surrounding the award of the Prize and there are a number of world banking institutions with reported multi-trillion dollar, off-balance-sheet derivative positions). However, since these are notional amounts (and often involve double-counting), they are meaningless for assessing either the importance or the risk-exposure to derivative securities. Nevertheless, it is enough to say here that, properly measured, derivatives are ubiquitous throughout the world financial system and that they are used widely by non-financial firms and sovereigns as well as by institutions in virtually every part of their financing and risk-managing activities. Some observers see the extraordinary growth in the use of

\(^2\text{Notional amounts typically represent either the total value of the underlying asset on which payments on the derivative is determined (e.g. interest-rate swap contracts) or the exercise price on an option. The value of the derivative contract itself is often a small fraction of its notional amount.}\)
derivatives as fad-like, but a more likely explanation is the vast saving in transactions costs derived from their use. The cost of implementing financial strategies for institutions using derivatives can be one-tenth to one-twentieth of the cost of executing them in the underlying cash-market securities. The significance of reducing spread costs in financing can be quite dramatic for corporations and for sovereigns: for instance, not long ago, a 1 percent (i.e., 100-basis-point) reduction in debt-spread cost on Italian government debt would have reduced the deficit by an amount equal to 1.25 percent of the gross domestic product of Italy.

Further improved technology, together with growing breadth and experience in the applications of derivatives, should continue to reduce transactions costs as both users and producers of derivatives move along the learning curve. Like retail depositors with automatic-teller machines in banks, initial resistance by institutional clients to contractual agreements can be high, but once customers use them they tend not to return to the traditional alternatives for implementing financial strategies.

A central process in the past two decades has been the remarkable rate of globalization of the financial system. Even today, inspection of the diverse financial systems of individual nation-states would lead one to question how effective integration across geopolitical borders could have realistically taken place since those systems are rarely compatible in institutional forms, regulations, laws, tax structures, and business practices. Still, significant integration did take place. This was made possible in large part by derivative securities functioning as “adapters.” In general, the flexibility created by the widespread use of contractual agreements, other derivatives, and specialized institutional designs provides an offset to dysfunctional institutional rigidities. More specifically, derivative-security contracting

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3See Perold (1992) for a case study illustrating the savings in transactions costs, taxes, and custodial fees from using derivatives instead of the cash market. Scholes (1976) provides an early analysis of the effect of taxes on option prices.

4Scholes and Wolfson (1992) develop the principles of security and institutional design along these lines. See also Perold (1992) and Merton (1993, 1995). Inspection of the weekly International Financing Review will find the widespread and varied applications of financial engineering, derivatives, special-purpose vehicles and securities for private-sector and sovereign financing in every part of the world.
technologies provide efficient means for creating cross-border interfaces among otherwise incompatible domestic systems, without requiring widespread or radical changes within each system. For that reason, implementation of derivative-security technology and markets within smaller and emerging-market countries may help form important gateways of access to world capital markets and global risk-sharing. Such developments and changes are not limited only to the emerging-market countries with their new financial systems. Derivatives and other contracting technologies are likely to play a significant role in the financial engineering of the major transitions required for European Monetary Union and for the major restructuring of financial institutions in Japan.

With this introduction as background, I turn now to the key conceptual and mathematical framework underlying the option-pricing model and its subsequent applications.

2.2 General Derivation of Derivative-Security Pricing

I understand that it is customary in these lectures for the Laureates to review the background and the process leading up to their discoveries. Happily, there is no need to do so here since that has been done elsewhere in Black (1989), Bernstein (1992, Ch. 11) Merton and Scholes (1995), and Scholes (1998). Instead, I briefly summarize. My principal contribution to the Black-Scholes option-pricing theory was to show that the dynamic trading strategy prescribed by Black and Scholes to offset the risk exposure of an option would provide a perfect hedge in the limit of continuous trading. That is, if one could trade continuously without cost, then following their dynamic trading strategy using the underlying traded asset and the riskless asset would exactly replicate the payoffs on the option. Thus, in a continuous-trading financial environment, the option price must satisfy the Black-Scholes formula or else there would be an opportunity for arbitrage profits. To demonstrate this limit-case result, I applied the tools developed in my earlier work (1969; 1971) on the continuous-time theory of portfolio selection. My 1973 paper also extended the applicability of the Black-Scholes model to allow for stochastic interest rates on the riskless asset, dividend payments on the underlying asset, a changing exercise price,
American-type early-exercise of the option, and other “exotic” features such as the “down-and-out” provision on the option. I am also responsible for naming the model, “the Black-Scholes Option-Pricing Model.”

The derivations of the pricing formula in both of our 1973 papers make the following assumptions:

I) “Frictionless” and “continuous” markets: there are no transactions costs or differential taxes. Markets are open all the time and trading takes place continuously. Borrowing and short-selling are allowed without restriction. The borrowing and lending rates are equal.

II) Underlying asset-price dynamics: let \( V = V(t) \) denote the price at time \( t \) of a limited-liability asset, such as share of stock. The posited dynamics for the instantaneous returns can be described by an Itô-type stochastic differential equation with continuous sample paths given by

\[
dV = [\alpha V - D_1(V,t)]dt + \sigma V dZ
\]

where: \( \alpha \equiv \) instantaneous expected rate of return on the security; \( \sigma^2 \equiv \) instantaneous variance rate, which is assumed to depend, at most, on \( V(t) \) and \( t \) (i.e., \( \sigma^2 = \sigma^2(V,t) \); \( dZ \) is a Wiener process; and \( D_1 \equiv \) dividend payment flow rate. With limited liability, to avoid arbitrage, \( V(t) = 0 \) for all \( t \geq t^* \) if \( V(t^*) = 0 \). Hence \( D_1 \) must satisfy \( D_1(0,t) = 0 \). Other than a technical requirement of bounded variation, \( \alpha \) can follow a quite general stochastic process, dependent on \( V \), other security prices, or state variables. In particular, the assumed dynamics permit a mean-reverting process for the underlying asset’s returns.

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My 1970 working paper was the first to use the “Black-Scholes” label for their model (cf. Merton, 1992, p. 379). This same paper was given at the July 1970 Wells Fargo Capital Market Conference, since made “famous” (or notorious) by Bernstein (1992, p. 223) as the one at which I “… inconveniently overslept …” the morning session and missed the Black and Scholes presentation. The second instance naming their model was in the 1971 working-paper version of Merton (1973a). Samuelson (1972) is the first published usage: both in the main text and in my appendix to that paper which derives the model and refers to it as the “Black-Scholes formula.” The formula is cited in Leonard (1971) and Baldwin (1972), the earliest theses to apply the model. Somewhat ironically, all these references to the “Black-Scholes model” appear before the actual publication of either Black and Scholes (1972) or (1973).
III) Default-free bond-price dynamics: bond returns are assumed
to be described by Itô stochastic processes with continuous
sample paths. In the original Black and Scholes formulation
and for exposition convenience here, it is assumed that the
riskless instantaneous interest rate, \( r(t) = r \), is a constant
over time.

IV) Investor preferences and expectations: investor preferences
are assumed to prefer more to less. All investors are assumed
to agree on the function \( \sigma^2 \) and on the Itô process character-
ization for the return dynamics. It is not assumed that they
agree on the expected rate of return, \( \alpha \).

V) Functional dependence of the option-pricing formula: the
option price is assumed to be a twice-continuously differenti-
table function of the asset price, \( V \), default-free bond prices,
and time.

In the particular case of a nondividend-paying asset \( (D_1 = 0) \) and a
constant variance rate, \( \sigma^2 \), these assumptions lead to the Black-Scholes
option-pricing formula for a European-type call option with exercise
price \( L \) and expiration date \( T \), written as

\[
C(V,t) = VN(d) - L \exp(-r(T-t))N(d - \sigma \sqrt{T-t})
\]

where \( d = (\ln[V/L] + [r + \sigma^2/2](T-t)]/\sigma \sqrt{T-t}) \) and \( N( ) \) is the cum-
ulative density function for the standard normal distribution.

Subsequent research in the field proceeded along three dimensions:
applications of the technology to other than financial options (which is
discussed in the next section); empirical testing of the pricing formula,
which began with a study using over-the-counter data from a dealer’s
book obtained by Black and Scholes (1972); attempts to weaken the
assumptions used in the derivation, and thereby to strengthen the foun-
dation of the applications developed from this research. The balance of
this section addresses issues of the latter dimension.

Early concerns raised about the model’s theoretical foundation came
from Long (1974) and Smith (1976), who questioned Assumption V:
namely, how does one know that the option prices do not depend on
other variables than the ones assumed (for instance, the price of beer), and why should the pricing function be twice-continuously differentiable? These concerns were resolved in an alternative derivation in Merton (1977b) which shows that Assumption V is a derived consequence, not an assumption, of the analysis.⁶

A broader, and still open, research issue is the robustness of the pricing formula in the absence of a dynamic portfolio strategy that exactly replicates the payoffs to the option security. Obviously, the conclusion on that issue depends on why perfect replication is not feasible as well as on the magnitude of the imperfection. Continuous trading is, of course, only an idealized prospect, not literally obtainable; therefore, with discrete trading intervals, replication is at best only approximate. Subsequent simulation work has shown that within the actual trading intervals available and the volatility levels of speculative prices, the error in replication is manageable, provided, however, that the other assumptions about the underlying process obtain. Cox and Ross (1976) and Merton (1976a, b) relax the continuous sample-path assumption and analyze option pricing using a mixture of jump and diffusion processes to capture the prospect of non-local movements in the underlying asset’s return process.⁷ Without a continuous sample path, replication is not possible and that rules out a strict no-arbitrage derivation. Instead, the derivation of the option-pricing model is completed by using equilibrium asset pricing models such as the Intertemporal CAPM (Merton, 1973b) and the

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⁶As another instance of early questioning of the core model, a paper I refereed argued that Black-Scholes must be fundamentally flawed because a different valuation formula is derived from the replication argument if the Stratonovich (1968) stochastic calculus is used for modeling instead of the Itô calculus. My report showed that while the paper’s mathematics were correct, its economics were not: A Stratonovich-type formulation of the underlying price process implies that traders have a partial knowledge about future asset prices that the non-anticipating character of the Itô process does not. The “paradox” is thus resolved because the assumed information sets are essentially different and hence, so should the pricing formulas.

⁷Since a discontinuous sample-path price process for the underlying asset rules out perfect hedging even with continuous trading but a continuous-sample-path process with stochastic volatility does not, there is considerable interest in testing which process fits the data better. See Rosenfeld (1980), an early developer of such tests and Wiggins (1987).
Arbitrage Pricing Theory (Ross, 1976a). This approach relates back to the original way in which Black and Scholes derived their model using the classic Sharpe-Lintner CAPM. There has developed a considerable literature on the case of imperfect replication (cf. Bertsimas et al. (1997), Breeden (1984), Davis (1997), Figlewski (1989), Föllmer and Sondermann (1986), and Romano and Touzi (1997)).

On this occasion, I re-examine the imperfect-replication problem for a derivative security linked to an underlying asset that is not continuously available for trading in an environment in which some assets are tradable at any time. As is discussed in the section to follow, non-tradability is the circumstance for several important classes of applications that have evolved over the last quarter century, which include among others, the pricing of financial guarantees such as deposit and pension insurance and the valuation of non-financial or “real” options. Since the Black-Scholes model was derived by assuming that the underlying asset is continuously traded, questions have been raised about whether the pricing formula can be properly applied in those applications. The derivation follows along the lines presented in Merton (1977b, 1997b) for the perfect-replication case.

A derivative security has contractually determined payouts that can be described by functions of observable asset prices and time. These payout functions define the derivative. We express the terms as follows:

Let $W(t) =$ price of a derivative security at time $t$.

If $V(t) \geq V(t)$ for $0 \leq t < T$, then $W(t) = f[V(t), t]$.

If $V(t) \leq V(t)$ for $0 \leq t < T$, then $W(t) = g[V(t), t]$ (2.2)

If $t = T$, then $W(T) = h[V(T)]$

For $0 \leq t \leq T$, the derivative security receives a payment flow rate specified by $D_2(V, t)$. The terms as described in (2.2) are to be interpreted

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8 The important Breeden (1979) Consumption-based Capital Asset Pricing Model, which was not published at the time of these papers, can also be used to complete those models.

9 See Black (1989) and Scholes (1998). Fischer Black always maintained with me that the CAPM-version of the option-model derivation was more robust because continuous trading is not feasible and there are transactions costs. As noted in Merton (1973a, p. 116) the discrete-time Samuelson and Merton (1969) model also gives the Black-Scholes formula under special conditions.
as follows: the first time that $V(t) \geq \bar{V}(t)$ or $V(t) \leq \underline{V}(t)$, the owner of the derivative must exchange it for cash according to the schedule in (2.2). If no such events occur for $t < T$, then the security is redeemed at $t = T$ for cash according to (2.2). $T$ is called the maturity date (or expiration date, or redemption date) of the derivative. The derivative security is thus defined by specifying the contingent payoff functions $f, g, h, D_2$, and $T$. In some cases, the schedules or the boundaries $\bar{V}(t)$ and $\underline{V}(t)$ are contractually specified; in others, they are determined endogenously as part of the valuation process, as in the case of the early-exercise boundary for an American-type option.

By arbitrage restrictions, the derivative security will have limited liability if and only if $g \geq 0, h \geq 0, f \geq 0$, and $D_2(0,t) = 0$.

If (as drawn in Figure 2.1) the boundaries $\underline{V}(t)$ and $\bar{V}(t)$ are continuous functions, then because $V(t)$ has a continuous sample path in $t$ by Assumption II, one has that (i) if $V(t) < \underline{V}(t)$ for some $t$, then there is a $\bar{t}, \bar{t} < t$, so that $V(\bar{t}) = \underline{V}(\bar{t})$ and (ii) if $V(t) > \bar{V}(t)$ for some $t$, then there is a $\bar{t}, \bar{t} < t$, so that $V(\bar{t}) = \bar{V}(\bar{t})$. Hence, in this case, the
inequalities for $V$ can be neglected in (2.2) and the only relevant region for analysis is $V(t) \leq V(t) \leq V(t), 0 \leq t \leq T$.

With the derivative-security characteristics fully specified, we turn now to the fundamental production technology for hedging the risk of issuing a derivative security and for evaluating the cost of its production. To locate the derivation in a more substantive framework, I posit a hypothetical financial intermediary that creates derivative securities in principal transactions for its customers by selling them contracts which are its obligation. It uses the capital markets or transactions with other institutions to hedge the contractual liabilities so created by dynamically trading in the underlying securities following a strategy designed to reproduce the cash flows of the issued contracts as accurately as it can. If the intermediary cannot perfectly replicate the payoffs to the issued derivative, it either obtains adequate equity to bear the residual risks of its imperfectly hedged positions or it securitizes those positions by bundling them into a portfolio for a special-purpose financial vehicle which it then sells either in the capital market or to a consortium of other institutions in a process similar to the traditional reinsurance market. Although surely a caricature, the following description is nevertheless not far removed from real-world practice.

The objective is to find a feasible, continuous-trading portfolio strategy constructed from all available traded assets including the riskless asset that comes “closest” to satisfying the following four properties: if $P(t)$ denotes the value of the portfolio at time $t$, then for $0 \leq t \leq T$:

(i) at $t$, if $V(t) = V(t)$, then $P(t) = g[V(t), t]$
(ii) at $t$, if $V(t) = V(t)$, then $P(t) = f[V(t), t]$
(iii) for each $t$, the payout rate on the portfolio is $D_2(V, t) dt$
(iv) at $t = T$, $P(T) = h[V(T)]$

Call this portfolio the “hedging portfolio” for the derivative security defined by (2.2). That portfolio is labeled as “portfolio (*)”. In the special, but important, case in which the portfolio meets the above conditions exactly, the hedging portfolio is called the “replicating portfolio” for the derivative security.

Bertsimas et al. (1997) study the complementary problem of “closeness” of dynamic replication where they assume that one can trade
in the underlying asset but that trading is not continuous. They apply stochastic dynamic programming to derive optimal strategies to minimize mean-squared tracking error. These strategies are then employed in simulations to estimate quantitatively how close one can get to dynamic completeness.

Determine the optimal hedging portfolio in two steps: first, find the portfolio strategy constructed from all continuously traded assets that has the smallest “tracking error” in replicating the returns on the underlying asset. For the underlying asset with price $V$, call this portfolio, the “$V$-Fund.” In the second step, derive the hedging portfolio for the derivative security as a dynamic portfolio strategy mixing the $V$-Fund with the riskless asset.

