An econometric model of serial correlation and illiquidity in hedge fund returns ☆

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Abstract

The returns to hedge funds and other alternative investments are often highly serially correlated. In this paper, we explore several sources of such serial correlation and show that the most likely explanation is illiquidity exposure and smoothed returns. We propose an econometric model of return smoothing and develop estimators for the smoothing profile as well as a smoothing-adjusted Sharpe ratio. For a sample of 908 hedge funds drawn from the TASS database, we show that our estimated smoothing coefficients vary considerably across hedge-fund style categories and may be a useful proxy for quantifying illiquidity exposure.
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1. Introduction

One of the fastest growing sectors of the financial services industry is the hedge fund or alternative investments sector. Long the province of foundations, family

offices, and high net-worth investors, hedge funds are now attracting major institutional investors such as large state and corporate pension funds and university endowments. Efforts are underway to make hedge fund investments available to individual investors through more traditional mutual-fund investment vehicles. One of the main reasons for such interest is the performance characteristics of hedge funds. Often known as high-octane investments, many hedge funds have yielded double-digit returns to their investors and, in some cases, in a fashion that seems uncorrelated with general market swings and with relatively low volatility. Most hedge funds accomplish this by maintaining both long and short positions in securities (hence the term "hedge fund") which, in principle, gives investors an opportunity to profit from both positive and negative information while, at the same time, providing some degree of market neutrality because of the simultaneous long and short positions.

However, several recent empirical studies have challenged these characterizations of hedge fund returns, arguing that the standard methods of assessing their risks and rewards are misleading. For example, Asness et al. (2001) show in some cases where hedge funds purport to be market neutral, i.e., funds with relatively small market betas, including both contemporaneous and lagged market returns as regressors and summing the coefficients yields significantly higher market exposure. Moreover, in deriving statistical estimators for Sharpe ratios of a sample of mutual and hedge funds, Lo (2002) shows that the correct method for computing annual Sharpe ratios based on monthly means and standard deviations can yield point estimates that differ from the naive Sharpe ratio estimator by as much as 70%.

These empirical properties have potentially significant implications for assessing the risks and expected returns of hedge fund investments. We can trace them to a single common source of significant serial correlation in their returns.

This is surprising because serial correlation is often (though incorrectly) associated with market inefficiencies, implying a violation of the Random Walk Hypothesis and the presence of predictability in returns. This seems inconsistent with the popular belief that the hedge fund industry attracts the best and the brightest fund managers in the financial services sector. In particular, if a fund manager's returns are predictable, the implication is that the manager's investment policy is not optimal. If the manager's returns next month can be reliably forecasted as positive, the fund manager should increase positions this month to take advantage of this forecast, and vice versa for the opposite forecast. By taking advantage of such predictability, the fund manager will eventually eliminate it, along the lines of Samuelson's (1965) original "proof that properly anticipated prices fluctuate randomly." Given the outsize financial incentives of hedge fund managers to produce profitable investment strategies, the existence of significant unexploited sources of predictability seems unlikely.

In this paper, we argue that in most cases, serial correlation in hedge fund returns is not due to unexploited profit opportunities, but is more likely the result of illiquid securities that are contained in the fund. For example, these illiquid securities can include securities that are not actively traded and for which market prices are not always readily available. In such cases, the reported returns of funds containing

illiquid securities will appear to be smoother than true economic returns (returns that fully reflect all available market information concerning those securities) and this, in turn, will impart a downward bias on the estimated return variance and yield positive serial return correlation.

The prospect of spurious serial correlation and biased sample moments in reported returns is not new. Such effects are available in the literature on "nonsynchronous trading", which refers to security prices recorded at different times but which are erroneously treated as if they were recorded simultaneously. For example, the daily prices of financial securities quoted in the *Wall Street Journal* are usually "closing" prices, prices at which the last transaction in each of those securities occurred on the previous business day. If the last transaction in security A occurs at 2:00 pm and the last transaction in security B occurs at 4:00 pm, then included in B's closing price is information not available when A's closing price was set. This can create spurious serial correlation in asset returns since economy-wide shocks will be reflected first in the prices of the most frequently traded securities, with less frequently traded stocks responding with a lag. Even when there is no statistical relation between securities A and B, their reported returns will appear to be serially correlated and cross-correlated simply because we have mistakenly assumed that they are measured simultaneously. One of the first to recognize the potential impact of nonsynchronous price quotes was Fisher (1966). Since then more explicit models of nontrading have been developed by Atchison et al. (1987), Dimson (1979), Shanken (1987), Cohen et al. (1978, 1979, 1983a, b, 1986), Kadlec and Patterson (1999), Lo and MacKinlay (1988, 1990), and Scholes and Williams (1977). Campbell et al. (1997, Chapter 3) provide a more detailed review of this literature.

However, this literature focuses exclusively on equity market-microstructure effects as the sources of nonsynchronicity (closing prices that are set at different times, or prices that are stale) where the temporal displacement is on the order of minutes, hours, or, in extreme cases, several days. For such applications, Lo and MacKinlay (1988, 1990) and Kadlec and Patterson (1999) show that nonsynchronous trading cannot explain all of the serial correlation in weekly returns of equal- and value-weighted portfolios of US equities during the past three decades.

In the context of hedge funds, we argue in this paper that serial correlation is the outcome of illiquidity exposure, and while nonsynchronous trading could be one symptom or by-product of illiquidity, it is not the only aspect of illiquidity that affects hedge fund returns. Even if prices are sampled synchronously, they can still yield highly serially correlated returns if the securities are not actively traded. In fact, for most hedge funds, returns are computed on a monthly basis, hence the pricing or mark-to-market of a fund's securities typically occurs synchronously on the last day of the month. Therefore, although our formal econometric model of illiquidity is similar to those in the nonsynchronous trading literature, the motivation is considerably broader, including linear extrapolation of prices for thinly traded securities, the use of smoothed broker–dealer quotes, trading restrictions arising from control positions and other regulatory requirements, and, in some cases, deliberate performance-smoothing behavior. Thus, the corresponding interpretations of our parameter estimates must be modified accordingly.

Regardless of the particular mechanism by which hedge fund returns are smoothed and serial correlation is induced, the common theme and underlying driver is illiquidity exposure, and although we argue that the sources of serial correlation are spurious for most hedge funds, nevertheless, the economic impact of serial correlation can be quite real. For example, spurious serial correlation yields misleading performance statistics such as volatility, Sharpe ratio, correlation, and market-beta estimates. Such statistics are commonly used by investors to determine whether or not they will invest in a fund, how much capital to allocate to a fund, what kinds of risk exposures they are bearing, and when to redeem their investments. Moreover, spurious serial correlation can lead to wealth transfers between new, existing, and departing investors, in much the same way that using stale prices for individual securities to compute mutual-fund net-asset-values can lead to wealth transfers between buy-and-hold investors and day-traders (see, for example, Boudoukh et al., 2002).

In this paper, we develop an explicit econometric model of smoothed returns and derive its implications for common performance statistics such as the mean, standard deviation, and Sharpe ratio. We find that the induced serial correlation and impact on the Sharpe ratio can be quite significant even for mild forms of smoothing. We estimate the model using historical hedge fund returns from the TASS Database, and show how to infer the true risk exposures of a smoothed fund for a given smoothing profile. Our empirical findings are quite intuitive. Funds with the highest serial correlation tend to be the more illiquid funds (e.g., emerging market debt, fixed income arbitrage, etc.), and after correcting for the effects of smoothed returns, some of the most successful types of funds tend to have considerably less attractive performance characteristics.

Before describing our econometric model of smoothed returns, we provide a brief literature review in Section 2 and then consider other potential sources of serial correlation in hedge fund returns in Section 3. We show that these other alternatives (time-varying expected returns, time-varying leverage, and incentive fees with high-water marks) are unlikely to generate the magnitudes of serial correlation observed in the data. We develop a model of smoothed returns in Section 4. We then derive its implications for serial correlation in observed returns, and we propose several methods for estimating the smoothing profile and smoothing-adjusted Sharpe ratios in Section 5. We apply these methods to a data set of 908 hedge funds spanning the period from November 1977 to January 2001 and summarize our findings in Section 6. We conclude in Section 7.

2. Literature review

Thanks to the availability of hedge-fund returns data from sources such as AltVest, Hedge Fund Research (HFR), Managed Account Reports (MAR), and TASS, a number of empirical studies of hedge funds are available. For example, Ackermann et al. (1999), Agarwal and Naik (2000b, c), Edwards and Caglayan (2001), Fung and Hsieh (1999, 2000, 2001), Kao (2002), and Liang (1999, 2000, 2001) provide comprehensive empirical studies of historical hedge-fund performance using various hedge-fund databases. Agarwal and Naik (2000a), Brown and Goetzmann

(2001), Brown et al. (1997, 1999, 2000, 2001), Fung and Hsieh (1997a, b), and Lochoff (2002) present more detailed performance attribution and "style" analysis for hedge funds. None of these empirical studies focuses directly on the serial correlation in hedge-fund returns or the sources of such correlation.