Let $S_i(t)$ denote the price of continuously traded asset $i$ at time $t$. There are $n$ such risky assets plus the riskless asset which are traded continuously. The dynamics for $S_i$ are assumed to follow a continuous-sample-path Itô process given by

$$dS_i = \alpha_i S_i dt + \sigma_i S_i dZ_i, \quad i = 1, \ldots, n$$

(2.3)

where $\alpha_i$ is the instantaneous expected rate of return on asset $i$; $dZ_i$ is a Wiener process; $\sigma_{ij}$ is the instantaneous covariance between the returns on $i$ and $j$ [that is, $(dS_i/S_i)(dS_j/S_j) = \sigma_{ij} dt$ and $\sigma_{ii} = \sigma_i^2$]; let $\eta_i$ be defined as the instantaneous correlation between $dZ_i$ and $dZ$ in Assumption II such that $dZ_i dZ = \eta_i dt$. Let $S(t)$ denote the value of the $V$-Fund portfolio and let $w_i(t)$ denote the fraction of that portfolio allocated to asset $i, i = 1, \ldots, n$, at time $t$. The balance of the portfolio’s assets are invested in the riskless asset. The dynamics for $S$ can be written as

$$dS = [\mu S - D_1(V, t)] dt + \delta S dq$$

(2.4)

where $\mu = r + \sum_{i=1}^n w_i(t)[\alpha_i - r], \delta^2 = \sum_{i=1}^n \sum_{j=1}^n w_i(t)w_j(t)\sigma_{ij}$ and $dq = \left[\sum_{i=1}^n w_i(t)\sigma_i dZ_i\right]/\delta$.

To create the $V$-Fund, the $w_i$ are chosen so as to minimize the unanticipated part of the difference between the return on the underlying asset and the traded portfolio’s return. That is, at each point in time, the portfolio allocation is chosen so as to minimize the instantaneous variance of $dS/S - dV/V$. As shown in Merton (1992,
Theorem 15.3; p. 501) the portfolio rule that does this is given by

$$ w_i(t) = \sigma \sum_{k=1}^{n} v_{ki} \sigma_k \eta_k, \quad i = 1, \ldots, n. \tag{2.5} $$

where $v_{ki}$ is the $k$-th $i$-th element of the inverse of the variance-covariance matrix of the returns on the $n$ risky continuously traded assets. From Merton (1992, p. 502), the instantaneous correlation between the returns on the $V$-Fund and the underlying asset, $\rho \, dt = dZdq$, can be written as

$$ \rho = \left( \sum_{k=1}^{n} \sum_{i=1}^{n} v_{ki} \sigma_k \sigma_i \eta_k \eta_i \right)^{1/2} \tag{2.6} $$

and

$$ \delta = \rho \sigma. \tag{2.7} $$

The dynamics of the tracking error can thus be written as

$$ dS/S - dV/V = (\mu - \alpha)dt + \theta db \tag{2.8} $$

where $\theta^2 = (1 - \rho^2)\sigma^2$ and the Wiener process $db = (\rho dq - dZ)/\sqrt{1 - \rho^2}$. As shown in Merton (1992, eq. 15.51), it follows that

$$ dS_i/S_i \, db = 0, \quad i = 1, \ldots, n. \tag{2.9} $$

That is, the tracking error in (2.8) is uncorrelated with the returns on all traded assets, which is a consequence of picking the portfolio strategy that minimizes that error.

With this, we now proceed with a “cookbook-like” derivation of the production process for our hypothetical financial intermediary to best hedge the cash flows of the derivative securities it issues. The derivation begins with a description of the activities for the intermediary’s quantitative-analysis (“quant”) department which is responsible for gathering the variance-covariance information necessary to use (2.5) to construct and maintain the $V$-Fund portfolio. It is also assigned the responsibility to solve the following linear parabolic partial differential equation for $F[V, t]$

$$ 0 = 1/2\sigma^2 (V, t)V^2 F_{11}[V, t] + \left[ rV - D_1(V, t) \right] F_1[V, t] $$

$$ - rF[V, t] + F_2[V, t] + D_2(V, t) \tag{2.10} $$
2.2. General Derivation of Derivative-Security Pricing

subject to the boundary conditions: for $\underline{V}(t) \leq V \leq \overline{V}(t)$ and $t < T$,

$$F[\overline{V}(t), t] = f[\overline{V}(t), t] \geq 0 \quad (2.11)$$

$$F[\underline{V}(t), t] = g[\underline{V}(t), t] \geq 0 \quad (2.12)$$

$$F[V, T] = h[V] \geq 0 \quad (2.13)$$

where $F_{11} \equiv \partial^2 F/\partial V^2, F_1 \equiv \partial F/\partial V$; and $F_2 \equiv \partial F/\partial t$. Note that the non-negativity conditions in (2.11)–(2.13) together with $D_2(0, t) = 0$ implies that the derivative security has limited liability. As a mathematical question, this is a well-posed problem, and a solution to (2.10)–(2.13) exists and is unique.

Having solved for the function $F[V, t]$, the quant department has the prescribed ongoing tasks at each time $t(0 \leq t \leq T)$ to:

(i) ask the trading desk for the prices of all traded assets necessary to determine the price $S(t)$ of the V-Fund and the best estimate of the current price of the underlying asset, $V(t)$;

(ii) compute from the solution to (2.10)–(2.13) compute $M(t) \equiv F_1[V(t), t]V(t)$;

(iii) tell the trading desk that the strategy of portfolio (*) requires that $M(t)$ be invested in the V-Fund for the period $t$ to $t + dt$;

(iv) compute $Y(t) \equiv F[V(t), t]$ and store $Y(t)$ in the intermediary’s data files for (later) analysis of the time series (i.e., stochastic process) $Y(t)$.

The prescription for the execution or trading-desk activities of the intermediary is as follows: At time $t = 0$, give the trading desk $P(0)$ as an initial funding (investment) for portfolio (*) which contains the V-Fund asset and the riskless asset. Let $P(t)$ denote the value of portfolio (*) at $t$, after having made any prescribed cash distribution (payment) from the portfolio. The trading desk has the job at each time $t(0 \leq t \leq T)$ to:

(a) determine the current prices of the underlying asset, $V(t)$ and all individual traded assets held in the V-Fund, and send that price information to the quant department;
(b) pay a cash distribution of $D_2[V(t),t]dt$ to the customer holding the derivative security; by selling securities in the portfolio (if necessary);
(c) compute the value of the balance of the portfolio, $P(t)$;
(d) receive instructions on $M(t)$ from the quant department;
(e) readjust the portfolio allocation so that $M(t)$ is now invested in the $V$-Fund and $[P(t) - M(t)]$ is invested in the riskless asset.

It follows that the dynamics for the value of portfolio (*) are given by

$$dP = M(t)\frac{dS}{S} + M(t)\frac{D_1(V,t)}{S} dt + [P - M(t)]r dt - D_2(V,t) dt$$

(2.14)

where

$$M(t)\frac{dS}{S} = \text{price appreciation}$$

$$M(t)\frac{D_1(V,t)}{S} dt = \text{dividend payments received into the portfolio}$$

$$[P - M(t)]r dt = \text{interest earned by the portfolio}$$

$$D_2(V,t) dt = \text{cash distribution to customer}$$

Noting that $M(t) = F_1[V,t]V$, one has by substitution from (2.4) into (2.14) that the dynamics of $P$ satisfy

$$dP = F_1[V,t]V dS/S + F_1[V,t]V D_1(V,t)/S$$

$$+ (P - F_1[V,t]V)r dt - D_2(V) dt$$

$$= [F_1V(\mu - r) + rP - D_2] dt + F_1V\delta dq$$

(2.15)

Return now to the quant department to derive the dynamics for $Y(t)$. From (iv), one has that $Y(t) = F[V,t]$ for $V(t) = V$. Because $F$ is the solution to (2.10)–(2.13), $F$ is a twice-continuously differentiable function of $V$ and $t$. Therefore, we can apply Itô’s lemma, so that for $V(t) = V$,

$$dY = F_1[V,t]dV + F_2[V,t]dt + 1/2F_{11}[V,t](dV)^2$$

$$= [1/2\sigma^2V^2F_{11} + F_1(\alpha V - D_1) + F_2] dt + F_1V\sigma dZ$$

(2.16)
2.2. General Derivation of Derivative-Security Pricing

because \((dV)^2 = \sigma^2 V^2 dt\). Because \(F[V, t]\) satisfies (2.10), one has that

\[
1/2\sigma^2 V^2 F_{11} - D_1 F_1 + F_2 = rF - rVF_1 - D_2 \tag{2.17}
\]

Substituting (2.17) into (2.16), one can rewrite (2.16) as

\[
dY = [F_1(\alpha - r)V + rF - D_2] dt + F_1 V \sigma dZ \tag{2.18}
\]

Note that the calculation of \(Y(t)\) and its dynamics by the quant department in no way requires knowledge of the time-series of values for portfolio (*), \(\{P(t)\}\), that are calculated by the trading desk. Putting these two time-series together, we define \(Q(t) \equiv P(t) - Y(t)\). It follows that \(dQ = dP - dY\). Substituting for \(dP\) from (2.15) and for \(dY\) from (2.18), rearranging terms using (2.8), one has that

\[
dQ = rQ dt + F_1 V (dS/S - dV/V)
\]

\[
= (rQ + F_1 V[\mu - \alpha]) dt + F_1 V \theta db. \tag{2.19}
\]

At this point, we digress to examine the special case in which perfect replication of the return on the underlying asset obtains (i.e., \(\rho = 1\) and there is no tracking error). In that case, equation (2.19) reduces to an ordinary differential equation \((\dot{Q}/Q = r)\) with solution

\[
Q(t) = Q(0) \exp(rt) \tag{2.20}
\]

where \(Q(0) = P(0) - Y(0) = P(0) - F[V(0),0]\). Therefore, if the initial funding provided to the trading desk for portfolio (*) is chosen so that \(P(0) = F[V(0),0]\), then from (2.20), \(Q(t) \equiv 0\) for all \(t\) and

\[
P(t) = F[V(t),t] \tag{2.21}
\]

By comparison of (2.11)–(2.13) with (2.2), one has from (2.21) that the (*)-portfolio strategy generates the identical payment flows and terminal (and boundary) values as the derivative security described at the outset of this analysis. That is, for a one-time, initial investment of \(\$F[V(0),0]\), a feasible portfolio strategy has been found that exactly replicates the payoffs to the derivative security. Thus, \(\$F[V(0),0]\) is the cost to the intermediary for producing the derivative. If the derivative security is traded, then to avoid (“conditional”) arbitrage (conditional on \(\sigma, r, D_1\)), its price must satisfy

\[
W(t) = P(t) = F[V(t),t]. \tag{2.22}
\]
Since the absence of arbitrage opportunities is a necessary condition for equilibrium, it follows that equilibrium prices for derivative securities on continuously tradable underlying assets must satisfy (2.22). This is, of course, the original Black-Scholes result and the \( V \)-Fund degenerates into a single asset, the underlying asset itself. However, note that (2.22) obtains without assuming that the derivative-pricing function is a twice-continuously differentiable function of \( V \) and \( t \). The smoothness of the pricing function is instead a derived conclusion.

Note further that the development of the (*)-portfolio strategy did not require that the derivative security (defined by (2.2)) actually trades in the capital market. The (*)-portfolio strategy provides the technology for “manufacturing” or synthetically creating the cash flows and payoffs of the derivative security if it does not exist. That is, if one describes a state-contingent schedule of outcomes for a portfolio (i.e. specifies \( f,g,h,D_2,T,\overline{V}(t),\overline{V}(t) \)), then the (*)-portfolio strategy provides the trading rules to create this pattern of payouts and it specifies the cost of implementing those rules. The cost of creating the security at time \( t \) is thus \( F[V(t),t] \). Moreover, if the financial-services industry is competitive, then price equals marginal cost, and (2.22) obtains as the formula for equilibrium prices of derivatives sold directly by intermediaries.

Returning from this digression to the case of imperfect replication, one has, by construction of the process for \( Y \), that \( Q = P - Y \) is the cumulative arithmetic tracking error for the hedging portfolio. By inspection of (2.19), the instantaneous tracking error for the derivative security is perfectly correlated with the tracking error of the \( V \)-Fund. Hence, from (2.9), it follows that the tracking error for the hedging portfolio is uncorrelated with the returns on all continuously traded assets. Using this lack of correlation with any other traded asset, I now argue that in this case the replication-based valuation can be used for pricing the derivative security even though replication is not feasible.

As we know, in all equilibrium asset-pricing models, assets that have only non-systematic or diversifiable risk are priced to yield an expected return equal to the riskless rate of interest. The condition satisfied by the tracking error component of the hedging portfolio satisfies an even stronger no-correlation condition than either a zero-beta asset in the
CAPM, a zero multibeta asset of the Intertemporal CAPM, or a zero factor-risk asset of the Arbitrage Pricing Theory. Thus, by any of those theories, the equilibrium condition from either (2.8) or (2.19) is that

\[ \mu = \alpha. \]  

(2.23)

If (2.23) obtains, it follows immediately that the equilibrium price for the derivative security is \( F[V(t), t] \), the same formula “as if” the underlying assets traded continuously. And as a consequence, the Black-Scholes formula would apply even in those applications in which the underlying asset is not traded.

As is well known from the literature on incomplete markets, (2.23) need not obtain if the creation of the new derivative security helps complete the market for a large enough subset of investors that the incremental dimension of risk spanned by this new instrument is “priced” as a systematic risk factor with an expected return different from the riskless interest rate. Markets tend to remain incomplete with respect to a particular risk either because the cost of creating the securities necessary to span that risk exceeds the benefits, or because non-verifiability, moral-hazard, or adverse-selection problems render the viability of such securities untenable. Generally, major macro risks for which significant pools of investors want to manage their exposures are not controllable by any group of investors, and it is unlikely that any group would have systematic access to materially better information about those risks. Hence, the usual asymmetric-information and incentive reasons given for market failure do not seem to be present. In systems with well-developed financial institutions and markets and with today’s financial technology, it is thus not readily apparent what factors make the cost of developing standardized derivative markets (e.g., futures, swaps, options) prohibitive if, in large scale, there is a significant premium latently waiting to be paid by investors who currently participate in the markets. On a more prosaic empirical note, in most applications of the option-pricing model, the “residual” or tracking-error variations are likely to be specific to the underlying project, firm, institution, or person, and thereby they are unlikely candidates for macro-risk surrogates. These observations support the prospects for (2.23) to obtain.
However, the risk need not be macro in scope in order to be significant to one investor or a small group of investors. Obvious examples of such risks would be various firm- or person-specific components of human capital, including death and disability risks. To make a case for instruments with these types of exposures to be priced with a risk premium, incomplete-market models often focus on the “incipient-demand” (or “maximum reservation”) price or risk premium that an investor would pay to eliminate a risk that is not covered in the market by the existing set of securities. In the abstract, that price, of course, can be quite substantial. However, arguments along these lines to explain financial product pricing implicitly assume a rather modest and static financial-services sector. A classic example is life insurance. Risk-averse individuals with families may, if necessary, be willing to pay a considerable premium for life insurance, well in excess of the actuarial mortality risk, even after taking into account moral-hazard and person-specific informational asymmetries. Moreover, if the analysis further postulates a financial sector so crude that bilateral contracts between risk-averse individuals are the only way to obtain such insurance, then the equilibrium price for such insurance in that model can be so large that few, if any, contracts are created. But, such models are a poor descriptor of the real world. If the institutions and markets were really that limited, the incentives for change and innovation would be enormous. Modern finance technology and experience in implementing it provide the means for such change. And if, instead, one admits into the model just the classic mechanism for organizing an “insurance” institution (whether government-run or private-sector) to take advantage of the enormous diversification benefits of pooling such risks and subdividing them among large numbers of participants, then the equilibrium price equals the “supply” price of such insurance contracts which approaches the actuarial rate.

As is typical in analyses of other industries, the equilibrium prices of financial products and services are more closely linked to the costs of the efficient producers than to the inefficient ones (except perhaps as a very crude upper bound to those prices). Furthermore, the institutional structure of the financial system is neither exogenous nor fixed. In theory and in practice, that structure changes in response to chang-
ing technology and to profit-opportunities for creating new products and existing products more efficiently. As discussed at length elsewhere (Merton, 1992, pp. 457–467; 535–536), a financial sector with a rich and well-developed structure of institutions can justify a “quasi-dichotomy” modeling approach to the pricing of real and financial assets that employs “reduced-form” equilibrium models with a simple financial sector in which all agents are assumed to be minimum-cost information processors and transactors. However, distortions of insights into the real world can occur if significant costs for the agents are introduced into that model while the simple financial sector is retained as an unchanged assumption. Put simply, high transaction and information costs for most of the economy’s agents to directly create their own financial products and services does not imply that equilibrium asset prices are influenced by those high costs, as long as there is an efficient financial-service industry with low-cost, reasonably competitive producers.

In considering the preceding technical analysis, one might wonder if there are relevant situations in which the price is observable but trade in the asset cannot take place? One common class of real-world instances is characterized as follows: consider an insurance company that has guaranteed the financial performance of the liabilities of a privately-held opaque institution with a mark-to-market portfolio of assets. The market value of that portfolio (corresponding to $V$ in the analysis here) is provided to the guarantor on a continuous basis, but the portfolio itself cannot be traded by the guarantor to hedge its exposure because it does not know the assets held within the portfolio. Elsewhere (Merton, 1997a), I have developed a model using an alternative approach of incentive-contracting combined with the derivative-security technology to analyze the problem of contract guarantees for an opaque institution. It is nevertheless the case that discontinuous tradability of an asset is often accompanied by discontinuous observations of its price. And so, the combination of the two warrants attention. Hence, I complete this section with consideration of how to modify the valuation formula if the price of the underlying asset $V$ is not continuously observable.
Suppose that in the example adopted in this section, the price of the underlying asset is observed at \( t = 0 \) and then again at the maturity of the derivative contract, \( t = T \). In between, there is neither direct observation nor inferential information from payouts on the asset. Hence, \( D_1(V,t) = 0 \), and the derivative security has no payouts or interim "stopping points" prior to maturity [as specified in (2.11) and (2.12)] contingent on \( V(t) \). It is however known that the dynamics of \( V \) are as described in Assumption II with a covariance structure with available traded assets sufficiently well specified to construct the \( V \)-Fund according to (2.5). Define the random variable \( X(t) \equiv V(t)/S(t) \), the cumulative proportional tracking error, with \( X(0) = 1 \). By applying Itô's lemma, one has from (2.8), (2.9), and (2.23) that the dynamics for \( X \) can be written as

\[
dX = \theta X \, db. \tag{2.24}
\]

It follows from (2.24) that the distribution for \( X(t) \), conditional on \( X(0) = 1 \), is lognormal with the expected value of \( X(t) \) equal to 1 and the variance of \( \ln[X(t)] \) equal to \( \theta^2 t \). The partial differential equation for \( F \), corresponding to (2.10), that determines the hedging strategy, uses as its independent variable the best estimate of \( V(t) \), which is \( S(t) \), and it is written as

\[
0 = 1/2 \delta^2 S^2 F_{11}[S,t] + rSF_{1}[S,t] - rF[S,t] + F_{2}[S,t], \tag{2.25}
\]

subject to the terminal-time boundary condition that for \( S(T-) = S \),

\[
F[S,T] = E\{h(SX)\} \tag{2.26}
\]

where \( h \) is as defined in (2.13), \( X \) is a lognormally distributed random variable with \( E\{X\} = 1 \) and variance of \( \ln[X] \) equal to \( \theta^2 T \) and \( E\{T\} \) is the expectation operator over the distribution of \( X \).

Condition (2.26) reflects the fact that for all \( t < T \), the best estimate of \( V(t) \) is \( S(t) \). However, at \( t = T \), \( V(T) \) is revealed and the value of \( S \) “jumps” by the total cumulative tracking error of \( X(T) \) from its value \( S \) at \( t = T - \) to \( S(T) = V(T) \). The effect of the underlying asset price not being observable is perhaps well-illustrated by comparing the solution for the European-type call option with the classic
Black-Scholes solution given here in (2.1). The solution to (2.25) and (2.26) with \( h(V) = \max[0, V - L] \) is given by, for \( 0 < t < T \),

\[
F[S,t] = SN(u) - L \exp(-r[T - t])N(u - \sqrt{\gamma})
\]  
(2.27)

where \( u = (\ln[S/L] + r[T - t] + \gamma/2)/\sqrt{\gamma}, \gamma = \delta^2(T - t) + \theta^2 T, \) and \( N(\cdot) \) is the cumulative density function for the standard normal distribution.

By inspection of (2.1) and (2.27), the key difference in the option-pricing formula with and without continuous observation of the underlying asset price is that the variance over the remaining life of the option does not go to zero as \( t \) approaches \( T \), because of the “jump” event at the expiration date corresponding to the cumulative effect of tracking error.

This section has explored conditions under which the Black-Scholes option-pricing model can be validly applied to the pricing of assets with derivative-security-like structures, even when the underlying asset-equivalent is neither continuously traded nor continuously observable. A fuller analysis of this question would certainly take account of the additional tracking error that obtains as a consequence of imperfect dynamic trading of the \( V \)-Fund portfolio, along the lines of Bertsimas et al. (1997). However, a more accurate assessment of the real-world impact should also take into account other risk-management tools that intermediaries have to reduce tracking error. For instance, as developed in analytical detail in Merton (1992, pp. 450–457) intermediaries need only use dynamic trading to hedge their net derivative-security exposures to various underlying assets. For a real-world intermediary with a large book of various derivative products, netting, which in effect extends the capability for hedging to include trading in securities with “non-linear” pay-off structures, can vastly reduce the size and even the frequency of the hedging transactions necessary to achieve an acceptable level of tracking error. Beyond this, as part of their optimal risk management, intermediaries can “shade” their bid and offer prices among their various products to encourage more or less customer activity in different products to help manage their exposures. The limiting case when the net positions of customer exposures leaves the intermediary with no exposure is called a “matched book.”
2.3 Applications of the Option-Pricing Technology

Open the financial section of a major newspaper almost anywhere in the world and you will find pages devoted to reporting the prices of exchange-traded derivative securities, both futures and options. Along with the vast over-the-counter derivatives market, these exchange markets trade options and futures on individual stocks, stock-index and mutual-fund portfolios, on bonds and other fixed-income securities of every maturity, on currencies, and on commodities including agricultural products, metals, crude oil and refined products, natural gas, and even electricity. The volume of transactions in these markets is often many times larger than the volume in the underlying cash-market assets. Options have traditionally been used in the purchase of real estate and the acquisition of publishing and movie rights. Employee stock options have long been granted to key employees and today represent a significantly growing proportion of total compensation, especially for the more highly paid workers in the United States. In all these markets, the same option-pricing methodology set forth in the preceding section is widely used both to price and to measure the risk exposure from these derivatives (cf. Jarrow and Rudd (1983) and Cox and Rubinstein (1985)). However, financial options represent only one of several categories of applications for the option-pricing technology.

In the late 1960s and early 1970s when the basic research leading to the Black-Scholes model was underway, options were seen as rather arcane and specialized financial instruments. However, both Black and Scholes (1972, 1973) and I (Merton, 1970, 1974) recognized early on in the research effort that the same approach used to price options could be applied to a variety of other valuation problems. Perhaps the first major development of this sort was the pricing of corporate liabilities, the “right-hand side” of the firm’s balance sheet. This approach to valuation treated the wide array of instruments used to finance firms such as debentures, convertible bonds, warrants, preferred stock, and common stock (as well as a variety of hybrid securities) as derivative securities with their contractual payouts ultimately dependent on the value of the overall firm. In contrast to the standard fragmented valuation methods
of the time, it provided a unified theory for pricing these liabilities. Because application of the pricing methodology does not require a history of trading in the particular instrument to be evaluated, it was well-suited for pricing new types of financial securities issued by corporations in an innovating environment. Applications to corporate finance along this line developed rapidly.\(^\text{10}\)

"Option-like" structures were soon seen to be lurking everywhere; thus there came an explosion of research in applying option-pricing theory which still continues. Indeed, I could not do full justice to the list of contributions accumulated over the past 25 years even if this entire paper were devoted to that endeavor. Fortunately, a major effort to do just that is underway and the results will soon be available (Jin et al., forthcoming). The authors have generously shared their findings with me. And so, I can convey here some sense of the breadth of applications and be necessarily incomplete without harm.

The put option is a basic option which gives its owner the right to sell the underlying asset at a specified ("exercise") price on or before a given ("expiration") date. When purchased in conjunction with ownership of the underlying asset, it is functionally equivalent to an insurance policy that protects its owner against economic loss from a decline in the asset's value below the exercise price for any reason, where the term of the insurance policy corresponds to the expiration date. Hence, option-pricing theory can be applied to value insurance contracts. An early insurance application of the Black-Scholes model was to the pricing of loan guarantees and deposit insurance (cf. Merton, 1977a). A contract that insures against losses in value caused by default on promised payments on a contract in effect is equivalent to a put option on the contract with an exercise price equal to the value of the contract if it were default-free. Loan and other contract guarantees, collectively called credit derivatives, are ubiquitous in the private sector. Indeed, whenever a debt instrument is purchased in which there is any chance that the promised payments will not be made, the purchaser is not only lending money but also in effect issuing a loan guarantee as

\(^{10}\)See Merton (1992, pp. 423–427) for an extensive list of references. See also Hawkins (1982) and Brennan and Schwartz (1985a) and the early empirical testing by Jones et al. (1984).
a form of self-insurance. Another private-sector application of options analysis is in the valuation of catastrophic-insurance reinsurance contracts and bonds.\textsuperscript{11} Dual funds and exotic options provide various financial insurance and minimum-return-guarantee products.\textsuperscript{12}

Almost surely, the largest issuer of such guarantees are governments. In the United States, the Office of the Management of the Budget is required by law to value those guarantees. The option model has been applied to assess deposit insurance, pension insurance, guarantees of student loans and home mortgages, and loans to small businesses and some large ones as well.\textsuperscript{13} The application to government activities goes beyond just providing guarantees. The model has been used to determine the cost of other subsidies including farm-price supports and through-put guarantees for pipelines.\textsuperscript{14} It has been applied to value licenses issued with limiting quotas such as for taxis or fisheries or the right to pollute and to value the government’s right to change those quotas.\textsuperscript{15} Government sanctions patents. The decision whether to spend the resources to acquire a patent depends on the value of the patent which can be framed as an option-pricing problem. Indeed, even on something that is not currently commercial, one may acquire the patent for its “option value,” should economic conditions change in an unexpected way.\textsuperscript{16} Paddock et al. (1988) show that option value can be a significant proportion of the total valuation of government-granted offshore drilling rights, especially when current and expected future economic conditions would not support development of the fields.

\textsuperscript{12} Brennan and Schwartz (1976), Ingersoll, Jr. (1976), Goldman et al. (1979), Gatto et al. (1980), and Stulz (1982). In an early real-world application, Myron Scholes and I developed the first options-strategy mutual fund in the United States, Money Market/Options Investments, Inc., in February 1976. The strategy which invested 90 percent of its assets in money market instruments and ten percent in a diversified portfolio of stock call options provided equity exposure on the upside with a guaranteed “floor” on the value of the portfolio. The return patterns from this and similar “floor” strategies were later published in Merton et al. (1978, 1982).
\textsuperscript{15} Anderson (1987) and Karpoff (1989).
\textsuperscript{16} Trigeorgis (1993).
2.3. Applications of the Option-Pricing Technology

Option-pricing analysis quantifies the government’s economic decision whether to build roads in less-populated areas depending on whether it has the policy option to abandon rural roads if they are not used enough.\textsuperscript{17}

Various legal and tax issues involving policy and behavior have been addressed using the option model. Among them is the valuation of plaintiffs’ litigation options, bankruptcy laws including limited-liability provisions, tax delinquency on real estate and other property as an option to abandon or recover the property by paying the arrears, tax evasion, and valuing the tax “timing” option for the capital-gains tax in a circumstance when only realization of losses and gains on investments triggers a taxable event.\textsuperscript{18}

In a recent preliminary study, the options structure has been employed to help model the decision of whether the Social Security fund should invest in equities (Smetters, 1997). As can be seen in the option formula of the preceding section, the value of an option depends on the volatility of the underlying asset. The Federal Reserve uses as one of its indicators of investor uncertainty about the future course of interest rates, the “implied” volatility derived from option prices on government bonds.\textsuperscript{19} In his last paper, published after his death, Black (1995) applies options theory to model the process for the interest rates that govern the dynamics of government bond prices. In another area involving central-bank concerns, Perold (1995) shows how the introduction of various types of derivatives contracts has helped reduce potential systemic-risk problems in the payment system from settlement exposures. The Black-Scholes model can be used to value the “free credit option” implicitly offered to participants, in addition to “float,” in markets with other than instantaneous settlement periods. See also Kupiec and White (1996). The prospective application of derivative-security technology to enhance central-bank stabilization policies in both interest rates and currencies is discussed in Merton (1995, 1997b).

\textsuperscript{17}Hamlett and Baumel (1990).
\textsuperscript{19}Nasar (1992). See Bodie and Merton (1995) for an overview article on implied volatility as an example of the informational role of asset and option prices.
In an application involving government activities far removed from sophisticated and relatively efficient financial markets, options analysis has been used to provide new insights into optimal government planning policies in developing countries. A view held by some in development economics about the optimal educational policy for less-developed countries is that once the expected future needs for labor-force composition are determined, the optimal education policy should be to pursue targeted training of the specific skills forecast and in the quantities needed. The alternative of providing either more general education and training in multiple skills or training in skills not expected to be used is seen as a “luxury” that poorer, developing countries could not afford. It, of course, was understood, that forecasts of future labor-training needs were not precise. Nevertheless, the basic prescription formally treated them as if they were. In Merton (1992), the question is revisited, this time with an explicit recognition of the uncertainty about future labor requirements embedded in the model. The analysis shows that the value of having the option to change the skill mix and skill type of the labor force over a relatively short period of time can exceed the increased cost in terms of longer education periods or less-deep training in any one skill. The Black-Scholes model is used to quantify that tradeoff. In a different context of the private-sector in a developed country, the same technique could be used to assess the cost-benefit tradeoff for a company to pay a higher wage for a labor force with additional skills not expected to be used in return for the flexibility to employ those skills if the unexpected happens.

The discussion of labor education and training decisions and litigation and taxes leads naturally into the subject of human capital and household decision-making. The individual decision as to how much vocational education to acquire can be formulated as an option-valuation problem in which the optimal exercise conditions reflect when to stop training and start working.\textsuperscript{20} In the classic labor-leisure tradeoff, one whose job provides the flexibility to increase or decrease the number of hours worked, and hence his total compensation, on relatively short notice, has a valuable option relative to those whose available

\textsuperscript{20} Dothan and Williams (1981).
work hours are fixed.\textsuperscript{21} Wage and pension-plan “floors” that provide for a minimum compensation, and even tenure for university professors (McDonald, 1974), have an option-like structure. Other options commonly a part of household finance are: the commitment by an institution to provide a mortgage to the house buyer, if he chooses to get one; the pre-payment right, after he takes the mortgage, that gives the homeowner the right to renegotiate the interest rate paid to the lender if rates fall;\textsuperscript{22} a car lease which gives the customer the right, but not the obligation, to purchase the car at a pre-specified price at the end of the lease.\textsuperscript{23} Health-care insurance contains varying degrees of flexibility, a major one being whether the consumer agrees in advance to use only a pre-specified set of doctors and hospitals (“HMO plan”) or he retains the right to choose an “out-of-plan” doctor or hospital (“point-of-service” plan). In the consumer making the decision on which to take and the health insurer assessing the relative cost of providing the two plans, each solves an option-pricing problem as to the value of that flexibility.\textsuperscript{24} Much the same structure of valuation occurs in choosing between “pay-per-view” and “flat-fee” payment for cable-television services.

Many of the preceding option-pricing applications do not involve financial instruments. The family of such applications is called “real” options. The most developed area for real-option application is investment decisions by firms.\textsuperscript{25} However, real-options analysis has also been applied to real-estate investment and development decisions.\textsuperscript{26} The common element for using option-pricing here is the same as in the preceding examples: the future is uncertain (if it were not, there would be no need to create options because we know now what we will do later) and in an uncertain environment, having the flexibility to decide what

\textsuperscript{21} Bodie et al. (1992).
\textsuperscript{22} Dunn and McConnell (1981) and Brennan and Schwartz (1985b).
\textsuperscript{23} Miller (1995).
\textsuperscript{24} Hayes et al. (1993) and Magiera and McLean (1996).
\textsuperscript{26} Smith (1984), Chi et al. (1986), Geltner and Wheaton (1989), Williams (1991), and Zinkhan (1991).
to do after some of that uncertainty is resolved definitely has value. Option-pricing theory provides the means for assessing that value.