However, several authors have examined the persistence of hedge-fund performance over various time intervals, and such persistence may be indirectly linked to serial correlation. For example, persistence in performance usually implies positively autocorrelated returns. Agarwal and Naik (2000c) examine the persistence of hedge-fund performance over quarterly, half-yearly, and yearly intervals by examining the series of wins and losses for two, three, and more consecutive time periods. Using net-of-fee returns, they find that persistence is highest at the quarterly horizon and decreases when moving to the yearly horizon. The authors also find that performance persistence, whenever present, is unrelated to the type of a hedge fund strategy. Brown et al. (1992) show that survivorship gives rise to biases in the first and second moments and cross-moments of returns, and apparent persistence in performance where there is dispersion of risk among the population of managers. However, using annual returns of both defunct and currently operating offshore hedge funds between 1989 and 1995, Brown et al. (1999) find virtually no evidence of performance persistence in raw returns or risk-adjusted returns, even after breaking funds down according to their returns-based style classifications. None of these studies considers illiquidity and smoothed returns as a source of serial correlation in hedge-fund returns.

Asness et al. (2001) show that lagged market returns are often significant explanatory variables for the returns of supposedly market-neutral hedge funds. This finding is closely related to serial correlation and smoothed returns, as we shall demonstrate in Section 4. In particular, we show that even simple models of smoothed returns can explain both serial correlation in hedge-fund returns and correlation between hedge-fund returns and lagged index returns, and our empirically estimated smoothing profiles imply lagged beta coefficients that are consistent with the lagged beta estimates reported in Asness et al. (2001). They derive their framework from the nonsynchronous trading literature, specifically the estimators for market beta for infrequently traded securities proposed by Dimson (1979), Scholes and Williams (1977), and Schwert (1977) (see Section 1 for additional references to this literature). A similar set of issues affects real-estate prices and price indexes, and Ross and Zisler (1991), Gyourko and Keim (1992), Fisher et al. (1994, 2003) propose various econometric estimators that have much in common with those in the nonsynchronous trading literature.

An economic implication of nonsynchronous trading that is closely related to the hedge-fund context is the impact of stale prices on the computation of daily net-asset-values (NAVs) of certain open-end mutual funds, e.g., Bhargava et al. (1998), Chalmers et al. (2001), Goetzmann et al. (2001), Boudoukh et al. (2002), Greene and Hodges (2002), and Zitzewitz (2003). In these studies, serially correlated mutual fund returns are traced to nonsynchronous trading effects in the prices of the securities contained in the funds, and although the correlation is spurious, it has real effects in the form of wealth transfers from a fund's buy-and-hold shareholders to those engaged in opportunistic buying and selling of shares based on forecasts of the

fund's daily NAVs. Although few hedge funds compute daily NAVs or provide daily liquidity, the predictability in some hedge-fund return series far exceeds levels found among mutual funds, hence the magnitude of wealth transfers attributable to hedge-fund NAV-timing is at times significant.

With respect to the deliberate smoothing of performance by managers, a recent study of closed-end funds by Chandar and Bricker (2002) concludes that managers seem to use accounting discretion in valuing restricted securities so as to optimize fund returns with respect to a passive benchmark. Because mutual funds are highly regulated entities that are required to disclose considerably more information about their holdings than hedge funds, Chandar and Bricker (2002) are able to perform a detailed analysis of the periodic adjustments (both discretionary and nondiscretionary) that fund managers made to the valuation of their restricted securities. Their findings suggest that performance smoothing is even more relevant in the hedge-fund industry which is not nearly as transparent, and that econometric models of smoothed returns can be an important tool for detecting such behavior and unraveling its effects on true economic returns.

3. Other sources of serial correlation

Before turning to our econometric model of smoothed returns in Section 4, we first consider four other potential sources of serial correlation in asset returns: (1) market inefficiencies; (2) time-varying expected returns; (3) time-varying leverage; and (4) incentive fees with high water marks.

Perhaps the most common explanation (at least among industry professionals and certain academics) for the presence of serial correlation in asset returns is a violation of the Efficient Markets Hypothesis, one of the central pillars of modern finance theory. As with so many of the ideas of modern economics, the origins of the Efficient Markets Hypothesis can be traced back to Samuelson (1965), whose contribution is neatly summarized by the title of his article: "Proof that properly anticipated prices fluctuate randomly." In an informationally efficient market, price changes must be unforecastable if they are properly anticipated, i.e., if they fully incorporate the expectations and information of all market participants. Fama (1970) operationalizes this hypothesis, which he summarizes in the well-known epithet "prices fully reflect all available information", by placing structure on various information sets available to market participants. This concept of informational efficiency has a wonderfully counter-intuitive and seemingly contradictory flavor to it. The more efficient the market, the more random the sequence of price changes generated by such a market, and the most efficient market of all is one in which price changes are completely random and unpredictable. This, of course, is not an accident of Nature but is the direct result of many active participants attempting to profit from their information. Unable to curtail their greed, an army of investors aggressively pounce on even the smallest informational advantages at their disposal, and in doing so, they incorporate their information into market prices and quickly eliminate the profit opportunities that gave rise to their aggression. If this

occurs instantaneously, which it must in an idealized world of "frictionless" markets and costless trading, then prices must always fully reflect all available information and no profits can be garnered from information-based trading (because such profits have already been captured).

In the context of hedge-fund returns, one interpretation of the presence of serial correlation is that the hedge-fund manager is not taking full advantage of the information or "alpha" contained in the manager's strategy. For example, if a manager's returns are highly positively autocorrelated, then it should be possible for the manager to improve performance by exploiting this fact. For example, in months where performance is good, the manager should increase bets in anticipation of continued good performance (due to positive serial correlation), and in months where performance is poor, the manager should reduce bets accordingly. The reverse argument can be made for the case of negative serial correlation. By taking advantage of serial correlation of either sign in his returns, the hedge-fund manager will eventually eliminate it along the lines of Samuelson (1965), i.e., properly anticipated hedge-fund returns should fluctuate randomly. And if this self-correcting mechanism of the Efficient Markets Hypothesis is at work among any group of investors in the financial community, it surely must be at work among hedge-fund managers, which consists of a highly trained, highly motivated, and highly competitive group of sophisticated investment professionals.

Of course, the natural counterargument to this somewhat naive application of the Efficient Markets Hypothesis is that hedge-fund managers cannot fully exploit such serial correlation because of transactions costs and liquidity constraints. But once again, this leads to the main thesis of this article: serial correlation is a proxy for illiquidity and smoothed returns.

There are, however, at least three additional explanations for the presence of serial correlation. One of the central tenets of modern financial economics is the necessity of some trade-off between risk and expected return, hence serial correlation may not be exploitable in the sense that an attempt to take advantage of predictabilities in fund returns can be offset by corresponding changes in risk, leaving the fund manager indifferent at the margin between the current investment policy and other alternatives. Specifically, Leroy (1973), Rubinstein (1976), and Lucas (1978) demonstrate conclusively that serial correlation in asset returns need not be the result of market inefficiencies, but could be the result of time-varying expected returns, which is perfectly consistent with the Efficient Markets Hypothesis.[2] If an investment strategy's required expected return varies through time (because of

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[2] Grossman (1976) and Grossman and Stiglitz (1980) go even further. They argue that perfectly informationally efficient markets are an *impossibility*, for if markets are perfectly efficient, the return to gathering information is nil, in which case there would be little reason to trade and markets would eventually collapse. Alternatively, the degree of market *inefficiency* determines the effort investors are willing to expend to gather and trade on information, hence a nondegenerate market equilibrium will arise only when there are sufficient profit opportunities, i.e., inefficiencies, to compensate investors for the costs of trading and information-gathering. The profits earned by these attentive investors can be viewed as economic rents that accrue to those willing to engage in such activities. Who are the providers of these rents? Black (1986) gives a provocative answer in proposing that they are "noise traders", individuals who trade on what they think is information but is in fact merely noise.
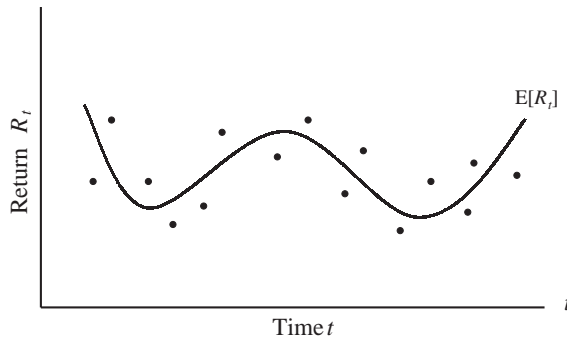
Fig. 1. Time-varying expected returns can induce serial correlation in asset returns.

changes in its risk exposures, for example), then serial correlation can be induced in realized returns without implying any violation of market efficiency (see Fig. 1). We examine this possibility more formally in Section 3.1.