The major categories of options within project-investment valuations are: the option to initiate or expand, the option to abandon or contract, and the option to wait, slow-down, or speed-up development. There are “growth” options which involve creating excess capacity as an option to expand and research and development as creating the opportunity to produce new products and even new businesses, but not the obligation to do so if they are not economically viable.27

A few examples: For real-world application of the options technology in valuing product development in the pharmaceutical industry, see Nichols (1994). In the generation of electric power, the power plant can be constructed to use a single fuel such as oil or natural gas or it can be built to operate on either. The value of that option is the ability to use the least-cost, available fuel at each point in time and the cost of that optionality is manifest in both the higher cost of construction and less-efficient energy conversion than with the corresponding specialized equipment. A third example described in Luehrman (1992) comes from the entertainment industry and involves the decision about making a sequel to a movie: the choices are: either to produce both the original movie and its sequel at the same time, or wait and produce the sequel after the success or failure of the original is known. One does not have to be a movie-production expert to guess that the incremental cost of producing the sequel is going to be less if the first path is followed. While this is done, more typically the latter is chosen, especially with higher-budget films. The economic reason is that the second approach provides the option not to make the sequel (if, for example, the original is not a success). If the producer knew (almost certainly) that the sequel will be produced, then the option value of waiting for more information is small and the cost of doing the sequel separately is likely to exceed the benefit. Hence, once again, we see that the amount of uncertainty is critical to the decision, and the option-pricing model provides the means for quantifying the cost/benefit tradeoff. As a last example, Baldwin

27 Kester (1984), McLaughlin and Taggart (1992), and Faulkner (1996).
and Clark (1999) develop a model for designing complex production systems focused around the concept of modularity. They exemplify their central theme with several industrial examples which include computer and automobile production. Modularity in production provides options. In assessing the value of modularity for production, they employ an option-pricing type of methodology, where complexity in the production system is comparable to uncertainty in the financial one.\(^{28}\)

In each of these real-option examples as with a number of the other applications discussed in this section, the underlying “asset” is rarely traded in anything approximating a continuous market and its price is therefore not continuously observable either. For that reason, this paper, manifestly focused on applications, devotes so much space to the technical section on extending the Black-Scholes option-pricing framework to include non-tradability and non-observability.

### 2.4 Future Directions of Applications

As I suggested at the outset, innovation is a central force driving the financial system toward greater economic efficiency with considerable economic benefit having accrued from the changes since the time that the option-pricing papers were published. Indeed, much financial research and broad-based practitioner experience developed over that period have led to vast improvements in our understanding of how to apply the new financial technologies to manage risk. Moreover, we have seen how wide ranging are the applications of our technology for pricing and measuring the risk of derivatives. Nevertheless, there still remains an intense uneasiness among managers, regulators, politicians, the press, and the public over these new derivative-security activities and their perceived risks to financial institutions. And this seems to be the case even though the huge financial disruptions, such as the

\(^{28}\) See also He and Pindyck (1992). On an entirely different application, Kester’s (1984) analysis of whether to develop products in parallel or sequentially could be applied to the evaluation of alternative strategies for funding basic scientific research: is it better to support N different research approaches simultaneously or just to support one or two and then use the resulting outcomes to sequence future research approaches? See also Merton (1992, p. 426).
savings-and-loan debacle of the 1980s in the United States and the current financial crises in Asia and some emerging markets, appear to be the consequence of the more traditional risks taken by institutions such as commercial, real-estate, and less-developed-country lending, loan guarantees, and equity-share holdings.

One conjecture attributes this uneasiness to the frequently cited instances of individual costly events that are alleged to be associated with derivatives, such as the failure of Barings Bank, Procter and Gamble’s losses on complex interest-rate contracts, the financial distress of Orange County, and so forth. Perhaps. But, as already noted, derivatives are ubiquitous in the financial world and thus, they are likely to be present in any financial circumstance, whether or not their use has anything causal to do with the resulting financial outcomes. However, even if all these allegations were valid, the sheer fact that we are able to associate individual names with these occurrences instead of mere numbers (“XYZ company” instead of “475–500 thrifts” as the relevant descriptor) would suggest that these are relatively isolated events – unfortunate pathologies rather than indicators of systemic flaws. In contrast, the physiology of this financial technology, that is, how it works when it works as it should, is not the subject of daily reports from around the globe but is essentially taken for granted.

An alternative or supplementary conjecture about the sources of the collective anxiety over derivatives holds that they are a part of a wider implementation of financial innovations which have required major changes in the basic institutional hierarchy and in the infrastructure to support it. As a result, the knowledge base now required to manage and oversee financial institutions differs greatly from the traditional training and experience of many financial managers and government regulators. Experiential changes of this sort are threatening. It is difficult to deal with change that is exogenous to our traditional knowledge base and framework and thus comes to seem beyond our control. Decreased understanding of the new environment can create a sense of greater risk even when the objective level of risk in the

\[\text{Miller (1997) provides a cogent analysis refuting many of the specific-case allegations of derivatives misuse.}\]
system remains unchanged or is actually reduced. If so, we should start to deal with the problem now since the knowledge gap may widen if the current pace of financial innovation, as some anticipate, accelerates into the 21st century. Moreover, greater complexity of products and the need for more rapid decision-making will probably increase the reliance on models, which in turn implies a growing place for elements of mathematical and computational maturity in the knowledge base of managers. Dealing with this knowledge gap offers considerable challenge to private institutions and government as well as considerable opportunity to schools of management and engineering and to university departments of economics and mathematics.

There are two essentially different frames of reference for trying to analyze and understand changes in the financial system. One perspective takes as given the existing institutional structure of financial service providers, whether governmental or private-sector, and examines what can be done to make those institutions perform their particular financial services more efficiently and profitably. An alternative to this traditional institutional perspective – and the one I favor – is the functional perspective, which takes as given the economic functions served by the financial system and examines what is the best institutional structure to perform those functions.\(^{30}\) The basic functions of a financial system are essentially the same in all economies, which makes them far more stable, across time and across geopolitical borders, than the identity and structure of the institutions performing them. Thus, a functional perspective offers a more robust frame of reference than an institutional one, especially in a rapidly changing financial environment. It is difficult to use institutions as the conceptual “anchor” for analyzing the evolving financial system when the institutional structure is itself changing significantly, as has been the case for the past two decades and as appears likely to continue well into the future. In contrast, in the functional perspective, institutional change is endogenous, and may therefore prove especially useful in predicting the future direction of

\(^{30}\) For elaboration on the functional perspective, see Merton (1993, 1995), Crane et al. (1995), and Bodie and Merton (1998).
financial innovation, changes in financial markets and intermediaries, and regulatory design.\textsuperscript{31}

The successful private-sector and governmental financial service providers and overseers in the impending future will be those who can address the disruptive aspects of innovation in financial technology while still fully exploiting its efficiency benefits. What types of research and training will be needed to manage financial institutions? The view of the future here as elsewhere in the economic sphere is clouded with significant uncertainties. With this in mind, I nevertheless try my hand at a few thoughts on the direction of change for product and service demands by users of the financial system and the implications of those changes for applications of mathematical financial modeling.

The household sector of users in the more fully developed financial systems has experienced a secular trend of disaggregation in financial services. Some see this trend continuing with existing products such as mutual funds being transported into technologically less-developed systems. Perhaps so, especially in the more immediate future, with the widespread growth of relatively inexpensive Internet accessibility. However, deep and wide-ranging disaggregation has left households with the responsibility for making important and technically complex micro financial decisions involving risk (such as detailed asset allocation and estimates of the optimal level of life-cycle saving for retirement) – decisions that they had not had to make in the past, are not trained to make in the present, and are unlikely to execute efficiently even with attempts at education in the future. The low-cost availability of the Internet does not solve the “principal-agent” problem

\textsuperscript{31} During the last 25 years, finance theory has been a good predictor of future changes in finance practice. That is, when theory seems to suggest that something “should be there” and it isn’t, practice has evolved so that it is. The “pure” securities developed by Arrow (1953) that so clearly explain the theoretical function of financial instruments in risk bearing were nowhere to be found in the real world until the broad development of the options and derivative-security markets. It is now routine for financial engineers to disaggregate the cash flows of various securities into their elemental Arrow-security component parts and then to reaggregate them to create securities with new patterns of cash flows. For the relation between options and Arrow securities and the application of the Black-Scholes model to the synthesis and pricing of Arrow securities, see Ross (1976b), Banz and Miller (1978), Breeden and Litzenberger (1978), Duffie and Huang (1986), and Merton (1992, pp. 443–450).
with respect to financial advice dispensed by an agent. That is why I believe that the trend will shift toward more integrated financial products and services, which are easier to understand and more tailored toward individual profiles. Those products and services will include not only the traditional attempt to achieve an efficient risk-return tradeoff for the tangible-wealth portfolio but will also integrate human-capital considerations, hedging, and income and estate tax planning into the asset-allocation decisions. Beyond the advisory role, financial service providers will undertake a role of principal to create financial instruments that eliminate “short-fall” or “basis” risk for households with respect to targeted financial goals such as tuition for children’s higher education and desired consumption-smoothing throughout the lifecycle (e.g., preserving the household’s standard of living in retirement, cf. Modigliani, 1986). The creation of such customized financial instruments will be made economically feasible by the derivative-security pricing technology that permits the construction of custom products at “assembly-line”-levels of cost. Paradoxically, making the products more user-friendly and simpler to understand for customers will create considerably more complexity for the producers of those products. Hence, financial-engineering creativity and the technological and transactional bases to implement that creativity, reliably and cost-effectively, are likely to become a central competitive element in the industry. The resulting complexity will require more elaborate and highly quantitative risk-management systems within financial service firms and a parallel need for more sophisticated approaches to government oversight. Neither of these can be achieved without greater reliance on mathematical financial modeling, which in turn will be feasible only with continued improvements in the sophistication and accuracy of financial models.

Non-financial firms currently use derivative securities and other contractual agreements to hedge interest rate, currency, commodity, and even equity price risks. With improved lower-cost technology and learning-curve experience, this practice is likely to expand. Eventually, this alternative to equity capital as a cushion for risk could lead to a major change of corporate structures as more firms use hedging
to substitute for equity capital; thereby moving from publicly traded shares to closely-held private shares.

The preceding section provides examples of current applications of the options technology to corporate project evaluation: the evaluation of research-and-development projects in pharmaceuticals and the value of flexibility in the decision about sequel production in the movie industry. The big potential shift in the future, however, is from tactical applications of derivatives to strategic ones. For example, a hypothetical oil company with crude oil reserves and gasoline and heating-oil distribution but no refining capability could complete the vertical integration of the firm by using contractual agreements instead of physical acquisition of a refinery. Thus, by entering into contracts that call for the delivery of crude oil by the firm on one date in return for receiving a mix of refined petroleum products at a pre-specified later date, the firm in effect creates a synthetic refinery. Real-world strategic examples in natural gas and electricity are described in Harvard Business School case studies, “Enron Gas Services” (1994) and “Tennessee Valley Authority: Option Purchase Agreements” (1996), by Tufano. There is some evidence that these new financial technologies may even lead to a revisiting of the industrial-organization model for these industries.

It is no coincidence that the early strategic applications are in energy- and power-generation industries that need long-term planning horizons and have major fixed-cost components on a large scale with considerable uncertainty. Since energy and power generation are fundamental in every economy, this use for derivatives offers mainline applications in both developed and developing countries. Eventually, such use of derivatives may become standard tools for implementing strategic objectives.

A major requirement for the efficient broad-based application of these contracting technologies in both the household and non-financial-firm sectors will be to find effective organizational structures

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32 See Kester (1984), Myers (1984), and Bowman and Hurry (1993) on the application of option-pricing theory to the evaluation of strategic decisions.
for ensuring contract performance, which includes global clarification and revisions of the treatment of such contractual agreements in bankruptcy. The need for assurances on contract performance is likely to stimulate further development of the financial-guarantee business for financial institutions. Such institutions will have to improve the efficiency of collateral management further as assurance for performance. As we have seen, one early application of the option-pricing model focuses directly on the valuation and risk-exposure measurement of financial guarantees.

A consequence of all this prospective technological change will be the need for greater analytical understanding of valuation and risk management by users, producers, and regulators of derivative securities. Furthermore, improvements in efficiency from derivative products will not be effectively realized without concurrent changes in the financial “infrastructure” – the institutional interfaces between intermediaries and financial markets, regulatory practices, organization of trading, clearing, settlement, other back-office facilities, and management-information systems. To perform its functions as both user and overseer of the financial system, government will need to innovate and make use of derivative-security technology in the provision of risk-accounting standards, designing monetary and fiscal policies, implementing stabilization programs, and overseeing financial-system regulation.

In summary, in the distant past, applications of mathematical models had only limited and sidestream effects on finance practice. But in the last quarter century since the publication of the Black-Scholes option-pricing theory, such models have become mainstream to practitioners in financial institutions and markets around the world. The option-pricing model has played an active role in that transformation. It is safe to say that mathematical models will play an indispensable role in the functioning of the global financial system.

Even this brief discourse on the application to finance practice of mathematical models in general and the option-pricing model in particular would be negligently incomplete without a strong word of caution about their use. At times we can lose sight of the ultimate purpose of the models when their mathematics become too interesting.
The mathematics of financial models can be applied precisely, but the models are not at all precise in their application to the complex real world. Their accuracy as a useful approximation to that world varies significantly across time and place. The models should be applied in practice only tentatively, with careful assessment of their limitations in each application.

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Derivatives in a Dynamic Environment*

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3.1 Introduction

The trading of financial derivatives on organized exchanges has exploded since the early 1970s. The trading of off-exchange financial derivatives on the so-called “Over-the-Counter” or “OTC” market has exploded since the mid-1980s. Academic and applied research on financial derivatives, which was initiated by the Black-Scholes and Merton option-pricing research in the late 1960s and early 1970s, also has exploded. As a result, three industries have blossomed: an exchange industry in derivatives, an OTC industry in synthetic products, and an academic industry in derivative research, populated by scientists in and out of academic institutions. The academic industry has seen a growth of research and course offerings in economics and business schools in the

* © The Nobel foundation 1997 used with permission.
1 I would like to thank Robert C. Merton for many years of fruitful and exciting discussions on these topics. In addition, I would like to thank my many colleagues at the University of Chicago, MIT, and Stanford University, for their ideas and stimulating discussions. Most important has been the support and involvement of Merton H. Miller in my career: I owe him a tremendous debt and cherish his friendship. I miss Fischer Black; I miss his friendship, his insights, and his good humor.

417
areas of options, futures, risk management, financial engineering, and by marrying institutional and derivative modeling, a richer approach to financial intermediation and innovation under uncertainty. In addition, business schools and economics departments have competition from new research and courses in mathematics departments and engineering schools in the mathematics of derivative pricing and alternative stochastic processes, and in law schools in understanding contracting under uncertainty.

The academic industry has produced myriad innovative research papers following the fundamental insights of Black and Scholes (1973) and Merton (1973). The Chicago Board of Trade and Chicago Mercantile Exchange initiated the exchange industry by developing financial options and futures contracts on securities; they have spawned the growth of many new derivative exchanges around the world. The first was the Chicago Board of Trade’s sponsorship of the Chicago Board Options Exchange in 1973. Moreover, some exchanges such as the Options Market in Stockholm have transported the technology used to trade derivatives to other markets in Europe and Asia. These exchanges succeed by producing the derivative contracts that add value for individuals and institution around the world. The OTC industry, which has grown to prominence since the mid-1980s, first in the United States and now in every corner of the globe, is now larger in size than the exchange industry. Financial institutions in the OTC industry offer customized derivative products to meet the specific needs of each of their clients; the exchange industry offers standardized products to reach a richer cross-section of demand.

Each industry requires original research to understand the pricing and production costs of the products and financial services that they bring to their clients. Derivative research is quickly transferred to practice; moreover, practice stimulates both academic and applied research. Some of the best research is conducted outside of academic institutions and academics have found a home in each of the three industries. Graduates from mathematics, computer sciences, engineering and physics compete and cooperate with those trained in financial economics for research and structuring positions in both the exchange and OTC industry.
It is difficult to define financial derivatives in a dynamic environment. The purest among us might argue that any security is a derivative if its price dynamics depend on the dynamics of some other underlying asset or asset and time. This broad definition allows not only for what currently exists but also what new derivative instruments will be developed in the future with enhanced understanding and changing production costs. The popular press, however, tends to limit the definition to include financial options, futures and forward contracts either traded on an exchange or issued in the OTC industry. In the future they may come to be called financial products.

The will of Alfred Nobel states, in part, that the Nobel Prize shall be awarded for an “important discovery or invention.” Fischer Black’s and my discovery was how to price options and to provide a way to manage risk. Robert Merton developed an important alternative proof of the pricing technology and extended the approach in many directions including how to price options with dividends, how to price options when the interest rate is not constant, and how to apply a more general structure to price many other contingent contracts.

Black and Scholes have over the years been accused of inventing derivatives, at least those derivative products that have been claimed to have had bad economic consequences for their users. It is seldom remembered that these contracts have two sides: if a buyer loses, the ultimate seller might gain. Only if losses cause dead-weight costs is there a net loss to society. It is said that “every successful idea has a thousand fathers, and every failure is an orphan.” Over the years we have been granted both distinctions. We did not, however, invent derivatives. Options existed in many financial contracts prior to the Black-Scholes and Merton pricing technologies. Options were noted to have been traded on the Amsterdam Stock Exchange in the late 17th century and traded on the Chicago Board of Trade into the early 1930s. As described in Cootner (1964), research in option pricing goes back as far as Bachelier’s Ph.D. thesis in 1900. Although they are not generally thought of as options, myriad securities and investment decisions have been made in the past and are being made currently that could be evaluated using the Black-Scholes technology. The technology was an invention that facilitated multiple inventions in each of the
three industries. What I did not realize at the time of the invention was how the technology would evolve and how it would be used to produce new types of securities with imbedded options at lower cost than could be accomplished prior to the development of the technology. This enhanced the efficiency set of demanders and suppliers of capital, not only in the United States but also around the world.

With the Christmas season approaching at these Nobel award ceremonies in Stockholm, I will invoke the “Past,” the “Present” and the “Future,” as Charles Dickens did in “A Christmas Carol,” to describe the evolution of derivatives in a dynamic environment. I do not mean to draw too fine a parallel, however. That is, I am not implying that through the eyes of Mr. Scrooge the past for derivatives was bright with hope and innocence, and the present is dark and foreboding and the future, without changing our ways, presents a bleak picture. On the contrary, many in academics and those in practice have often asked for a glimpse of the past; that is, how we developed the technology and the model, and the past gives insights into the present. It was a time of innocence. It was a time of discovery. It is a tradition at these Nobel award ceremony lectures to describe the age of innocence. I am honored and thank the Nobel Prize Committee for this wonderful opportunity to do so. I wish that Fischer Black were alive today to share this honor with us.

Twenty-five years is a tender age for the new academic and the new exchange industry. Fifteen years is still a young age for the new OTC industry. The Present, which I date from the late 1980s to current time, shows an industry that has experienced growing pains, and many, including regulators, are worried that it still has not come of age. And, will the Future be bleak? No, there will be failures, but the industries will thrive because derivative instruments will provide progressively lower-cost solutions to investor and entity problems than will competing alternatives. These lower-cost solutions will involve the unbundling and repackaging of coarse financial products into their constituent parts to satisfy client demands. The process will continue to evolve as advances in information technology drive down the cost of providing alternative and more productive solutions.
3.2 The Past

Although Black (1989) and Bernstein (1992) described their version of the development of the option-pricing technology, this lecture, however, gives me an opportunity to add to their description through my recollection of things past. My formal training at the University of Chicago was in financial economics, statistics and economics. At Chicago, Merton Miller, the 1990 Nobel laureate, and Eugene Fama, a prolific scholar, stimulated my excitement in economics and a new branch of economics, which has come to be called financial economics. I also owe a similar and considerable debt to my fellow classmates at Chicago, most notably, Jack Gould, Michael Jensen and Richard Roll.

The three stands of financial economics that most set the tone for my future research were arbitrage and the notion of substitutes; the efficient markets hypothesis; and the capital asset pricing model. These strands gave a mathematical basis to the models of finance, in a general equilibrium framework. Modigliani and Miller (see Miller (1988) for a retrospective) were making profound breakthroughs that provided a general equilibrium model for corporate finance. Their arbitrage arguments, which demonstrated how a firm’s value was independent of how it financed its activities, had a profound effect on the way I analyze and model many problems in economics. The Fama (1965) and Samuelson (1965b) efficient-markets hypothesis that states that, in a well-functioning capital market, the dynamics of asset prices are described by a submartingale and that the best estimate of the value of a security is today’s price, was revolutionary to economics. Their insights gave me an important framework to think about the dynamics of asset prices and how markets adjust to “news.” It set the stage for empirical testing of how information was incorporated into security prices and gave me a vehicle to apply my statistics and computer skills in a financial economics context. For example, Jensen (1968) had used the concepts of the efficient-markets hypothesis to test whether mutual

\[2\] I am sure that Fischer Black would pick at least the efficient-markets hypothesis and the capital asset pricing model as most influential in the development of his thinking in financial economics.
funds, which were professionally managed and spent considerable sums of money to discover undervalued assets, could outperform randomly-selected investments on a risk-adjusted basis. Roll (1970) had tested the efficiency of the bond market controlling for changes in expected returns impounded in bond prices. In my own work (Scholes (1972)), I used the concepts of efficient markets and substitutes (arbitrage) to test the extent to which security prices were influenced by the size of the sales of large blocks of securities or by changes in the information set. Following on the work of Markowitz (1952, 1959), Sharpe (1964), and Lintner (1965) the capital asset pricing model provided a general equilibrium model of asset prices under uncertainty. This became the fundamental model for measuring the risk of a security. It was elegant to condense the required relative rates of return on securities into a simple reduced-form equation that depended only on their “betas,” a measure of their relative contribution to the risk of the optimal portfolio.

The capital structure models, the efficient-markets hypothesis, and the capital asset pricing model had the common central themes of arbitrage and market equilibrium: securities with similar economic risk had to exhibit similar returns to prevent arbitrage profits. This principle applies to all securities, whether they are common stock, bonds, or hybrid instruments. Through participation in seminar presentations at the University of Chicago, I became generally aware of the nature of warrants and convertible bonds. I was unaware, however, of the interest in warrant pricing and the research that had been conducted at MIT on this topic in the 1960s even though I became an Assistant Professor at the Sloan School of Management at MIT in 1968.

I did not meet Robert Merton until the spring of 1969, when he was interviewing for a position at the Sloan School. We began interacting in the fall of 1969. We talked then about his current research on dynamic applications of the capital asset pricing model with changing opportunity sets and my work, at the time, including tests of the capital asset pricing model. We did not talk about warrant pricing even though Fischer Black and I were working on the problem. I guess we did not appreciate that each of us had an interest in this research area.

In the summer of 1968, a research project at Wells Fargo Bank in San Francisco convinced me that the passive management of assets
could be a viable contender to actively managed portfolios. In my report to Wells Fargo Bank, I recommended that they initiate passive investment strategies and offer them to their clients, the forerunner to so-called “index funds.” That fall, on the suggestion of Michael Jensen, I had lunch with Fischer Black, who he had met because of Jensen’s research on mutual funds. Fischer was employed at Arthur D. Little, a consulting firm in Cambridge, Mass. We had several other lunches that fall and Fischer suggested that he was thinking of leaving Arthur D. Little to start his own consulting firm. At about the same time, John McQuown at Wells Fargo Bank asked whether I wanted to conduct research that would describe the tradeoff of risk and return in the market as a forerunner to introducing passive investment strategies to their clients. Without research, they were not willing to offer index-fund-like products to clients. Being an Assistant Professor, I was restricted as to the number of days I could consult. I asked Fischer whether he wanted to join forces on the project. It was obvious from our lunch discussions that he had very similar ideas, and was starting a research project with Jensen on measuring risk and return. We joined forces and worked together to test the capital asset pricing model (see Black et al. (1972)). We developed the concept of the zerobeta portfolio to test the model. If we could create a zero beta-minimum variance portfolio by buying low beta stocks and selling high beta stocks and achieve realized returns on this portfolio that were significantly different from the interest rate this would violate the predictions of the original capital asset pricing model.

In the winter of 1969, I agreed to direct the Masters thesis of an MIT graduate student who had garnered a time series of warrant and underlying stock prices and wanted to apply the capital asset pricing model to value the warrants. I read all of the articles relating to warrant pricing in Paul Cootner’s book of readings on The Random Character of Stock Prices (1964). One included paper, by Case Sprenkle and dated 1960, seemed the most relevant to me, but Sprenkle used an exogenously determined discount rate to discount the expected terminal value of the warrant to its present value.

What seemed apparent was that the expected return of the warrant could not be constant for each time period because the risk of the
warrant changed with changes in the stock price and with changes in
time to maturity. For example, if the warrant was far “in-the-money,”
that is, the underlying stock price was far above the exercise price, and
the warrant was almost sure to be exercised, its price would change
almost dollar for dollar with a change in the underlying stock price.
The percentage change in the value of the warrant, however, would be
greater than the percentage change in the value of the common stock
because the warrant was a leveraged instrument. On the other hand, if
the warrant was “out-of-the-money,” that is, the underlying stock price
was less than the exercise price, the warrant price would move far less
than dollar for dollar with price of the common stock (for example, $.5
for $1 move in the common). The percentage change in the price of the
warrant, however, would be even greater than that of the in-the-money
warrant.

As a result, the expected return on the warrant could not be con-
stant each period if the beta of the stock was constant each period.
I thought about using the capital asset pricing model to establish a
zero-beta portfolio of common stock and warrants by selling enough
shares of common stock per each warrant held each period to create
a zero-beta portfolio. Given I could create a zero-beta portfolio, the
expected return on the net investment in this portfolio would be equal
to the riskless rate of interest. I knew that I would have to change the
number of shares of stock each period to retain my zero-beta portfolio.
But, after working on this concept, off and on, I still couldn’t figure out
analytically how many shares of stock to sell short to create a zero-beta
portfolio.

Fischer and I continued working on tests of the capital asset pricing
model and the development of investment products based on the impli-
cations of our research throughout 1969. In the summer or early fall
of 1969, I discussed with Fischer my earlier experience with warrants,
my attempt at creating the zero-beta portfolio, and my inability to
determine the changing number of shares needed each period to create
the zero-beta portfolio. He described to me his research on warrants
and that he was frustrated in his inability to progress further than he
had to that time. He showed me a sheet of paper, which described the
relation between the return on the warrant and the underlying stock.
Following on earlier work by Jack Treynor, Fischer had used a Taylor Series expansion of \( w(x,t) \), where “\( w \)” is the warrant price, “\( x \)” is the current stock price and “\( t \)” is time to maturity to show the relation between the change in the warrant price as a function of the change in the price of the common stock and a decrease in the time to maturity of the option. Ignoring terms of second order with regard to time, over a short period of time, this expansion was:

\[
\Delta w(x,t) = w_1 \Delta x + w_2 \Delta t + 1/2w_{11}\Delta x^2 \Delta t
\]

where \( \Delta \) is the change symbol, and the subscripts refer to partial derivatives with respect to the first or second arguments.

Not surprisingly, Fischer had used the capital asset pricing model to describe the relation between the expected return on the warrant and the market and the expected return on the common stock and the market. By substituting for the change in the warrant price as a function of changes in the stock price and time in the warrant asset pricing relation, it became obvious on how to create a zero-beta portfolio that would have an expected rate of return equal to the interest rate (for we assumed a constant interest rate).

Consider the returns over a very short period of time on two alternative investment strategies: under (1), we acquire the warrant, and enough bonds earning at interest, \( r \), per period, such that our investment in strategy (1) is the same as in alternative strategy (2), in which we buy \( w_1 \) of stock. The following is the investment and the return on these two alternative strategies:

<table>
<thead>
<tr>
<th>Investment</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Buy warrant</td>
<td>( w_1 \Delta x + w_2 \Delta t + 1/2w_{11}\Delta x^2 \Delta t )</td>
</tr>
<tr>
<td>Bonds</td>
<td>( r\Delta t(w_1x - w) )</td>
</tr>
<tr>
<td>(2) Buy stock</td>
<td>( w_1x )</td>
</tr>
<tr>
<td></td>
<td>( w_1\Delta x )</td>
</tr>
</tbody>
</table>

The investment was constructed to be the same in strategy (1) and strategy (2). The risk appears to be the same in both strategies. The only uncertain term is \( \Delta x \), the change in the stock price, and the total uncertainty due to changes in the stock price is the same in both strategies. \( \Delta x^2 \) involves the change in the stock price squared, a form of
Derivatives in a Dynamic Environment

variance, which as $\Delta t$ becomes small approaches $x^2 \sigma^2$, the stock price squared times the instantaneous variance of the underlying returns on the common stock, which is assumed to be constant.

Since the risk is the same and the investment is the same under both strategies, to prevent arbitrage, the returns must be the same over a short period of time. After equating the returns on strategy (1) with the returns on strategy (2), and substituting for $\Delta x^2$, we find:

$$-rw + w_1 xr + w_2 + 1/2 w_{11} x^2 \sigma^2 = 0$$

This is the Black-Scholes differential equation. The initial condition for a warrant or call option is that $w(x,t^*) = \text{Max}(x - c, 0)$, where $t^*$ is the maturity date of the option, and $c$ is the exercise price of the option. The only required inputs to value the option, other than its initial conditions, are $r$, the interest rate, and $\sigma^2$, the variance rate per unit time on the returns on the underlying stock. We were both amazed that the expected rate of return on the underlying stock did not appear in the differential equation.

Although the number of shares needed to create a zero-beta portfolio each period was $w_1$ it was not obvious to us how to find $w_1$. The next step in solving the problem was to realize that since the warrant valuation depended only on the variability of returns and not the expected return on the underlying common stock, it was arbitrary what expected return was assumed for the underlying common stock. The same warrant valuation equation would result because we had hedged out the risk of the common stock in establishing the zero-beta portfolio or the alternative replicating portfolio, as above. We assumed that the expected return on the common stock was equal to the interest rate over the next short period of time, or in terms of the capital-asset pricing model that the common stock had a zero beta.

With the assumption of constant return and variance of return, the distribution of returns on the underlying stock at expiration of the warrant would be lognormally distributed. We used the Sprenkle formulation to find the terminal value of the warrant using a constant interest rate as the expected rate of return on the stock. But, we wanted the present value of the warrant. The key here was to realize that although the warrant would have greater price variability than the underlying
3.2. The Past 427

If we assume that the stock had a zero beta, the warrant would have a zero beta. If the warrant had a zero beta each period of time, the warrant had also to return the interest rate, \( r \), each period of time.

If we had decided to value the warrant using the actual expected return on the common stock or, for that matter, any other appreciation rate, the discount rate to value the warrant would depend on time and changes in the stock price. It does not for the zero-beta case. Using Sprenkle’s formula with the assumption that the expected return on common stock and the discount rate for the warrant was equal to a constant interest rate, we obtained the Black-Scholes option-pricing formula.

\[
w(x,t) = xN(d_1) - c e^{r(t-t^*)}N(d_2)
\]

where \( N(d) \) is the cumulative normal density function, \( c \) is the exercise price, \( t^* \) is the expiration date, and \( t \) is the current date, \( t^* - t \) is the remaining number of periods in the life of the option, and “e” is the exponential operator. Lastly,

\[
d_1 = \frac{\ln x/c + (r + 1/2\sigma^2)(t^* - t))}{\sigma \sqrt{t^* - t}}
\]

\[
d_2 = d_1 - \sigma \sqrt{t^* - t}
\]

We checked the formula against the differential equation and, as we expected, it fit. We were sure that we had the correct formula for valuing call options or warrants, the right to buy an asset at a fixed price, the exercise price, at maturity of the right, its expiration date. With minor adjustments we could value a put option, the right to sell an asset under similar terms.

From the formula, we were finally able to compute \( w_1 \), which was equal to \( N(d_1) \), the required number of shares to hedge the option. The number of shares will change over time and as the price of the underlying security changes with respect to the exercise. But, given the assumptions it is a known quantity each time period.

The formulation also suggests the technology necessary to price any contingent claim that depends on an underlying asset’s price (or even other state variables) and time. This is so even with differential known pay-outs, such as dividends on a stock or coupons on a bond, that are
Derivatives in a Dynamic Environment

not received by the option holder. The technology suggests that what is necessary is to hedge the stochastic term, \( \Delta x \), to create an alternative investment that is riskless. Merton (1973) formalized these relations.

We had spent a considerable amount of time working to finish up several other papers including our paper on testing the capital asset pricing model. As a result, we finished a draft of the paper sometime in the winter of 1970. We did not know whether our formulation was exact, but intuitively we thought investors could diversify away any residual risk that was left. For larger price changes in the common stock, the hedge position of being long the option and short the appropriate number of shares of stock would tend to make money whether the market went up or down, and would lose money on small changes in the market portfolio. The risk of the position appeared to be independent of market risk. We programmed the model and tried to understand the sensitivity of the price of the option to changes in the stock price, time to maturity, volatility, and the interest rate.