Another possible source of serial correlation in hedge-fund returns is time-varying leverage. If managers change the degree to which they leverage their investment strategies, and if these changes occur in response to lagged market conditions, this is tantamount to time-varying expected returns. We consider this case in Section 3.2.

Finally, we investigate one more potential explanation for serial correlation: the compensation structure of the typical hedge fund. Because most hedge funds charge an incentive fee coupled with a high-water mark that must be surpassed before incentive fees are paid, this path dependence in the computation for net-of-fee returns can induce serial correlation. We develop a formal model of this phenomenon in Section 3.3.

The analysis of Sections 3.1–3.3 shows that time-varying expected returns, time-varying leverage, and incentive fees with high-water marks can all generate serial correlation in hedge-fund returns, but none of these effects can plausibly generate serial correlation to the degree observed in the data, e.g., 30–50% for monthly returns. Therefore, illiquidity and smoothed returns are more likely sources of serial correlation in hedge-fund returns.

3.1. Time-varying expected returns

Let R_t denote a hedge fund's return in month t, and suppose that its dynamics are given by the following time-series process:

$$R_t = \mu_1 I_t + \mu_0 (1 - I_t) + \varepsilon_t, \tag{1}$$

where ε_t is assumed to be independently and identically distributed (IID) with mean 0 and variance σ_ε^2, and I_t is a two-state Markov process with transition matrix

$$P \equiv \begin{array}{c} I_t = 1 \\ I_t = 0 \end{array} \begin{array}{cc} I_{t+1} = 1 & I_{t+1} = 0 \\ \begin{pmatrix} p & 1 - p \\ 1 - q & q \end{pmatrix} \end{array} \tag{2}$$

and μ_0 and μ_1 are the equilibrium expected returns of fund i in states 0 and 1, respectively. This is a particularly simple model of time-varying expected returns in which we abstract from the underlying structure of the economy that gives rise to Eq. (1), but focus instead on the serial correlation induced by the Markov regime-switching process Eq. (2).[3] In particular, observe that

$$P^k = \frac{1}{2-p-q}\begin{pmatrix} 1-q & 1-p \\ 1-q & 1-p \end{pmatrix} + \frac{(p+q-1)^k}{2-p-q}\begin{pmatrix} 1-p & -(1-p) \\ -(1-q) & 1-q \end{pmatrix} \tag{3}$$

assuming that $|p+q-1|<1$, hence the steady-state probabilities and moments for the regime-switching process I_t are

$$P^\infty = \begin{pmatrix} \pi_1 \\ \pi_0 \end{pmatrix} = \begin{pmatrix} \dfrac{1-q}{2-p-q} \\[2ex] \dfrac{1-p}{2-p-q} \end{pmatrix}, \tag{4}$$

$$\mathrm{E}[I_t] = \frac{1-q}{2-p-q} \tag{5}$$

and

$$\mathrm{Var}[I_t] = \frac{(1-p)(1-q)}{(2-p-q)^2}. \tag{6}$$

These, in turn, imply the following moments for R_t:

$$\mathrm{E}[R_t] = \mu_1 \frac{1-q}{2-p-q} + \mu_0 \frac{1-p}{2-p-q}, \tag{7}$$

$$\mathrm{Var}[R_t] = (\mu_1 - \mu_0)^2 \frac{(1-p)(1-q)}{(2-p-q)^2} + \sigma_{\varepsilon_i}^2, \tag{8}$$

$$\rho_k \equiv \mathrm{Corr}[R_{t-k}, R_t] = \frac{(p+q-1)^k}{1 + \sigma_\varepsilon^2 / \left[(\mu_1 - \mu_0)^2 \frac{(1-p)(1-q)}{(2-p-q)^2}\right]}. \tag{9}$$

By calibrating the parameters μ_1, μ_0, p, q, and σ_ε^2 to empirically plausible values, we can compute the serial correlation induced by time-varying expected returns using Eq. (9).

Observe from Eq. (9) that the serial correlation of returns depends on the squared difference of expected returns, $(\mu_1 - \mu_0)^2$, not on the particular values in either regime. Moreover, the absolute magnitudes of the autocorrelation coefficients ρ_k are monotonically increasing in $(\mu_1 - \mu_0)^2$, i.e., the larger the difference in expected returns between the two states, the more serial correlation is induced. Therefore, we begin our calibration exercise by considering an extreme case where $|\mu_1 - \mu_0|$ is 5% per month, or 60% per year, which yields rather dramatic shifts in regimes. To

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[3] Cecchetti and Mark (1990), Goodwin (1993), Hamilton (1989, 1990, 1996), Kandel and Stambaugh (1991), and Turner et al. (1989) provide examples of dynamic general equilibrium models that yield a Markov-switching process for asset prices, and econometric methods to estimate such processes.

complete the calibration exercise, we fix the unconditional variance of returns at a particular value, say $(20\%)^2/12$ (which is comparable to the volatility of the S&P 500 over the past 30 years), vary p and q, and solve for the values of σ_ε^2 that are consistent with the values of p, q, $(\mu_1 - \mu_0)^2$, and the unconditional variance of returns.

Panel A of Table 1 reports the first-order autocorrelation coefficients for various values of p and q under these assumptions, and we see that even in this most extreme case, the largest absolute magnitude of serial correlation is only 15%. Panel B of Table 1 shows that when the unconditional variance of returns is increased from 20% to 50% per year, the correlations decline in magnitude with the largest absolute correlation of 2.4%. And Panel C illustrates the kind of extreme parameter values needed to obtain autocorrelations that are empirically relevant for hedge-fund returns. These values show a difference in expected returns of 20% per month or 240% per year, and probabilities p and q that are either both 80% or higher, or both 20% or lower. Given the implausibility of these parameter values, we conclude that time-varying expected returns (at least of this form) are not the most likely explanation for serial correlation in hedge-fund returns.

3.2. Time-varying leverage

Another possible source of serial correlation in hedge-fund returns is time-varying leverage. Since leverage directly affects the expected return of any investment strategy, this can be considered a special case of the time-varying expected returns model of Section 3.1. Specifically, if L_t denotes a hedge fund's leverage ratio, then the actual return R_t^o of the fund at date t is given by

$$R_t^o = L_t R_t, \tag{10}$$

where R_t is the fund's unlevered return.[4] For example if a fund's unlevered strategy yields a 2% return in a given month, but 50% of the funds are borrowed from various counterparties at fixed borrowing rates, the return to the fund's investors is approximately 4%, hence the leverage ratio is two (assuming that the borrowing rate is zero for simplicity).

The specific mechanisms by which a hedge fund determines its leverage can be quite complex and often depend on a number of factors including market volatility, credit risk, and various constraints imposed by investors, regulatory bodies, banks, brokers, and other counterparties. But the basic motivation for typical leverage dynamics is the well-known trade-off between risk and expected return. By increasing its leverage ratio, a hedge fund boosts its expected returns proportionally, but also increases its return volatility and, eventually, its credit risk or risk of default. Therefore, counterparties providing credit facilities for hedge funds will impose some ceiling on the degree of leverage they are willing to provide. More importantly, as

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[4] For simplicity, and with little loss in generality, we have ignored the borrowing costs associated with leverage in our specification of Eq. (10). Although including such costs will obviously reduce the net return, the serial correlation properties will be largely unaffected because the time variation in borrowing rates is not significant relative to R_t and L_t.

Table 1

First-order autocorrelations of returns from a two-state Markov model of time-varying expected returns

This table shows first-order autocorrelation coefficients of returns from a two-state Markov model of time-varying expected returns, $R_t = \mu_1 I_t + \mu_0(1 - I_t) + \varepsilon_t$, where $p \equiv \text{Prob}(I_{t+1} = 1 \mid I_t = 1)$, $q \equiv \text{Prob}(I_{t+1} = 0 \mid I_t = 0)$, μ_1 and μ_0 are the monthly expected returns in states 1 and 0, respectively, and $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ and σ_ε^2 is calibrated to fix the unconditional variance $\text{Var}[R_t]$ of returns at a prespecified level.