We presented a draft of the paper at a conference on capital market theory sponsored by Wells Fargo Bank in July of 1970. Later that summer, on vacation together, Fischer and I worked out the applications of the option-pricing framework to the pricing of risky debt and other capital structure issues. We viewed the common stock of a company with debt in its capital structure as a call option. The equity holders have an option to buy back the firm from the debt holders by paying off the face amount of the debt at its maturity. The equity holders will not buy back the firm from the debt holders at the maturity of the debt if the face amount of the debt is less than the value of the firm’s assets; they will turn the remaining assets of the firm over to the debt holders. For us, it was exciting to realize that the equity of a corporation was an option, and that our framework applied far more broadly than the valuation of warrants or put and call options. The methodology provided a systematic approach that relied on arbitrage to value the capital structures of firms, and to understand how management decisions affect the relative values of debt and equity in the firm’s capital structure.

As it turned out, Robert Merton, who had written an earlier paper on the valuation of warrants with Paul Samuelson (see Samuelson
and Merton, 1969) following on Samuelson's (Samuelson (1965a)) early work on warrant valuation, had expected to attend our session at the Wells Fargo conference, but he overslept and missed the session. In the winter and spring of 1970, Fischer and I searched the academic literature to determine how close others were to our invention. Fischer and I realized that the Samuelson and Merton paper contained an equilibrium model to value warrants but they did not value the warrant continuously. Given that there was friendly rivalry between the two teams, Fischer and I wanted to progress, on our own, as far as we could prior to the conference.

After the conference, in the early fall, Robert Merton and I discussed the Black-Scholes valuation methodology and option-valuation formula. He was not convinced that using the capital asset pricing model framework was sufficient. It seemed to him that, as the interval of adjustment of the hedge became closer to continuous time, there might still be covariance between the hedged position and the market return. Merton and I speculated that one way that there would be no such covariance would be if the return on the option was perfectly correlated with the return on the stock in continuous time, and therefore the hedge was exact. Merton later proved that the hedged position in continuous time was riskless, and that the replicating portfolio argument was exact. Fischer and I used this derivation in the final version of the paper because it relied on arbitrage and not on any underlying model of capital market equilibrium. However, we still presented the capital asset pricing model derivation of the model, as it provided us with the many insights we used to unlock the puzzles of option pricing.

Merton (1973) then started working on his paper on various aspects of the option formula. He incorporated his alternative proof of the option-pricing model. He also showed that the right to exercise a call option prior to maturity is not valuable for a non-dividend paying stock, but valuable for a put option on a similar stock. He also showed how to incorporate changes in interest rates into the valuation methodology, and generalized the formula to handle other state variables.

To our dismay, when we submitted a version of our paper entitled “A Theoretical Valuation Formula for Options, Warrants, and Other Securities” (October, 1970) to the Journal of Political Economy, it was
rejected without review. The *Review of Economics and Statistics* also rejected the paper. Fischer felt that the paper was rejected because he was not an academic; I felt that I was an unknown Assistant Professor and the paper would not be considered to be broad enough for those academic journals. With the help of Merton Miller and Eugene Fama, who took an interest in the paper and stepped in on our behalf, the *Journal of Political Economy* agreed to consider the paper if we revised it and broadened its applicability. We had planned to publish the corporate-finance-capital structure applications in a subsequent paper but we broadened the original paper and showed how corporate liabilities could be viewed as options. The final version of the paper was published in 1973 in the *Journal of Political Economy* under the title “The Pricing of Options and Corporate Liabilities.”

### 3.2.1 The Aftermath

In Black and Scholes (1972), we tested the option pricing model using data recorded in a transactions diary of a broker in the over-the-counter options market provided to us by another Master’s student at MIT. The diary listed the prices at which he sold options to his clients. Using simple estimates of the volatility, the model generally performed well. It produced profits if one could buy options at diary-market prices if the model indicated a higher value than the market and sell options at diary-market prices if the model indicated a lower value than the market. Each trading day, we assumed that the model could be used to determine the hedge ratio, the delta, and undertook these hedges (assuming that transaction costs were zero). We regressed the daily returns on our hedged portfolio on the market returns. As expected, the portfolio returns were uncorrelated with the market returns, but the model produced substantial and significant abnormal profits. The market appeared to ignore information available in the historical data on estimating volatility.

When we assumed that we could buy the undervalued and sell the overvalued options at model prices and hedge out the underlying stock risk, we incurred significant losses. These losses were incurred because using simple estimates of the volatility ignored information on future
volatility that the market was using to price the options. When actual realized volatility over the life of the option was used to compute the model prices, buying undervalued and selling overvalued options at model prices generated returns that were insignificantly different from zero.

It became apparent to us that the transaction costs of dealing in the over-the-counter market were quite large. The dealers would only sell options. As a result, the market in put and call options was quite small. The world was about to change. In the early 1970s, various studies were commissioned to provide an economic justification for a new options exchange. As a result, The Chicago Board Options Exchange (CBOE) was born in 1973 almost simultaneously with the publication of the Black-Scholes and Merton option-pricing papers. The reduction in transaction costs and the transparency of the market were justifications enough for the subsequent success of the options market.

It is ironic that these empirical tests of the Black-Scholes model were published in the *Journal of Finance* in the proceedings volume of the *American Finance Association Meetings*, in May of 1972, a full year prior to the publication of the model itself. Although we did not present it as such, it is ironic that the methodology in the paper is generally the same as that used today by financial entities to manage the risks of their trading positions and to measure the performance of their traders.

### 3.2.2 Historical Notes: From Theory to Practice

Both the derivative exchange industry and the derivative academic industry grew significantly from 1973 to 1985. Financial economists started to interact with a broad set of practitioners and this led to a cross-fertilization of ideas among the participants in both industries.

The option-pricing technology was adopted simply because it reduced transaction costs. For without a model, traders could neither price securities with imbedded options with sufficient accuracy to compete against other traders with models, nor could they reduce the risk of their positions to employ their capital efficiently at a low enough cost to compete with other traders. Although it is hard to prove, I do
think that the success of the CBOE and other exchanges, in part, can be attributable to option-pricing models. As traders became familiar with these models, bid-offer spreads narrowed. As traders became more familiar with risk-management techniques they could take on larger position sizes to support the market. With a deeper and more efficient market, investors began to use options to facilitate their own investment strategies.

In those formative years, notable extensions and additions to the basic framework include important contributions by Black (1975, 1976), Banz and Miller (1978), Breeden and Litzenbeger (1978), Brennan and Schwartz (1979), Cox and Ross (1976), Cox et al. (1979), Geske (1979), Harrison and Kreps (1979), Magrabe (1978), Merton (see Merton, 1992a), Parkinson (1977), Richard (1978), Ross (1976), Rubinstein (1976), Scholes (1976), Sharpe (1978), and Vasicek (1977).

In 1971, Fischer Black left Boston to become a Professor at the University of Chicago. In 1972, Robert Merton and I became consultants to Donaldson, Lufkin and Jenrette (DLJ) to build mathematical models to price so-called “Down-and-Out Options” and to build their options technology to price call options in the event of a launch of the CBOE. Leo Pomerance, the head of the DLJ options group, was an options trader from the old school; he traded OTC options using intuition and experience without regard to a formal model. He later became the first chairman of the CBOE. At DLJ we forged a marriage of the old-time trader types, with their mental set, with young mathematical modeling types, with their model assumptions, to add value for the firm.

The spread of the option-valuation technology was rapid once the CBOE launched its first contracts on listed securities. Initially, many of the older-market-wise traders rejected using a model to price options. And, initially prices were not in line with prices predicted from using the Black-Scholes model adjusting for dividends and using relatively simple estimates of volatility to price the options. This left an opportunity for younger model-based traders to step in and profit from price discrepancies in the market. They used the model to price options and to determine the appropriate hedge ratios to reduce the risk of their positions. Generally, retaining these risks had zero present value
because traders had little expertise in forecasting stock returns. By so doing they could undertake larger positions and enhance their profits by concentrating on risks that could add to their profits.

In Galai (1975), Dan Galai, one of my Ph.D. students as the University of Chicago, where I had returned in 1973, tested the pricing of options on the CBOE in the first year of its existence using the model with simple historical estimates of the volatility. He found that the profits on trading options; that is, buying undervalued contracts and selling overvalued contracts each day to maintain a neutral risk position, were even greater than those found in our original tests. His strategies could achieve greater profits by reducing positions if the prices of options return to model values prior to the expiration of the contract. Transaction costs could have reduced actual trading profits for other than the option dealers.

By the end of the first year of trading options, it was no longer possible to use simple estimates of the historical volatility to spot opportunities in the market. Many of the clearing firms, who financed the positions of the option traders, used the model and the hedge ratios to determine the net risk of each of these traders. Fischer had started a service to provide option prices and the share-equivalent positions (hedge ratios) on each of the options traded on the various exchanges. He used a more sophisticated estimate of volatility to price the options. He combined historical estimates adjusted for changes in stock-market levels, with the volatility implied by the prices of options. As is true even to this day, as the market-price level of securities increases relative to a previous level over a relatively short period of time, the volatility of stocks tends to fall. This result is due, in part, to a reduction in the leverage of the underlying equities. Given the rate of interest, the price of the common stock, the dividend yields, the exercise price, and the maturity date of the option, the model can be used to infer the implied volatility that the market is using to price the option. In fact, even today, options are described in terms of implied volatility. Traders are asked whether they want to buy or sell volatility. Although his volatility estimates held some cache for a while, advanced computer technology made Fischer’s pricing sheets obsolete after a few years.
In fact, Texas Instruments marketed a hand-held calculator in 1977 that gave the Black-Scholes model values and hedge ratios. When I asked them for royalties, they replied that our work was in the public domain; when I asked, at least, for a calculator, they suggested that I buy one. I never did. Robert Merton and I continued to consult for DLJ. Working with Mathew Gladstein, we decided that the time might be appropriate to provide investors with a fund that protected their downside risk but allowed for some upside participation in the performance of the stock market. To achieve this goal, we decided that the assets of the fund would be held in two parts: in any six-month period, 90% of the assets would be held in U.S. Treasury bills and 10% of the assets would be used to buy a diversified portfolio of call options. In several papers, Merton et al. (1978, 1982) simulated the performance characteristics of such a strategy using the underlying stocks of the options that were traded in the various markets. The return characteristics were as predicted by the theory: losses were truncated and gains were less than a direct investment in the underlying stocks. The returns on the strategy were non-linearly related to the market. We always stressed the role of options as insurance. In early 1976, we attempted to launch Money Market/Options Investments under the auspices of Phoenix Investment Counsel of Boston. Unfortunately, the fund raised only a small amount of money. Our simulation results indicated that fully-covered investment strategies; that is, a sale of an option on a position in the underlying stock (for example, long 100 shares of IBM and short a call option to buy 100 shares of IBM), would provide returns of only the premium received on the option approximately 60% of the time. Call-option holders would call their stock away if the stock price were above the exercise price on expiration of the contract and would not exercise call options if the stock price were below the exercise price at expiration. This strategy produces higher current income but with the possibility of capital losses just like a high-yield bond. The expected return, however, was less than the expected return on the money market-options strategy. At about this time other investment companies marketed the fully-covered strategy, which we thought exhibited inferior return characteristics for most investors, and naturally, to our dismay, were quite
successful because of the promises of higher income (but at the unadvertised expense of expected capital losses).

3.3 The Present

The past twenty years has seen a transformation of the entire financial services industry, first in the United States, and now around the world. During the 1970s and 1980s, regulations divided the activities of financial institutions into separate market segments. In the U.S., commercial banks handled deposits and made commercial loans; investment banks were involved in mergers, acquisitions and underwriting; brokerage companies sold stocks and bonds; savings and loans, along with banks, initiated and held mortgages; and, insurance companies sold life, and property and casualty insurance products. Many of the regulations were directed at preserving the profitability of these institutions by restricting competition, mainly at the expense of the users of these services. Each institution had a product focus; for example deposits, or life insurance, or commercial loans. No financial company served a broad range of its clients’ financial needs.

As happens at times, it is not possible for regulators to protect the profitability of the industries they regulate. In the U.S., mutual funds competed with banks in providing deposit services after banks were not permitted to pay market interest rates on deposits in the early 1970s. The growth of institutional investors managing pension funds and mutual funds forced the abolition of fixed commission rates to trade securities in the U.S. and around the world. The larger brokerage firms evolved to compete against the banks and the savings and loan associations in packaging and repackaging mortgages to broaden the extent of the market. Banks started to compete with brokerage, investment banks, and insurance companies in financing commercial real estate and financing highly-leveraged mergers and acquisitions, so-called “Leveraged Buyouts.” It is nearly impossible to maintain regulations that restrict activities in one industry when new competitors not subject to costly regulations are attacking the profitable businesses of that industry.
The driving force behind today’s tidal wave of financial innovation has been the reduction in the cost of computer and communications technology. This lower-cost technology has led to a globalization of the product and financial markets. Corporate and institutional needs have become more complex. Investors are demanding more services. Technology brings new competitors to the market who can offer similar and expanded services at lower cost than existing competitors can. The growth of lower cost providers of brokerage services such as Schwab, Fidelity, and Internet brokers are examples.

Financial service firms today must decide which clients to serve, determine those clients’ needs and then decide which products and services add value for their clients as well as their own shareholders. Firms that were quite similar fifteen years ago have become very different today. For instance, J.P. Morgan, a wholesale bank, today differs far more from Citicorp, a retail bank than it did in the early 1980s. Conversely, firms that were quite different from each other fifteen years ago have become quite similar today. It is hard to distinguish the activities of today’s J.P. Morgan from UBS, from Goldman Sachs or Merrill Lynch. Meanwhile, A.I.G. and Travelers, both insurance companies, these days offer a number of services that are similar to those offered by Goldman Sachs or UBS. Investment banks no longer merely structure and advise in transactions but instead have moved to a more packaged, integrated, convenient financial-solutions approach, directed at solving the complex problems of their clients around the world.

The many advances in financial theory have enabled financial services firms to meet those complex needs more effectively and at lower cost than was possible previously. The marriage of business school and economics department graduates with engineers, mathematicians, physicists and computer scientists has led to more efficient and lower-cost financial engineering solutions to client problems.

To date, the major growth in the use of derivatives has been fueled by trends toward securitization and the increased understanding of the role that derivatives can play in the unbundling, packaging, and transferring of risks. No longer do financial service firms only sell the same products they buy from clients. Instead, they break the products down into their component parts and either sell the parts or recombine
them into new and hybrid custom-tailored financial instruments. And, this unbundling and repackaging is only in the beginning stages of evolution.

With information asymmetries between clients and their financial service providers, it would be prohibitively expensive for a client to develop close relations with many financial service firms. To be productive, the financial service firm must learn the needs of its clients and understand their businesses. As a result, it can be expert to only a select client list. Clients find it inefficient to “shop” widely for new financial service firms. Creating custom-tailored solutions strengthens relations between the financial services firm and its clients. It would be too costly for each client to replicate the specialized expertise required to engineer financial theory solutions; such talent would be underutilized most of the time. It would be analogous to every corporation maintaining an entire full-service law firm on its premises. Notwithstanding many regulators’ fears, it is not likely that all financial service firms will disappear if left to compete in the global arena. Product standardization will erode profits more quickly than in the past because more diverse entities, such as General Electric or Enron or accounting firms can compete in providing financial services using financial technology. New competition will enter various markets from global competitors. Although inefficient financial service firms will disappear more quickly than in the past, their clients will obtain more value-enhancing and less costly services from the remaining financial service firms. Financial products are becoming so specialized that, for the most part, it would be prohibitively expensive to trade them in organized markets.

Financial service firms have become the leaders in using derivatives in their risk-management programs. Using information and option-pricing technology, financial services firms can not only value their commitments, such as guarantees and other derivative contracts, but also are moving to understand the sensitivities of their holdings to various market factors. They can decide what risks to transfer and what risks to retain.

Tables 3.1 and 3.2 show the growth of derivative contracts trading since 1986 in both the exchange industry and in the OTC industry. The cells of the tables contain the “notional principal amounts outstanding”
Table 3.1 Markets for Selected Derivative Financial Instruments: Notional Principal Amounts Outstanding: 1986–96 (In billions of U.S. dollars)

<table>
<thead>
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<tbody>
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<td>Interest rate futures</td>
<td>370.0</td>
<td>487.7</td>
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<td>2,913.0</td>
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<td>39.2</td>
<td>59.5</td>
<td>48.0</td>
<td>50.2</td>
<td>56.5</td>
<td>62.9</td>
<td>71.1</td>
<td>75.6</td>
<td>55.6</td>
<td>43.2</td>
<td>46.5</td>
</tr>
<tr>
<td>Stock market index futures</td>
<td>14.5</td>
<td>17.8</td>
<td>27.1</td>
<td>41.3</td>
<td>69.1</td>
<td>76.0</td>
<td>79.8</td>
<td>110.0</td>
<td>127.3</td>
<td>172.2</td>
<td>198.6</td>
</tr>
<tr>
<td>Stock market index options¹</td>
<td>37.8</td>
<td>27.7</td>
<td>42.9</td>
<td>70.7</td>
<td>93.7</td>
<td>132.8</td>
<td>158.6</td>
<td>229.7</td>
<td>238.3</td>
<td>329.3</td>
<td>380.2</td>
</tr>
<tr>
<td>Total</td>
<td>618.3</td>
<td>729.9</td>
<td>1,304.8</td>
<td>1,766.9</td>
<td>2,290.4</td>
<td>3,519.3</td>
<td>4,634.4</td>
<td>7,771.1</td>
<td>8,862.5</td>
<td>9,188.2</td>
<td>9,884.6</td>
</tr>
<tr>
<td>North America</td>
<td>518.1</td>
<td>578.1</td>
<td>951.7</td>
<td>1,155.8</td>
<td>1,268.5</td>
<td>2,151.7</td>
<td>2,694.7</td>
<td>4,358.6</td>
<td>4,819.5</td>
<td>4,849.6</td>
<td>4,839.7</td>
</tr>
<tr>
<td>Europe</td>
<td>13.1</td>
<td>13.3</td>
<td>177.7</td>
<td>251.0</td>
<td>461.2</td>
<td>710.1</td>
<td>1,114.3</td>
<td>1,777.9</td>
<td>1,831.7</td>
<td>2,241.6</td>
<td>2,831.7</td>
</tr>
<tr>
<td>Asia-Pacific</td>
<td>87.0</td>
<td>138.5</td>
<td>175.4</td>
<td>360.0</td>
<td>560.5</td>
<td>657.0</td>
<td>823.5</td>
<td>1,606.0</td>
<td>2,171.8</td>
<td>1,990.1</td>
<td>2,154.0</td>
</tr>
<tr>
<td>Other</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
<td>1.8</td>
<td>28.7</td>
<td>39.5</td>
<td>106.8</td>
<td>59.3</td>
</tr>
</tbody>
</table>

¹ Calls plus puts.
Table 3.2 Notional Principal Value of Outstanding Interest Rate and Currency Swaps of the Members of the International Swaps and Derivatives Association, 1987–June 1996 (In billions of U.S. dollars)

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Interest rate swaps</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All counterparties</td>
<td>682.9</td>
<td>1,101.2</td>
<td>1,502.6</td>
<td>2,311.5</td>
<td>3,065.1</td>
<td>3,850.8</td>
<td>6,177.3</td>
<td>8,815.6</td>
<td>12,810.7</td>
<td>15,584.2</td>
</tr>
<tr>
<td>Interbank (ISDA member)</td>
<td>206.6</td>
<td>341.3</td>
<td>547.1</td>
<td>909.5</td>
<td>1,342.3</td>
<td>1,880.8</td>
<td>2,967.9</td>
<td>4,533.9</td>
<td>7,100.6</td>
<td>—</td>
</tr>
<tr>
<td>Financial Institutions</td>
<td>300.0</td>
<td>421.3</td>
<td>579.2</td>
<td>817.1</td>
<td>985.7</td>
<td>1,061.1</td>
<td>1,715.7</td>
<td>2,144.4</td>
<td>3,435.0</td>
<td>—</td>
</tr>
<tr>
<td>Governments$^1$</td>
<td>47.6</td>
<td>63.2</td>
<td>76.2</td>
<td>136.9</td>
<td>165.5</td>
<td>242.8</td>
<td>327.1</td>
<td>307.6</td>
<td>500.9</td>
<td>—</td>
</tr>
<tr>
<td>Corporations$^2$</td>
<td>128.6</td>
<td>168.9</td>
<td>295.2</td>
<td>447.9</td>
<td>571.7</td>
<td>666.2</td>
<td>1,166.6</td>
<td>1,829.8</td>
<td>1,774.2</td>
<td>—</td>
</tr>
<tr>
<td><strong>Currency swaps</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>All counterparties</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(adjusted for reporting of both sides)</td>
<td>182.8</td>
<td>319.6</td>
<td>449.1</td>
<td>577.5</td>
<td>807.2</td>
<td>860.4</td>
<td>899.6</td>
<td>914.8</td>
<td>1,197.4</td>
<td>1,294.7</td>
</tr>
<tr>
<td>Interest rate options$^3$</td>
<td>0.0</td>
<td>327.3</td>
<td>537.3</td>
<td>561.3</td>
<td>577.2</td>
<td>634.5</td>
<td>1,397.6</td>
<td>1,572.8</td>
<td>3,704.5</td>
<td>4,190.1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>865.6</td>
<td>1,657.1</td>
<td>2,459.0</td>
<td>3,450.3</td>
<td>4,449.5</td>
<td>5,345.7</td>
<td>8,474.5</td>
<td>11,303.2</td>
<td>17,712.6</td>
<td>21,068.9</td>
</tr>
</tbody>
</table>

1 Including international institutions.
2 Including others.
3 Including caps, collars, floors, and swaptions.
for various categories of derivative contracts. For example, the face amount of stock market index options (including call and put options) at the end of 1986 stood at $37.8 billion and by the end of 1996 grew to $380.2 billion.\(^3\)

These tables indicate that the OTC market in derivatives has grown much faster than the exchange market in the last 10 years. In 1995, turnover on the major derivative exchanges around the world actually declined while OTC activity rose by 40 percent. The Bank for International Settlements estimated that outstanding OTC contracts exceeded $47.5 trillion in early 1995, much greater than the numbers reported in Table 3.2. To put these values into perspective, the value of all outstanding debt in Europe, Japan and North America totaled $25.8 trillion in 1995.

The growth of the OTC market will continue to outstrip the growth of the exchange market because the clients of the financial service firms need assistance to structure their financial programs. The current growth path is to provide more client-focused structured solutions to problems. Clients likely would find it less expensive to execute a program through their financial service firm than to execute it themselves in the exchange market. This is even more likely if the positions must be adjusted frequently to hedge risks.

Moreover, the relative growth of the OTC market is overstated because the exchange markets require that entities post margin on contracts each day. Futures contracts are settled at the end of each day. Forward contracts, such as swaps and options written by OTC firms such as caps and floors, are not settled each day. The product offerings are different. The financial service firms and their counterparts rely on each other’s credit. Most financial service firms can post collateral on

\(^3\)See “International Capital Markets, Developments, Prospects and Key Policy Issues,” International Monetary Fund (September 1997) for the source of these statistics. The notional amount outstanding is not an economic measure of the size of the market. The notional amount of a swap or an option is the amount on which the contract is based. It is not the value that the security would trade at in the market. For example, if a call option to buy $100,000 of a major-market index trades at $5,000, the notional amount is recorded as $100,000 while the economic value is only $5,000. The economic value of swaps and options might be as low as 2% and 5% of the notional outstanding amounts of the contracts. These statistics provide estimates of the growth of the derivative market.
OTC contracts, which is very similar to settling the contracts as in the case of posting margin on an exchange. These entities will use either the financial futures and options markets or the OTC markets. They will use the industry that provides services at lower cost. Many entities, however, are not indifferent to posting collateral. In particular, many OTC swap and option contracts have a financial entity and a corporation as counterparts because the corporation is willing to pay the financial entity to post margin for it in the futures or options market. That is, the financial services firm enters into a swap with a corporate entity, which does not post collateral, and the financial service firm hedges its market risk by entering into an offsetting swap with another financial service firm or by using the exchange-derivatives industry. In either case, the financial service firm posts collateral or margin on its transactions. It is the lower cost producer of margin services. Financial service firms have the capacity and the personnel to undertake the pricing of credit risk and can handle these transactions; many corporate entities currently do not.

To the extent that collateral is not posted on the obligation, one consequence of these transactions is that the financial service firm and the corporation are exposed to each other’s credit risk. The corporation buying a call option on an underlying debt instrument from the financial service firm has credit risk to the extent of the value of the call option. The financial service firm can fail to honor its obligation to deliver the underlying debt instrument. The financial service firm holding a put option, issued by the corporation, on the same debt instrument is a creditor of the corporation to the extent of the value of the put option. A swap contract, which states that the corporation will receive a fixed rate of interest on an underlying debt instrument and pay a floating rate of interest, is equivalent to the corporation being long a call option and short a put option.

The other major reason that the OTC industry will continue to grow faster than the exchange industry is that financial service firms and others need only to hedge the remaining factor risks of their portfolio positions, which is a far smaller amount than their gross contracting with their clients. Moreover, depending on the costs, they can hedge either with another financial service firm or in the exchange industry.
The financial service firm that hedges factor risks retains the remaining risk, the so-called “basis risk,” of its net positions.

Another reason for the growth of the OTC market has been that the outstanding amounts in Table 3.2 do not necessarily represent net exposures. It might be less expensive for a corporation or a financial service firm to enter into an offsetting derivative contract with another counterparty than it would be to unwind the initial contract. If it does, the contract volume increases but the net exposure falls.

3.3.1 The Present: The Pathologies

From the perspective of market commentators, many regulators, and the public, derivatives tend to top the list of suspects when the stock market turns downward or when entities announce unanticipated financial losses. The press, the public and regulators fear derivatives, in part, because they are new and, in part, because their growth has appeared to be so explosive over the last ten years. Although they vastly overstate the economic exposure, notional amounts as high as $45 trillion cause worry. The press and others credit the market crash of 1987 to portfolio insurance, an attempt to dynamically replicate the returns on options. Even in this time, market pundits warn that forms of dynamic hedging could foster a severe market downturn. Widely publicized losses attributed to derivative trading in the 1990s include: the leveraged-derivative contracts issued by Bankers Trust to firms such as Procter and Gamble and Gibson Greeting (over $150 million in losses); the loss of $1.5 billion by Shell Sekiyu, the losses incurred by Orange County investing in inverse floaters; the bankruptcy of Metallgesellschaft and Barings Corporation (both over $1 billion in losses) and many other losses by financial service firms such as UBS, Salomon Brothers, etc. Obviously, many of these losses are overstated because there were gains made by the other side to these contracts: It is only the dead-weight costs to society that result in actual loss. For excellent discussions of the entire range of purported pathologies and an excellent review of the literature addressing these issues see Miller (1997), who argues that most, if not all, of the “diagnoses” of severe pathologies are misdiagnosed.
Yet, the growth of these industries depicted in Tables 3.1 and 3.2 clearly suggests that these instruments have added net value. It is hard to believe such growth could continue for so many years without value being realized by the clients of financial service firms, the shareholders of these firms and the exchanges. I have argued in Scholes (1995, 1996a) that the development of financial infrastructure might lag financial innovation. It is costly to develop controls and firm-wide understanding of new products that are in the prototype phase of their development. Prototypes are built using existing infrastructure. For an innovation as long lasting and profitable as derivatives, the OTC industry, the exchange industry and the academic industry find it profitable to build the infrastructure necessary to support them. Each of these industries has a vested interest in profiting from adding value that is sustainable, so each will attempt to invest in the cost-effective infrastructure necessary to preserve this value.

This is not to argue that we have seen the end of derivative failures. There will be losses sustained as in many other business activities. In 1997, the Governments of many countries in Asia could no longer support the losses of their financial institutions resulting from defaulted commercial and real estate loans. Although many banks had been economically bankrupt without the support of their Governments (that is; the value of their equity would have been effectively zero without Government implicit guarantees), the Governments allowed these banks to participate in any potential gains and sheltered them against bank runs by promising to pay off their depositors. These options were costly to society, and it will be difficult to prove that they were value enhancing. In part, other countries, through the grants made by the International Monetary Fund, as the lenders of last resort to these and other countries, may have written the put options that supported the activities of the financial institutions of the region and granting these options might have encouraged risk taking and even unprofitable activities.

Moreover, absent government guarantees, some brokerage houses and banks in Japan probably would be bankrupt, because while these entities had promised clients protection against loss on any decline in value of Japanese stocks, they did not hedge their commitments. Clients might have paid for these put options through higher commission rates
in Japan. These Japanese financial service entities, however, suffered severe losses when their clients exercised these put rights during the decline in the value of Japanese stocks in the 1990s. The entities currently are not required to value their commitments on a mark-to-the-market basis. As a result, neither regulators, nor investors nor even senior management could deduce the financial condition of the entities. In all probability, the extent of these losses could have been mitigated if risk management policies had been put in place.

3.4 The Future

The future will be a continuation of the present. Financial innovation will continue at the same or at even an accelerating pace because of the insatiable demand for lower-cost, more efficient solutions to client problems. Information and financial technology will continue to expand and so will the circle of understanding of how to use this technology. There is value to investing in education. Financial service firms will expand the use of this technology to manage their own activities. Otherwise, they will have to face mergers with other financial service entities. Although some would like to see derivatives wither in importance, they will not, for they have become essential mechanisms in the tool kit of financial innovation.

Scholes and Wolfson (1992) used the concepts of frictions and restrictions to illustrate how tax rules and other regulations affect investor and corporate behavior. As Merton (1992) argues, the functions of a financial system change far less than institutions. Institutions change because lower-cost solutions that reduce information asymmetries are found to facilitate transactions, to provide funding for large-scale investment projects, to transfer savings across borders and into the future, and provide more efficient risk-sharing and diversification mechanisms.

Most financial instruments are derivative contracts in one form or another. Black and Scholes (1973) pointed out that the equity holders of a firm with debt in its capital structure have an option to buy back the firm from its debt-holders at maturity of the debt. The high-yield bond (the so-called “junk bond”) is a riskier debt-option contract than
more-highly rated corporate debt. Corporate debt and equity contracts are derivative to underlying investments. Other lines of research on so-called “real options” indicate that even the investment decisions of firms are better understood by using an option framework rather than a more conventional present-value-analysis framework.

Standard debt and equity contracts are institutional arrangements or boxes. They provide particular cash flows to investors with their own particular risk and return characteristics. These institutional arrangements survive only because they provide lower cost solutions than competing alternative arrangements. Competitive opportunities evolve over time with changing frictions and restrictions. Because of information asymmetries and regulatory restrictions, investors might require a higher rate of return to hold these standard-form contracts than contracts (now and in the future) of alternative design but of similar risk. Time will continue to blur the distinctions between debt and equity.

The firm’s investment set is generally the composite of coarse bundles of payoffs. Firms issue claims to finance these activities, claims that themselves represent bundles of coarse cash flows. It will become more efficient for financial service firms to offer new derivative securities in various forms to break cash flows into finer gradients that can be tailored to the specific needs of demanders and suppliers of capital. In the process, dead-weight costs are mitigated, thereby reducing the cost of capital. The financial service firm can sell the newly created securities or retain them in whole or in part for its own account. It can create new products on its own name or use the OTC or exchange markets to hedge its risks. Given information asymmetries, it will use the lower-cost solution.

3.4.1 Investor Demands

In recent years, we have witnessed a movement from a limited number of investors holding an undiversified portfolio of their own home-country securities to many more investors holding diversified portfolios domestically and internationally. More and more investors around the world, who have never invested in financial products other than through social promises made by their governments, will become more willing to
select from a broad class of “mutual-fund” type offerings. Although the diversity of products has grown, few tools are in place other than in academic circles that allow investors to make informed portfolio allocation decisions. As Franco Modigliani, the 1985 Nobel laureate, has argued, individuals want to smooth consumption over the life cycle. If it were more cost efficient, investors would want to insure against contingencies, control risks more efficiently, and plan their investments efficiently to meet life-cycle needs. As information and financial technology become more easily available, financial service firms will repackage investments to meet these investor demands, and this will spur financial innovation. The financial service firm will offer products in its own name that promise specific risk and return patterns; these firms will also offer products in the form of mutual funds. The classifications of investment products into stock funds, bond funds, growth, income, etc. will diminish in importance with reduced costs to understand risk, return, and contingent payoffs.

Even today, the boxes that define institutional-fund arrangements have blurred. For example, if a pension fund manager wants to achieve a stock-index fund return, she can invest with an index-fund provider that buys a diversified portfolio of the index-fund stocks. She can achieve the same result, however, in myriad other ways including using another manager who might claim to have expertise in the bond market and can provide an enhanced return over the index-fund return. To achieve this, the enhanced manager might undertake a complicated strategy. He might hold a portfolio of undervalued corporate and government bonds, hedge the credit risk of the corporate bonds by selling stock short, and hedge the price risk of interest rate movements by using futures or options. He might buy stock-index futures to achieve the systematic risk exposure of the stock-index fund. Given costs, the manager might be able to produce a return in excess of that achieved by holding the index fund directly while the systematic risks of the two offerings would be exactly the same.

The exchange industry will compete with the OTC industry to provide investment products. If the exchange industry can provide efficient margining systems for those investors who can not post collateral in a cost efficient manner, those products that become standardized will
most likely be ideally suited for the exchange industry: it can address a larger set of participants in a lower-cost marketplace. The exchange industry complements the OTC industry; they will grow together.