ρ_1 (%)		q (%)								
		10	20	30	40	50	60	70	80	90
Panel A: $\|\mu_1 - \mu_0\| = 5\%$, $\text{Var}[R_t] = (20\%)^2/12$										
	10	−15.0	−13.1	−11.1	−9.0	−6.9	−4.8	−2.8	−1.1	0.0
	20	−13.1	−11.3	−9.3	−7.3	−5.3	−3.3	−1.5	0.0	0.7
	30	−11.1	−9.3	−7.5	−5.6	−3.6	−1.7	0.0	1.3	1.6
	40	−9.0	−7.3	−5.6	−3.8	−1.9	0.0	1.7	2.8	2.8
p (%)	50	−6.9	−5.3	−3.6	−1.9	0.0	1.9	3.5	4.6	4.2
	60	−4.8	−3.3	−1.7	0.0	1.9	3.8	5.5	6.7	6.0
	70	−2.8	−1.5	0.0	1.7	3.5	5.5	7.5	9.0	8.4
	80	−1.1	0.0	1.3	2.8	4.6	6.7	9.0	11.3	11.7
	90	0.0	0.7	1.6	2.8	4.2	6.0	8.4	11.7	15.0
Panel B: $\|\mu_1 - \mu_0\| = 5\%$, $\text{Var}[R_t] = (50\%)^2/12$										
	10	−2.4	−2.1	−1.8	−1.4	−1.1	−0.8	−0.5	−0.2	0.0
	20	−2.1	−1.8	−1.5	−1.2	−0.9	−0.5	−0.2	0.0	0.1
	30	−1.8	−1.5	−1.2	−0.9	−0.6	−0.3	0.0	0.2	0.3
	40	−1.4	−1.2	−0.9	−0.6	−0.3	0.0	0.3	0.5	0.4
p (%)	50	−1.1	−0.9	−0.6	−0.3	0.0	0.3	0.6	0.7	0.7
	60	−0.8	−0.5	−0.3	0.0	0.3	0.6	0.9	1.1	1.0
	70	−0.5	−0.2	0.0	0.3	0.6	0.9	1.2	1.4	1.4
	80	−0.2	0.0	0.2	0.5	0.7	1.1	1.4	1.8	1.9
	90	0.0	0.1	0.3	0.4	0.7	1.0	1.4	1.9	2.4
Panel C: $\|\mu_1 - \mu_0\| = 20\%$, $\text{Var}[R_t] = (50\%)^2/12$										
	10	−38.4	−33.5	−28.4	−23.0	−17.6	−12.3	−7.2	−2.9	0.0
	20	−33.5	−28.8	−23.9	−18.8	−13.6	−8.5	−3.8	0.0	1.9
	30	−28.4	−23.9	−19.2	−14.3	−9.3	−4.4	0.0	3.3	4.2
	40	−23.0	−18.8	−14.3	−9.6	−4.8	0.0	4.3	7.2	7.1
p (%)	50	−17.6	−13.6	−9.3	−4.8	0.0	4.7	9.0	11.8	10.7
	60	−12.3	−8.5	−4.4	0.0	4.7	9.6	14.1	17.1	15.4
	70	−7.2	−3.8	0.0	4.3	9.0	14.1	19.2	23.0	21.6
	80	−2.9	0.0	3.3	7.2	11.8	17.1	23.0	28.8	29.9
	90	0.0	1.9	4.2	7.1	10.7	15.4	21.6	29.9	38.4

market prices move against a hedge fund's portfolio, thereby reducing the value of the fund's collateral and increasing its leverage ratio, or as markets become more volatile and the fund's risk exposure increases significantly, creditors (and, in some cases, securities regulations) will require the fund to either post additional collateral or liquidate a portion of its portfolio to bring the leverage ratio back down to an acceptable level. As a result, the leverage ratio of a typical hedge fund varies through time in a specific manner, usually as a function of market prices and market

volatility. Therefore we propose a simple data-dependent mechanism through which a hedge fund determines its ideal leverage ratio.

Denote by R_t the return of a fund in the absence of any leverage, and to focus squarely on the ability of leverage to generate serial correlation, let R_t be IID through time, hence:

$$R_t = \mu + \varepsilon_t, \quad \varepsilon_t \text{ IID} \mathcal{N}(0, \sigma_\varepsilon^2), \tag{11}$$

where we have assumed that ε_t is normally distributed only for expositional convenience (other distributions can easily be used instead of the normal in the Monte Carlo simulation experiment described below). Given Eq. (10), the kth order autocorrelation coefficient of leveraged returns R_t^o is

$$\rho_k = \frac{1}{\text{Var}[R_t^o]} \big[\mu^2 \, \text{Cov}[L_t, L_{t+k}] + \mu \, \text{Cov}[L_t, L_{t+k}\varepsilon_{t+k}]$$
$$+ \mu \, \text{Cov}[L_{t+k}, L_t\varepsilon_t] + \text{Cov}[L_t\varepsilon_t, L_{t+k}\varepsilon_{t+k}] \big]. \tag{12}$$

Now suppose that the leverage process L_t is independently distributed through time and also independent of ε_{t+k} for all k. Then Eq. (12) implies that $\rho_k = 0$ for all $k \neq 0$, hence time-varying leverage of this sort will not induce any serial correlation in returns R_t^o.

However, as discussed above, leverage is typically a function of market conditions, which can induce serial dependence in L_t and dependence between L_{t+k} and ε_t for $k \geqslant 0$, yielding serially correlated observed returns R_t^o. To see how, we propose a simple but realistic mechanism by which a hedge fund could manage its leverage ratio. Suppose that, as part of its enterprise-wide, risk-management protocol, a fund adopts a policy of limiting the 95% Value-at-Risk of its portfolio to no worse than δ. For example, if $\delta = -10\%$, this policy requires managing the portfolio so that the probability of a loss greater than or equal to 10% of the fund's assets is at most 5%. If we assume that the only control variable available to the manager is the leverage ratio L_t and that unleveraged returns R_t are given by Eq. (11), this implies the following constraint on leverage:

$$\text{Prob}(R_t^o \leqslant \delta) \leqslant 5\%, \quad \delta \leqslant 0,$$

$$\text{Prob}(L_t R_t \leqslant \delta) \leqslant 5\%,$$

$$\text{Prob}\left(\frac{R_t - \mu}{\sigma} \leqslant \frac{\delta/L_t - \mu}{\sigma}\right) \leqslant 5\%,$$

$$\Phi\left(\frac{\delta/L_t - \mu}{\sigma}\right) \leqslant 5\%, \tag{13}$$

$$\delta/L_t \leqslant \sigma \Phi^{-1}(5\%), \tag{14}$$

$$\Rightarrow \ L_t \leqslant \frac{\delta}{\sigma \Phi^{-1}(5\%)}, \tag{15}$$

where, following common industry practice, we have set $\mu = 0$ in Eq. (13) to arrive at Eq. (14).[5] Now in implementing the constraint Eq. (15), the manager must estimate the portfolio volatility σ, which is typically estimated using some rolling window of historical data, hence the manager's estimate is likely to be time-varying but persistent to some degree. This persistence, and the dependence of the volatility estimate on past returns, will both induce serial correlation in observed returns R_t^o. Specifically, let

$$\hat{\sigma}_t^2 \equiv \frac{1}{n} \sum_{k=1}^{n} (R_{t-k} - \hat{\mu})^2, \quad \hat{\mu} \equiv \frac{1}{n} \sum_{k=1}^{n} R_{t-k} \tag{16}$$

and

$$L_t = \frac{\delta}{\hat{\sigma}_t \Phi^{-1}(5\%)}, \tag{17}$$

where we have assumed that the manager sets his leverage ratio L_t to the maximum allowable level subject to the VaR constraint Eq. (15).

To derive the impact of this heuristic risk management policy on the serial correlation of observed returns, we perform a Monte Carlo simulation experiment where we simulate a time series of 100,000 returns $\{R_t\}$ and implement the leverage policy Eq. (17) to obtain a time series of observed returns $\{R_t^o\}$, from which we compute its autocorrelation coefficients $\{\rho_k\}$. Given the large sample size, our estimate should yield an excellent approximation to the population values of the autocorrelation coefficients. This procedure is performed for the following combinations of parameter values: $n = 3, 6, 9, 12, 24, 36, 48, 60$; $12\mu = 5\%$; $\sqrt{12}\,\sigma = 10\%, 20\%, 50\%$, and $\delta = -25\%$, and Table 2 summarizes the results. Note that the autocorrelation of observed returns Eq. (12) is homogeneous of degree 0 in δ, hence we need only simulate our return process for one value of δ without loss of generality as far as ρ_k is concerned. Of course, the mean and standard of observed returns and leverage will be affected by our choice of δ, but because these variables are homogeneous of degree 1, we can obtain results for any arbitrary δ simply by rescaling our results for $\delta = -25\%$.

For a VaR constraint of -25% and an annual standard deviation of unlevered returns of 10%, the mean leverage ratio ranges from 9.52 when $n = 3$ to 4.51 when $n = 60$. For small n, there is considerably more sampling variation in the estimated standard deviation of returns, hence the leverage ratio (which is proportional to the reciprocal of $\hat{\sigma}_t$) takes on more extreme values as well and has a higher expectation in this case.

As n increases, the volatility estimator becomes more stable over time since each month's estimator has more data in common with the previous month's estimator, leading to more persistence in L_t as expected. For example, when $n = 3$,

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[5] Setting the expected return of a portfolio equal to 0 for purposes of risk management is often motivated by a desire to be conservative. Most portfolios will tend to have positive expected return, hence setting μ equal to 0 will generally yield larger values for VaR. However, for actively managed portfolios that contain both long and short positions, Lo (2002) shows that the practice of setting expected returns equal to 0 need not be conservative, but in some cases, can yield severely downward-biased estimates of VaR.

Table 2

Monte Carlo simulation results for time-varying leverage model with a VaR constraint

Each row corresponds to a separate and independent simulation of 100,000 observations of independently and identically distributed $\mathcal{N}(\mu, \sigma^2)$ returns R_t which are multiplied by a time-varying leverage factor L_t to generated observed returns $R_t^o \equiv L_t R_t$.

n	Return R_t^o		Leverage L_t		Return R_t^o			Leverage L_t		
	12 Mean (%)	$\sqrt{12}$ SD (%)	Mean	SD	ρ_1 (%)	ρ_2 (%)	ρ_3 (%)	ρ_1 (%)	ρ_2 (%)	ρ_3 (%)
Panel A: $12\mu = 5\%$, $\sqrt{12}\,\sigma = 10\%$, $\delta = -25\%$										
3	50.53	191.76	9.52	15.14	0.7	0.3	0.4	17.5	2.9	0.0
6	29.71	62.61	5.73	2.45	0.1	0.4	0.3	70.6	48.5	32.1
12	24.34	51.07	4.96	1.19	0.1	0.4	−0.3	88.9	78.6	68.8
24	24.29	47.27	4.66	0.71	0.3	0.1	−0.2	95.0	90.0	85.1
36	21.46	46.20	4.57	0.57	−0.2	0.0	0.1	96.9	93.9	90.9
48	22.67	45.61	4.54	0.46	0.3	−0.5	0.3	97.6	95.3	92.9
60	22.22	45.38	4.51	0.43	−0.2	0.0	0.2	98.2	96.5	94.7
Panel B: $12\mu = 5\%$, $\sqrt{12}\,\sigma = 20\%$, $\delta = -25\%$										
3	26.13	183.78	4.80	8.02	0.0	−0.1	0.0	13.4	1.9	−0.6
6	14.26	62.55	2.87	1.19	0.2	0.1	0.4	70.7	48.6	32.0
12	12.95	50.99	2.48	0.59	0.2	−0.1	0.1	89.1	79.0	69.4
24	11.58	47.22	2.33	0.36	0.2	0.0	0.1	95.2	90.4	85.8
36	11.23	46.14	2.29	0.28	−0.1	0.3	−0.3	97.0	94.0	90.9
48	11.00	45.63	2.27	0.24	0.2	−0.5	−0.1	97.8	95.5	93.3
60	12.18	45.37	2.26	0.21	0.1	−0.1	0.4	98.3	96.5	94.8
Panel C: $12\mu = 5\%$, $\sqrt{12}\,\sigma = 50\%$, $\delta = -25\%$										
3	9.68	186.59	1.93	3.42	−1.1	0.0	−0.5	14.7	1.8	−0.1
6	6.25	62.43	1.16	0.48	−0.2	0.3	−0.2	70.9	49.4	32.9
12	5.90	50.94	0.99	0.23	−0.1	0.1	0.0	89.0	78.6	69.0
24	5.30	47.29	0.93	0.15	0.2	0.3	0.4	95.2	90.5	85.7
36	5.59	46.14	0.92	0.12	−0.1	0.3	−0.2	97.0	94.1	91.1
48	4.07	45.64	0.91	0.10	−0.4	−0.6	0.1	97.8	95.7	93.5
60	5.11	45.34	0.90	0.08	0.4	0.3	−0.3	98.2	96.5	94.7

the average first-order autocorrelation coefficient of L_t is 43.2%, but increases to 98.2% when $n = 60$. However, even with such extreme levels of persistence in L_t, the autocorrelation induced in observed returns R_t^o is still only −0.2%. In fact, the largest absolute return-autocorrelation reported in Table 2 is only 0.7%, despite the fact that leverage ratios are sometimes nearly perfectly autocorrelated from month to month. This suggests that time-varying leverage, at least of the form described by the VaR constraint Eq. (15), cannot fully account for the magnitudes of serial correlation in historical hedge-fund returns.

3.3. Incentive fees with high-water marks

Yet another source of serial correlation in hedge-fund returns is an aspect of the fee structure commonly used in the hedge-fund industry, called an "incentive fee",

which is typically 20% of excess returns above a benchmark and subject to a high-water mark. This means that incentive fees are paid only if the cumulative returns of the fund are above water, i.e., if they exceed the *cumulative* return of the benchmark since inception. Bhattacharya and Pfleiderer (1985), Brown et al. (2002), Carpenter (2000), Carpenter et al. (2001), Elton and Gruber (2003), and Goetzmann et al. (1997) provide more detailed analyses of high-water marks and other incentive-fee arrangements in the context of delegated portfolio management. This type of nonlinearity can induce serial correlation in net-of-fee returns because of the path dependence inherent in the definition of the high-water mark. When the fund is below water the incentive fee is not charged, but over time, as the fund's cumulative performance rises above water, the incentive fee is reinstated, and the net-of-fee returns are reduced accordingly.

Specifically, denote by F_t the incentive fee paid to the manager in period t and for simplicity, set the benchmark to 0. Then

$$F_t \equiv \text{Max}\,[0, \gamma(X_{t-1} + R_t)], \quad \gamma > 0, \tag{18a}$$

$$X_t \equiv \text{Min}\,[0, X_{t-1} + R_t], \tag{18b}$$

where X_t is a state variable that is nonzero only when the manager is under water, in which case it measures the cumulative losses that must be recovered before an incentive fee is paid. The net-of-fee returns R_t^o are then given by

$$R_t^o = R_t - F_t = (1 - \gamma)R_t + \gamma(X_t - X_{t-1}) \tag{19}$$

which is clearly serially correlated due to the presence of the lagged state variable X_{t-1}.[6]

Because the high-water mark variable X_t is a nonlinear recursive function of X_{t-1} and R_t, its statistical properties are quite complex and difficult to derive in closed form. Therefore, we perform a Monte Carlo simulation experiment in which we simulate a time series of returns $\{R_t\}$ of length $T = 100{,}000$ where R_t is given by Eq. (11), compute the net-of-fee returns $\{R_t^o\}$, and estimate the first-order autocorrelation coefficient ρ_1. We follow this procedure for each of the combinations of the following parameter values: $12\,\mu = 5\%$, 10%, 15%, $\ldots$, 50%; $\sqrt{12}\,\sigma = 10\%$, 20%, $\ldots$, 50%, and $\gamma = 20\%$.

Table 3 summarizes the results of the simulations which show that although incentive fees with high-water marks do induce some serial correlation in net-of-fee returns, they are generally quite small in absolute value. For example, the largest absolute value of all the entries in Table 3 is only 4.4%. Moreover, all of the averages are negative, a result of the fact that all of the serial correlation in R_t^o is due to the first difference of X_t in Eq. (19). This implies that incentive fees with high-water

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[6]This is a simplified model of how a typical hedge fund's incentive fee is structured. In particular, Eqs. (18a) and (18b) ignore the fact that incentive fees are usually paid on an annual or quarterly basis whereas high-water marks are tracked on a monthly basis. Using the more realistic fee cycle did not have significant impact on our simulation results, hence we use Eqs. (18a) and (18b) for expositional simplicity. Also, some funds do pay their employees and partners monthly incentive compensation, in which case Eqs. (18a) and (18b) are the exact specification of their fee structure.

Table 3

First-order autocorrelation coefficients for Monte Carlo simulation of net-of-fee returns under an incentive fee with a high water mark

Each entry corresponds to a separate and independent simulation of 100,000 observations of independently and identically distributed $\mathcal{N}(\mu, \sigma^2)$ returns R_t, from which a 20% incentive fee $F_t \equiv$ Max$[0, 0.2 \times (X_{t-1} + R_t)]$ is subtracted each period to yield net-of-fee returns $R_t^o \equiv R_t - F_t$, where $X_t \equiv$ Min$[0, X_{t-1} + R_t]$ is a state variable that is non-zero only when the fund is under water, in which case it measures the cumulative losses that must be recovered before an incentive fee is paid.