### 3.4.2 Corporate Demands for Derivatives

Finance specialists have puzzled over the reasons why corporations hedge the risks of their cash flows. Under classical finance theory, it is often asked why shareholders of a firm pay it to incur costs to reduce risk when they can diversify on their own account. The firm’s managers should act as if the firm is risk neutral. Smith and Stulz (1985) provide three reasons, all tied to the cost of financial distress, why a firm might hedge its cash flows. Because of the convexity of the tax schedule, a firm might issue more debt only after hedging its cash flows to reduce its expected operating losses and the resultant loss of tax benefits. Because of bankruptcy costs associated with high levels of debt, the firm that hedges can use more debt to finance its activities. As in Froot et al. (1993), if the firm can hedge its cash flows, a reduction in the probability of financial distress reduces the expected costs of financial distress and, as a result, encourages investment in profitable projects that might have been foregone without such hedging. This argument is based on the observation that firms are reluctant to issue equity, and, instead, use retained earnings to finance investments before using the debt markets. Also, if a high-debt-to-equity firm were to become financially distressed it would not be possible to issue equity to finance business activities. Because owner-managers in smaller firms might not be able to diversify their holdings, hedging the cash flows of the firm might be a lower-cost alternative than selling off pieces of the firm to outsiders and using the proceeds to diversify. If corporations face these problems there are financial engineering solutions that might reduce their import.

The corporate use of derivatives is not limited to hedging. Some corporate financial strategists believe that they can outperform other market participants in forecasting the direction of interest rates or commodity prices. Stulz (1996) argues that firms that have such financial acumen can hedge their downside exposures by buying put options.
This allows the financial officers of the firm to become more active managers. By buying put options or by reducing systematic risks, they can use more leverage and increase their personal stakes by reducing the costs of financial distress. This tactical use of derivatives probably explains a significant part of the growth of the use of derivatives attributable to financial service firms and corporations, as shown in Tables 3.1 and 3.2.

As reported in Stulz (1996), empirical evidence gathered from surveys of corporations indicates that large corporations without debt in their capital structures hedge cash flows more so than smaller corporations. And, those corporations that do hedge lift their hedges from time to time or do not fully hedge their exposures. It is these tactical uses of derivatives, an attempt to “beat” the market using highly leveraged strategies, that have been the cause of most of the reported financial losses. Obviously, the successful tactical users of derivatives are most often absent in press reports. It is unlikely, however, that corporate officials, on average, can outperform other market participants.

Large firms hedge cash flows, in part, to smooth reported financial earnings with the hope those smoother earnings will boost their price-to-earnings ratios. This might be a value-enhancing strategy if market participants can not discern whether the variability in earnings is caused by the firm’s taking systematic exposures to market factors or by firm-specific risks.

The Present is still young. The Future will bring many new solutions to solve corporate problems. Many corporations and financial entities still need to learn and evaluate to what extent hedging and risk control can be beneficial to their activities. Smaller firms and product markets are just now becoming familiar with the risk control aspects of these financial instruments. It may be surprising that in the United States the top 8 banks account for 94% (almost $19 trillion) of the total outstanding notional amount in the OTC market as of the end of 1996. As of this date, the knowledge base or the financial acumen needed to financial engineer solutions to client problems is highly concentrated.

\[^{4}\text{See International Monetary Fund, op. cit.}\]
As in Scholes (1995, 1996a), I argue that corporations will use risk management techniques to reduce their level of equity capital, and, as a result of risk management techniques, some firms that would have gone public will remain private. Equity capital is an expensive form of financing. There are large differences between the knowledge base of insiders and outsiders. Insiders can not fully divulge their plans to outsiders for the fear that competitors will profit from this knowledge, and generally must sell shares at a discount. Moreover, tax and other considerations make the corporate form of undertaking activities in the U.S. and in other countries very expensive.

Equity is a risk-management device. It is an “all purpose” risk cushion. The more equity a firm has, the more it cushions itself against outcomes that require it to go the capital markets in adverse times or when it might have to divulge its confidential operating plans to outside parties. Hedging, on the other hand, is targeted risk control. Hedging requires more refined knowledge of the firm, and an understanding of the interaction of investment returns and financing alternatives. Moreover, it requires that the firm be able to warrant to others that it will maintain a strategy of hedging its activities to support higher levels of debt. But as the costs to hedge fall relative to the costs of equity, firms will substitute hedging for equity.

Moreover, hedging provides ancillary benefits as a measurement tool to help calibrate how the firm is making money. In a diverse, decentralized organization, management information systems might not divulge the true source of profits within the organization; that is, did profits arise because systematic risk exposures produced positive returns, or because the entity possessed superior skills? Standard accounting neither provides risk management reports which decompose profitability into profits from market forces and profits from managerial efforts, whether the firm is a manufacturing or a financial firm, nor does it describe the sensitivities of the firm’s profit and loss to market factors. As more entities use financial engineering skills, the current accounting system will be under considerable pressure for change, as will many of the current forms of regulations and restrictions.

Because of differences in the required knowledge of insiders and outsiders, the growth of the private equities market has reduced the
disclosure costs of becoming a public corporation. Private equity allows expert management teams to leverage their activities. Private equity, however, is still an inefficient form of financing compared with potential lower-cost solutions. Ways will be found through financial engineering to provide private entities with the advantages of the public market – risk sharing, liquidity, and pricing signals – while retaining the advantages of the private market – lower disclosure and agency costs. Financial engineering will foster the growth of the private corporation, and convert entities into alternative forms.

Many firms hedge interest rate movements, foreign currency exposures, or commodity price exposures. Firms will learn to use stock-index options or futures to reduce their risk exposures. The firm can reduce the beta of its own stock by hedging stock-market risks. Moreover, with this approach, the firm does not have to target risks. It can just hedge its own market risks or other factor risks, or the general stock-market risk that affect its stock price. This reduces the economic risks of the firm to firm-specific risks, or residual risks, and reduces the need for equity to cushion adverse outcomes.

I believe that the corporate form we know today will not be long-lived. With more knowledge and a better understanding of the power of financial engineering and of how to reduce asymmetric information costs, the costs of using financial engineering solutions will continue to fall. As more firms learn how to use these solutions, their profits will be enhanced and more investment will follow the increase in demand. Risk management is only a step in the direction of producing synthetic entities.

The firm of the future might be an organizational form far different from those used today. Some entities, such as electricity producers, aircraft manufacturers and users, natural resource producers and users, and financial service firms already are deciding what services to produce and what risks to retain; what services to rent and what risks to shed, based on their perceived competitive advantage.

Financial service firms are building large capital bases to make markets around the world, and to put into practice specific knowledge to engineer solutions for their clients on a global basis and to create
long-lived derivative products for issue in their own names. Their profits are made from modeling and understanding markets and providing value-added solutions for clients. A risk-management system provides information on what risks to keep and what risks to hedge. In addition, it provides a way to reduce information asymmetries between senior management and employees, and to provide the incentive system necessary to align the interests of the employees and the firm’s shareholders.

A risk-management system must also address how a financial service firm handles crisis situations. To preserve its franchise, a financial service firm can insure against adverse price movements or unforeseen contingencies by holding working capital as a reserve against adverse liquidity needs. Alternatively, the financial service firm might be able to buy options from the exchange markets or from the OTC markets (for example, lines of credit) at a lower dead-weight cost to insure against extreme price movements that could adversely affect its business. Maybe regulatory bodies, in effect, provide lower cost insurance.

Because of tax and regulatory costs, financial-service firms might find that working capital held in corporate form is too expensive relative to other alternatives. For example, clients of financial-service firms hold large quantities of passive wealth in mutual funds, insurance companies, pension funds, and various trusts as investment vehicles for individual savings. Financial-service firms and other entities must hold working capital to insure against adverse contingencies. With options and other forms of contingent capital arrangements, it will become possible to mobilize the capital in these client passive investment vehicles and reduce the dead-weight costs of the current system. This could lower the cost of capital of financial-service firms and the cost of providing financial services to their client base.

Once again, information and financial technology will expand to reduce the costs of information asymmetries. Understanding and developing markets for credit derivatives, understanding the implications of contingent capital options under asymmetric information, and understanding what is the most efficient mechanisms to hold capital will change organizational forms, the boxes, and blur the distinctions
between debt and equity, corporations and partnerships, and the
demanders and suppliers of capital. The evolution of option technology
will open up entire new institutional structures.

3.5 Conclusion

We started in the Past, the age of innocence, and we progressed to
the Present, the age of understanding, growth, and maturation. The
growth has not been without pains. A considerable amount of addi-
tional understanding and development awaits the users of the derivative
technology in the Future – the age of excitement. Advances in commu-
nications and computing technology will allow for greater reduction in
asymmetric information costs. The future growth of innovation using
the option-pricing technology will be as great or greater than in the
past. Organizational forms will change dramatically in a global envi-
ronment. The exchange industry will continue to grow; the OTC indus-
try will continue to grow, and the research necessary by academics and
practitioners to understand and to foster the evolution will be so great
that the academic industry will become more important than in the
past. The need for highly trained and skilled practitioners that under-
stand the technology will continue to increase on a global basis. None of
these three industries has retained its past form and none will retain its
present form. These economic organizational forms always will evolve
and respond to the demands of economic agents.

The capital asset pricing model, arbitrage and capital structure
models, and the efficient markets hypothesis introduced me to key
finance concepts that were the genesis of our development of the option-
pricing technology. In a world of information asymmetries, derivative
instruments provide lower-cost solutions to financial contracting prob-
lems in a dynamic environment and these lower-cost solutions enhance
economic efficiency.

3.6 Acknowledgements

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for their helpful suggestions on this lecture and for so much more.
Over the past thirty years, I have come to owe an incalculable debt to Paul A. Samuelson, my teacher, mentor, colleague, co-researcher, and friend. Try as I have (cf. Merton, 1983, 1992a), I cannot find the words to pay sufficient tribute to him. I dedicate this lecture to Paul and to the memory of Fischer Black. Copyright © Nobel Foundation.

3.7 References


4

Classification Categories

11000000 GENERAL THEORY
11010000 Rational Bounds on Derivative Prices
11020000 Binomial and Multinomial Models
11030000 Risk-Neutral Pricing
11031000 State-Price Densities and Equivalent Martingale Measures
11032000 Implied Binomial Trees
11033000 Implied Risk Aversion
11034000 Simulation and Monte Carlo Techniques
11040000 Dynamic Spanning and Market Completeness
11041000 Convergence Results
11042000 Approximation Results
11050000 Implied Volatility
11060000 Equilibrium Models
11061000 Stochastic Volatility Models
11062000 Jump Processes
11063000 Other Incomplete-Market Models
11064000 Discrete-Time Models
11065000 Term Structure of Interest Rates
11070000 Option Pricing and Hedging with Transaction Costs
11080000 Numerical Methods, Solution of PDE's
11090000 Nonparametric Pricing
11100000 Mathematics of Derivatives
11110000 Dynamic Hedging
11120000 Security Design
11130000 Optimal Exercise
Classification Categories

12000000 FINANCIAL-MARKET APPLICATIONS
12010000 Derivatives
12010100 Compound Derivatives
12010200 Exotic Derivatives
12010300 Fixed Income Derivatives
12010400 Foreign Exchange Derivatives
12010500 Commodity Derivatives
12010600 Equity Derivatives
12010700 Index Derivatives
12010800 Asset-Backed Securities
12010810 Mortgages and Mortgage-Backed Securities
12010820 Other Asset-Backed Securities
12010900 Swaps and Swaptions
12011000 Hybrid Securities
12011100 Credit Derivatives
12011200 Weather Derivatives
12020000 Corporate Finance
12020100 Risk Management
12020200 Debt, Equity, and Other Corporate Liabilities
12020210 Debt and Limited Liability
12020211 Bankruptcy and Default
12020212 Credit Spreads and Risk
12020213 Bond Indenture Provisions
12020220 Warrants
12020230 Convertible Securities
12020240 Optimal Capital Structure
12020241 SEO’s, IPO’s, underwriting
12020250 Corporate Control
12020251 Tender Offers
12020252 Corporate Restructuring
12020253 Corporate Diversification
12020300 Executive Compensation
12020310 Employee Stock Options
12020400 Venture Capital
12020500 Asset Leasing
12020510 Automobile Leasing
12030000 Insurance
12030100 Loan Guarantees
12030110 Expropriation Risk
12030120 Third World Loans
12030200 Non-Life Insurance
12030210 Home Equity Insurance
12030220 Inflation Insurance
12030230 Deposit Insurance
12030240 Health Insurance
12030250 College Tuition Insurance
12030260  Catastrophe Insurance
12030300  Portfolio Insurance
12030400  Life Insurance
12030410  Annuity Insurance
12030500  Re-insurance
12030600  Terrorism Insurance
12040000  Investment Management
12040100  Performance Analysis and Attribution
12040110  Asset Allocation and Market Timing
12040120  Risk Measurement
12040130  Portfolio Efficiency
12040140  Hedge Funds Analysis
12040200  Market Microstructure
12040210  Limit Orders
12040220  Bid-ask Spreads
12040230  Order Placement Strategies
12040240  Price Discreteness
12040250  Market Making
12040300  Derivative Trading Strategies
13000000  GENERAL ECONOMIC APPLICATIONS
13010000  Real Options
13010100  Investment under Uncertainty
13010110  Capacity Planning
13010120  Infrastructure Investment
13010200  Options to Invest
13010300  Research and Development
13010310  Product Design
13010320  Business Incubation
13010400  Inventories
13010500  Entry and Exit
13010600  Exhaustible Resources
13010610  Oil, Coal, Natural Gas, Mineral Deposits
13010620  Pollution Rights
13010700  Abandonment Options
13010800  Strategic Options
13010900  Patents
13011000  Vacant Land Options
13011100  Real Estate
13011200  Power Generation
13020000  Macroeconomics
13020100  Economic Derivatives
13020200  Stabilization Policies
13030000  Political Science
13040000  Defense
13050000  Labor Economics
13050100  Education
Classification Categories

13050200 Academic Tenure
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13060000 Agency Theory, Incentives and Contracts
13070000 Game Theory
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13090000 Trade
13090100 Trade Credit
13100000 Asset Pricing
13110000 Financial Innovation
13120000 Organizational Behavior
14000000 OTHER APPLICATIONS
14010000 Legal
14010100 Tax Delinquency
14010200 Taxation
14010300 Quota Licenses
14010400 Estate Tax
14010500 Regulation
14010600 Litigation Participation
14020000 Transportation
14030000 Biomedical Research
14040000 Entertainment
14050000 Agriculture
14050100 Farm Price Supports
14060000 Marketing
14070000 Environment
14080000 Social Sciences
14080100 Psychology
14090000 Accounting
14100000 Weather Derivatives
15000000 EMPIRICAL ANALYSIS
15010000 Estimation of Stochastic Processes
15010100 Estimation of Volatility
15010110 GARCH models
15010200 Jump Process Estimation
15020000 Tests of Derivative Pricing Models
15020100 Term Structure Tests
15020300 Implied Volatility Tests
15020400 Implied Risk Aversion Tests
15020500 Tests of Market Efficiency
15020510 Arbitrage Relations
15020520 Tests of Boundary Conditions
15020530 Nonparametric Tests
15020540 Option Exercise Tests
15060000 Forecasting
15070000 Empirical Studies
15070100 Event Studies
15070200 Market Crashes
15070300 Option Listing
15070400 Seasonalities
15070500 Announcement Reactions
15070600 Informational Transmission and Trading
16000000 HISTORICAL
11000000 GENERAL THEORY


11010000 Rational Bounds on Derivative Prices

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11100000 Mathematics of Derivatives


11110000 Dynamic Hedging


11120000 Security Design


Citations


11130000 Optimal Exercise


12000000 FINANCIAL-MARKET APPLICATIONS


12010000 Derivatives


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486 Citations


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12020000 Corporate Finance


12020100 Risk Management


12020200 Debt, Equity, and Other Corporate Liabilities


12020190 Debt and Limited Liability

12020211 Bankruptcy and Default
Citations 501


502  Citations


12020212 Credit Spreads and Risk


12020213 Bond Indenture Provisions


12020220 Warrants


12020230 Convertible Securities


12020240 Optimal Capital Structure


12020241 SEO’s, IPO’s, underwriting


12020250 Corporate Control


12020251 Tender Offers


12020252 Corporate Restructuring

Citations


12020300 Executive Compensation


Citations


12020310 Employee Stock Options


12020400 Venture Capital

12020500 Asset Leasing


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Citations


12030100 Loan Guarantees


12030110 Expropriation Risk


12030120 Third World Loans

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542  Citations

13030000 Political Science

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Citations


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15070000 Empirical Studies


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