ρ_1 (%)		$12\,\mu$ (%)									
		5	10	15	20	25	30	35	40	45	50
	10	−1.4	−2.5	−3.2	−3.4	−3.4	−3.2	−2.9	−2.4	−2.0	−1.5
	20	−1.6	−2.3	−2.9	−3.4	−3.8	−4.1	−4.3	−4.4	−4.4	−4.3
$\sigma \times \sqrt{12}$	30	−0.6	−1.1	−1.6	−2.1	−2.4	−2.8	−3.0	−3.3	−3.5	−3.6
(%)	40	−0.2	−0.7	−1.1	−1.4	−1.8	−2.1	−2.3	−2.6	−2.8	−3.0
	50	0.0	−0.3	−0.6	−0.9	−1.2	−1.5	−1.7	−1.9	−2.1	−2.3

marks are even less likely to explain the large positive serial correlation in historical hedge-fund returns.

4. An econometric model of smoothed returns

Having shown in Section 3 that other possible sources of serial correlation in hedge-fund returns are hard-pressed to yield empirically plausible levels of autocorrelation, we now turn to illiquidity and smoothed returns, which are the main focus of this study. Although illiquidity and smoothed returns are two distinct phenomena, it is important to consider them in tandem because one facilitates the other. For actively traded securities, both theory and empirical evidence suggest that in the absence of transactions costs and other market frictions, returns are unlikely to be very smooth.

As we argued in Section 1, nonsynchronous trading is a plausible source of serial correlation in hedge-fund returns. In contrast to the studies by Lo and MacKinlay (1988, 1990) and Kadlec and Patterson (1999) in which they conclude that it is difficult to generate serial correlations in weekly US equity portfolio returns much greater than 10% to 15% through nonsynchronous trading effects alone, we argue that in the context of hedge funds, significantly higher levels of serial correlation can be explained by the combination of illiquidity and smoothed returns, of which nonsynchronous trading is a special case. To see why, note that the empirical analysis in the nonsynchronous-trading literature is devoted exclusively to exchange-traded equity returns, not hedge-fund returns, hence their conclusions are not relevant in our context. For example, Lo and MacKinlay (1990) argue that securities would have to go without trading for several days on average to induce serial correlations of 30%, and they dismiss such nontrading intervals as unrealistic for most exchange-traded US equity issues. However, such nontrading intervals are considerably more realistic for the types of securities held by many hedge funds, e.g.,

emerging-market debt, real estate, restricted securities, control positions in publicly traded companies, asset-backed securities, and other exotic, over-the-counter (OTC) derivatives. Therefore, nonsynchronous trading of this magnitude is one plausible source of serial correlation observed in hedge-fund returns.

But even when prices are synchronously measured—as they are for many funds that mark their portfolios to market at the end of the month to strike a net-asset-value at which investors can buy into or cash out of the fund—there are several other channels by which illiquidity exposure can induce serial correlation in the reported returns of hedge funds. Apart from the nonsynchronous-trading effect, naive methods for determining the fair market value or ''marks'' for illiquid securities can yield serially correlated returns. For example, one approach to valuing illiquid securities is to extrapolate linearly from the most recent transaction price (which, in the case of emerging-market debt, could be several months ago), which yields a price path that is a straight line, or at best a series of straight lines. Returns computed from such marks will be smoother, exhibiting lower volatility and higher serial correlation than true economic returns, i.e., returns computed from mark-to-market prices where the market is sufficiently active to allow all available information to be impounded in the price of the security. Of course, for securities more easily traded and with deeper markets, mark-to-market prices are more readily available, extrapolated marks are not necessary, and serial correlation is therefore less of an issue. But for securities that are thinly traded, or not traded at all for extended periods of time, marking them to market is often an expensive and time-consuming procedure that cannot easily be performed frequently. Therefore, we argue in this paper that serial correlation serves as a proxy for a fund's liquidity exposure.

Even if a hedge-fund manager does not make use of any form of linear extrapolation to mark the portfolio's securities, the manager could still be subject to smoothed returns if the manager obtains marks from broker–dealers that engage in such extrapolation. For example, consider the case of a conscientious hedge-fund manager attempting to obtain the most accurate mark for a portfolio at month's end by getting bid/offer quotes from three independent broker–dealers for every security in such a portfolio, and then marking each security at the average of the three quote midpoints. By averaging the quote midpoints, the manager is inadvertently downward-biasing price volatility, and if any of the broker–dealers employ linear extrapolation in formulating their quotes (and many do, through sheer necessity because they have little else to go on for the most illiquid securities), or if they fail to update their quotes because of light volume, serial correlation will also be induced in reported returns.

Finally, a more prosaic channel by which serial correlation can arise in the reported returns of hedge funds is through ''performance smoothing'', which is the unsavory practice of reporting only part of the gains in months when a fund has positive returns so as to partially offset potential future losses and thereby reduce volatility and improve risk-adjusted performance measures such as the Sharpe ratio. For funds containing liquid securities easily marked to market, performance smoothing is more difficult and, as a result, less of a concern. Indeed, it is only for

portfolios of illiquid securities that managers and brokers have any discretion in marking their positions. Such practices are generally prohibited by various securities laws and accounting principles, and great care must be exercised in interpreting smoothed returns as deliberate attempts to manipulate performance statistics. After all, as we have discussed above, there are many other sources of serial correlation in the presence of illiquidity, none of which is motivated by deceit. Nevertheless, managers do have certain degrees of freedom in valuing illiquid securities (for example, discretionary accruals for unregistered private placements and venture capital investments), and Chandar and Bricker (2002) conclude that managers of certain closed-end mutual funds do use accounting discretion to manage fund returns around a passive benchmark. Therefore, the possibility of deliberate performance smoothing in the less regulated hedge-fund industry must be kept in mind in interpreting our empirical analysis of smoothed returns.

To quantify the impact of all of these possible sources of serial correlation, denote by R_t the true economic return of a hedge fund in period t, and let R_t satisfy the following linear single-factor model:

$$R_t = \mu + \beta \Lambda_t + \varepsilon_t, \quad \mathrm{E}[\Lambda_t] = \mathrm{E}[\varepsilon_t] = 0, \quad \varepsilon_t, \Lambda_t \sim \mathrm{IID}, \tag{20a}$$

$$\mathrm{Var}[R_t] \equiv \sigma^2. \tag{20b}$$

True returns represent the flow of information that would determine the equilibrium value of the fund's securities in a frictionless market. However, true economic returns are not observed. Instead, R_t^o denotes the reported or observed return in period t, and let

$$R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \cdots + \theta_k R_{t-k}, \tag{21}$$

$$\theta_j \in [0, 1], \quad j = 0, \ldots, k \tag{22}$$

and

$$1 = \theta_0 + \theta_1 + \cdots + \theta_k \tag{23}$$

which is a weighted average of the fund's true returns over the most recent $k + 1$ periods, including the current period.

This averaging process captures the essence of smoothed returns in several respects. From the perspective of illiquidity-driven smoothing, Eq. (21) is consistent with several models in the nonsynchronous trading literature. For example, Cohen et al. (1986, Chapter 6.1) propose a similar weighted-average model for observed returns. In particular, their specification for observed returns is: $r_{j,t}^o = \sum_{l=0}^{N}(\gamma_{j,t-l,l} r_{j,t-l} + \theta_{j,t-l})$ where $r_{j,t-l}$ is the true but unobserved return for security j in period $t - l$, the coefficients $\{\gamma_{j,t-l,l}\}$ are assumed to sum to 1, and $\theta_{j,t-l}$ are random variables meant to capture bid/ask bounce. The authors motivate their specification of nonsynchronous trading in the following way (p. 116): "Alternatively stated, the $\gamma_{j,t,0}, \gamma_{j,t,1}, \ldots, \gamma_{j,t,N}$ comprise a delay distribution that shows how the true return generated in period t impacts on the returns actually observed during t and the next N periods." In other words, the essential feature of nonsynchronous trading

is the fact that information generated at date t may not be fully impounded into prices until several periods later.

Alternatively, Eq. (21) can be viewed as the outcome of marking portfolios to simple linear extrapolations of acquisition prices when market prices are unavailable, or mark-to-model returns where the pricing model is slowly varying through time. And of course, Eq. (21) also captures the intentional smoothing of performance.

The constraint Eq. (23) that the weights sum to 1 implies that the information driving the fund's performance in period t will eventually be fully reflected in observed returns, but this process could take up to $k + 1$ periods from the time the information is generated.[7] This is a sensible restriction in the current context of hedge funds for several reasons. Even the most illiquid securities will trade eventually, and when that occurs, all of the cumulative information affecting that security will be fully impounded into its transaction price. Therefore the parameter k should be selected to match the kind of illiquidity of the fund. For example, a fund comprised mostly of exchange-traded US equities would require a much lower value of k than a private equity fund. Alternatively, in the case of intentional smoothing of performance, the necessity of periodic external audits of fund performance imposes a finite limit on the extent to which deliberate smoothing can persist.

In fact, if a fund allows investors to invest and withdraw capital only at pre-specified intervals, imposing lock-ups in between, and external audits are conducted at these same pre-specified intervals, then it can be argued that performance smoothing is irrelevant. For example, no investor should be disadvantaged by investing in a fund that offers annual liquidity and engages in annual external audits with which the fund's net-asset-value is determined by a disinterested third party for purposes of redemptions and new investments. However, there are at least two additional concerns that remain. Historical track records are still affected by smoothed returns, and estimates of a fund's liquidity exposure are also affected, both of which are important factors in the typical hedge-fund investor's overall investment process. Moreover, given the apparently unscrupulous role that the auditors at Arthur Andersen played in the Enron affair, there is the further concern of whether third-party auditors are truly objective and free of all conflicts of interest.

4.1. Implications for performance statistics

Given the smoothing mechanism outlined above, we have the following implications for the statistical properties of observed returns:

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[7] In Lo and MacKinlay's (1990) model of nonsynchronous trading, they propose a stochastic non-trading horizon so that observed returns are an infinite-order moving average of past true returns, where the coefficients are stochastic. In that framework, the waiting time for information to become fully impounded into future returns could be arbitrarily long (but with increasingly remote probability).

Proposition 1. *Under Eqs.* (21) *through* (23), *the statistical properties of observed returns are characterized by*

$$\mathrm{E}[R_t^o] = \mu, \tag{24}$$

$$\mathrm{Var}[R_t^o] = c_\sigma^2 \sigma^2 \leqslant \sigma^2, \tag{25}$$

$$\mathrm{SR}^o \equiv \frac{\mathrm{E}[R_t^o]}{\sqrt{\mathrm{Var}[R_t^o]}} = c_s \mathrm{SR} \geqslant \mathrm{SR} \equiv \frac{\mathrm{E}[R_t]}{\sqrt{\mathrm{Var}[R_t]}}, \tag{26}$$

$$\beta_m^o \equiv \frac{\mathrm{Cov}[R_t^o, \Lambda_{t-m}]}{\mathrm{Var}[\Lambda_{t-m}]} = \begin{cases} c_{\beta,m}\beta & \text{if } 0 \leqslant m \leqslant k, \\ 0 & \text{if } m > k, \end{cases} \tag{27}$$

$$\mathrm{Cov}[R_t^o, R_{t-m}^o] = \begin{cases} \left(\sum_{j=0}^{k-m}\theta_j\theta_{j+m}\right)\sigma^2 & \text{if } 0 \leqslant m \leqslant k, \\ 0 & \text{if } m > k, \end{cases} \tag{28}$$

$$\mathrm{Corr}[R_t^o, R_{t-m}^o] = \frac{\mathrm{Cov}[R_t^o, R_{t-m}^o]}{\mathrm{Var}[R_t^o]} = \begin{cases} \dfrac{\sum_{j=0}^{k-m}\theta_j\theta_{j+m}}{\sum_{j=0}^{k}\theta_j^2} & \text{if } 0 \leqslant m \leqslant k, \\ 0 & \text{if } m > k, \end{cases} \tag{29}$$

where

$$c_\mu \equiv \theta_0 + \theta_1 + \cdots + \theta_k, \tag{30}$$

$$c_\sigma^2 \equiv \theta_0^2 + \theta_1^2 + \cdots + \theta_k^2, \tag{31}$$

$$c_s \equiv 1 \Big/ \sqrt{\theta_0^2 + \cdots + \theta_k^2}, \tag{32}$$

$$c_{\beta,m} \equiv \theta_m. \tag{33}$$

Proposition 1 shows that smoothed returns of the form Eqs. (21) through (23) do not affect the expected value of R_t^o, but reduce its variance, hence boosting the Sharpe ratio of observed returns by a factor of c_s. From Eq. (27), we see that smoothing also affects β_0^o, the contemporaneous market beta of observed returns, biasing it towards 0 or market neutrality, and induces correlation between current observed returns and lagged market returns up to lag k. This provides a formal interpretation of the empirical analysis of Asness et al. (2001) in which many hedge funds are found to have significant lagged market exposure despite relatively low contemporaneous market betas.

Smoothed returns also exhibit positive serial correlation up to order k according to Eq. (29), and the magnitude of the effect is determined by the pattern of weights $\{\theta_j\}$. If, for example, the weights are disproportionately centered on a small number of lags, relatively little serial correlation will be induced. However, if the weights are evenly distributed among many lags, this will result in higher serial correlation.

A useful summary statistic for measuring the concentration of weights is

$$\xi \equiv \sum_{j=0}^{k} \theta_j^2 \in [0, 1] \tag{34}$$

which is simply the denominator of Eq. (29). This measure is well known in the industrial organization literature as the *Herfindahl index*, a measure of the concentration of firms in a given industry where θ_j represents the market share of firm j. Because $\theta_j \in [0, 1]$, ξ is also confined to the unit interval, and is minimized when all the θ_j's are identical, which implies a value of $1/(k + 1)$ for ξ, and is maximized when one coefficient is 1 and the rest are 0, in which case $\xi = 1$. In the context of smoothed returns, a lower value of ξ implies more smoothing, and the upper bound of 1 implies no smoothing, hence we shall refer to ξ as a "smoothing index".

In the special case of equal weights, $\theta_j = 1/(k + 1)$ for $j = 0, \ldots, k$, the serial correlation of observed returns takes on a particularly simple form

$$\text{Corr}[R_t^o, R_{t-m}^o] = 1 - \frac{m}{k + 1}, \quad 1 \leqslant m \leqslant k \tag{35}$$

which declines linearly in the lag m. This can yield substantial correlation even when k is small. For example, if $k = 2$ so that smoothing takes place only over the current quarter (i.e., this month and the previous two months), the first-order autocorrelation of monthly observed returns is 66.7%.

To develop a sense for just how much observed returns can differ from true returns under the smoothed-return mechanism Eqs. (21) through (23), denote by $\Delta(T)$ the difference between the cumulative observed and true returns over T holding periods, where we assume that $T > k$

$$\Delta(T) \equiv (R_1^o + R_2^o + \cdots + R_T^o) - (R_1 + R_2 + \cdots + R_T) \tag{36}$$

$$= \sum_{j=0}^{k-1} (R_{-j} - R_{T-j})\left(1 - \sum_{i=0}^{j} \theta_i\right). \tag{37}$$

Then we have:

Proposition 2. *Under Eqs. (21) through (23) and for $T > k$,*

$$E[\Delta(T)] = 0, \tag{38}$$

$$\text{Var}[\Delta(T)] = 2\sigma^2 \sum_{j=0}^{k-1} \left(1 - \sum_{l=0}^{j} \theta_l\right)^2 = 2\sigma^2 \zeta \tag{39}$$

and

$$\zeta \equiv \sum_{j=0}^{k-1} \left(1 - \sum_{l=0}^{j} \theta_l\right)^2 \leqslant k. \tag{40}$$

Proposition 2 shows that the cumulative difference between observed and true returns has 0 expected value, and its variance is bounded above by $2k\sigma^2$.
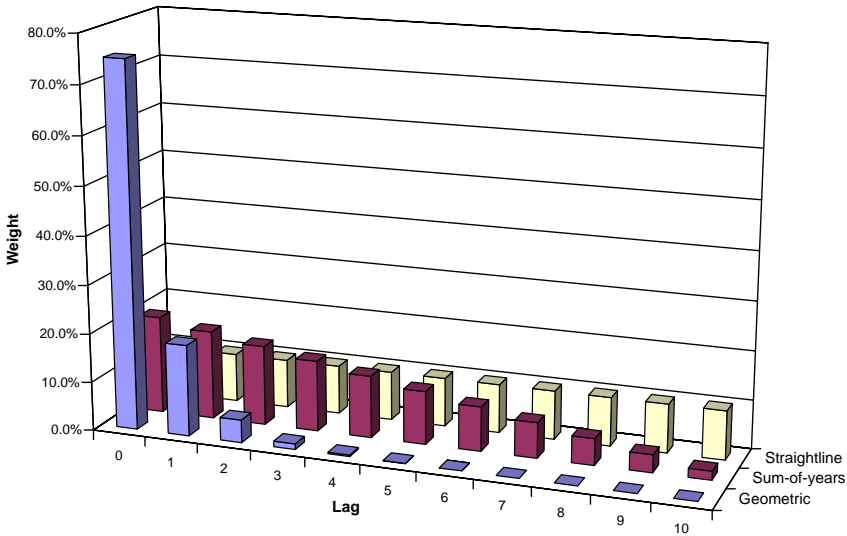
Fig. 2. Straightline, sum-of-years, and geometric smoothing profiles for $k = 10$.

4.2. Examples of smoothing profiles

To develop further intuition for the impact of smoothed returns on observed returns, we consider the following three specific sets of weights $\{\theta_j\}$ or smoothing profiles:[8]

$$\theta_j = \frac{1}{k+1} \quad \text{(straightline)}, \tag{41}$$

$$\theta_j = \frac{k+1-j}{(k+1)(k+2)/2} \quad \text{(sum-of-years)}, \tag{42}$$

$$\theta_j = \frac{\delta^j(1-\delta)}{1-\delta^{k+1}}, \quad \delta \in (0,1) \quad \text{(geometric)}. \tag{43}$$

The straightline profile weights each return equally. In contrast, the sum-of-years and geometric profiles weight the current return the most heavily, and then have monotonically declining weights for lagged returns, with the sum-of-years weights declining linearly and the geometric weights declining more rapidly (see Fig. 2).

More detailed information about the three smoothing profiles is contained in Table 4. The first panel reports the smoothing coefficients $\{\theta_j\}$, constants $c_{\beta,0}$, c_σ, c_s, ζ, and the first three autocorrelations of observed returns for the straightline profile for $k = 0, 1, \ldots, 5$. Consider the case where $k = 2$. Despite the relatively short

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[8] Students of accounting will recognize these profiles as commonly used methods for computing depreciation. The motivation for these depreciation schedules is not entirely without relevance in the smoothed-return context.

Table 4

Implications of three different smoothing profiles for observed betas, standard deviations, Sharpe ratios, and serial correlation coefficients for a fund with IID true returns

Straightline smoothing is given by $\theta_j = 1/(k+1)$; sum-of-years smoothing is given by $\theta_j = (k + 1 - j)/[(k+1)(k+2)/2]$; geometric smoothing is given by $\theta_j = \delta^j(1-\delta)/(1-\delta^{k+1})$. c_β, c_σ, and c_s denote multipliers associated with the beta, standard deviation, and Sharpe ratio of observed returns, respectively, ρ_j^o denotes the jth autocorrelation coefficient of observed returns, and ζ is proportional to the variance of the discrepancy between true and observed multi-period returns.

k	θ_0 (%)	θ_1 (%)	θ_2 (%)	θ_3 (%)	θ_4 (%)	θ_5 (%)	c_β	c_σ	c_s	ρ_1^o (%)	ρ_2^o (%)	ρ_3^o (%)	ρ_4^o (%)	ρ_5^o (%)	ζ (%)
Panel A: straightline smoothing															
0	100.0	—	—	—	—	—	1.00	1.00	1.00	0.0	0.0	0.0	0.0	0.0	—
1	50.0	50.0	—	—	—	—	0.50	0.71	1.41	50.0	0.0	0.0	0.0	0.0	25.0
2	33.3	33.3	33.3	—	—	—	0.33	0.58	1.73	66.7	33.3	0.0	0.0	0.0	55.6
3	25.0	25.0	25.0	25.0	—	—	0.25	0.50	2.00	75.0	50.0	25.0	0.0	0.0	87.5
4	20.0	20.0	20.0	20.0	20.0	—	0.20	0.45	2.24	80.0	60.0	40.0	20.0	0.0	120.0
5	16.7	16.7	16.7	16.7	16.7	16.7	0.17	0.41	2.45	83.3	66.7	50.0	33.3	16.7	152.8
Panel B: sum-of-years smoothing															
0	100.0	—	—	—	—	—	1.00	1.00	1.00	0.0	0.0	0.0	0.0	0.0	—
1	66.7	33.3	—	—	—	—	0.67	0.75	1.34	40.0	0.0	0.0	0.0	0.0	11.1
2	50.0	33.3	16.7	—	—	—	0.50	0.62	1.60	57.1	21.4	0.0	0.0	0.0	27.8
3	40.0	30.0	20.0	10.0	—	—	0.40	0.55	1.83	66.7	36.7	13.3	0.0	0.0	46.0
4	33.3	26.7	20.0	13.3	6.7	—	0.33	0.49	2.02	72.7	47.3	25.5	9.1	0.0	64.9
5	28.6	23.8	19.0	14.3	9.5	4.8	0.29	0.45	2.20	76.9	54.9	35.2	18.7	6.6	84.1
Panel C: geometric smoothing ($\delta = 0.25$)															
0	100.0	—	—	—	—	—	1.00	1.00	1.00	0.0	0.0	0.0	0.0	0.0	—
1	80.0	20.0	—	—	—	—	0.80	0.82	1.21	23.5	0.0	0.0	0.0	0.0	4.0
2	76.2	19.0	4.8	—	—	—	0.76	0.79	1.27	24.9	5.9	0.0	0.0	0.0	5.9
3	75.3	18.8	4.7	1.2	—	—	0.75	0.78	1.29	25.0	6.2	1.5	0.0	0.0	6.5
4	75.1	18.8	4.7	1.2	0.3	—	0.75	0.78	1.29	25.0	6.2	1.6	0.4	0.0	6.6
5	75.0	18.8	4.7	1.2	0.3	0.1	0.75	0.77	1.29	25.0	6.2	1.6	0.4	0.1	6.7
Panel D: geometric smoothing ($\delta = 0.50$)															
0	100.0	—	—	—	—	—	1.00	1.00	1.00	0.0	0.0	0.0	0.0	0.0	—
1	66.7	33.3	—	—	—	—	0.67	0.75	1.34	40.0	0.0	0.0	0.0	0.0	11.1
2	57.1	28.6	14.3	—	—	—	0.57	0.65	1.53	47.6	19.0	0.0	0.0	0.0	20.4
3	53.3	26.7	13.3	6.7	—	—	0.53	0.61	1.63	49.4	23.5	9.4	0.0	0.0	26.2
4	51.6	25.8	12.9	6.5	3.2	—	0.52	0.60	1.68	49.9	24.6	11.7	4.7	0.0	29.6
5	50.8	25.4	12.7	6.3	3.2	1.6	0.51	0.59	1.71	50.0	24.9	12.3	5.9	2.3	31.4

smoothing period of three months, the effects are dramatic. Smoothing reduces the market beta by 67%, increases the Sharpe ratio by 73%, and induces first- and second-order serial correlation of 67% and 33%, respectively, in observed returns. Moreover, the variance of the cumulative discrepancy between observed and true returns, $2\sigma^2\zeta$, is only slightly larger than the variance of monthly true returns σ^2, suggesting that it is difficult to detect this type of smoothed returns even over time.

As k increases, the effects become more pronounced. For $k = 5$, the market beta is reduced by 83%, the Sharpe ratio is increased by 145%, and the first three autocorrelation coefficients are 83%, 67%, and 50%, respectively. However, in this extreme case, the variance of the discrepancy between true and observed returns is approximately three times the monthly variance of true returns, in which case it is easier to identify smoothing from realized returns.

The sum-of-years profile is similar to, although somewhat less extreme than, the straightline profile for the same values of k because more weight is being placed on the current return. For example, even in the extreme case of $k = 5$, the sum-of-years profile reduces the market beta by 71%, increases the Sharpe ratio by 120%, induces autocorrelations of 77%, 55%, and 35%, respectively, in the first three lags, and has a discrepancy variance that is approximately 1.6 times the monthly variance of true returns.

Panels C and D of Table 4 contain results for the geometric smoothing profile for two values of δ, 0.25 and 0.50. For $\delta = 0.25$, the geometric profile places more weight on the current return than the other two smoothing profiles for all values of k, hence the effects tend to be less dramatic. Even in the extreme case of $k = 5$, 75% of current true returns are incorporated into observed returns, the market beta is reduced by only 25%, the Sharpe ratio is increased by only 29%, the first three autocorrelations are 25%, 6%, and 1% respectively, and the discrepancy variance is approximately 13% of the monthly variance of true returns. As δ increases, less weight is placed on the current observation and the effects on performance statistics become more significant. When $\delta = 0.50$ and $k = 5$, geometric smoothing reduces the market beta by 49%, increases the Sharpe ratio by 71%, induces autocorrelations of 50%, 25%, and 12%, respectively, for the first three lags, and yields a discrepancy variance that is approximately 63% of the monthly variance of true returns.

The three smoothing profiles have very different values for ζ in Eq. (40):

$$\zeta = \frac{k(2k+1)}{6(k+1)}, \tag{44}$$

$$\zeta = \frac{k(3k^2 + 6k + 1)}{15(k+1)(k+2)}, \tag{45}$$

and

$$\zeta = \frac{\delta^2(-1 + \delta^k(2 + 2\delta + \delta^k(-1 - 2\delta + k(\delta^2 - 1))))}{(\delta^2 - 1)(\delta^{k+1} - 1)^2}, \tag{46}$$

with the straightline and sum-of-years profiles implying variances for $\Delta(T)$ that grow approximately linearly in k, and the geometric profile implying a variance for $\Delta(T)$ that asymptotes to a finite limit (see Fig. 3).

The results in Table 4 and Fig. 3 show that a rich set of biases can be generated by even simple smoothing profiles, and even the most casual empirical observation suggests that smoothed returns are an important source of serial correlation in hedge-fund returns. To address this issue directly, we propose methods for
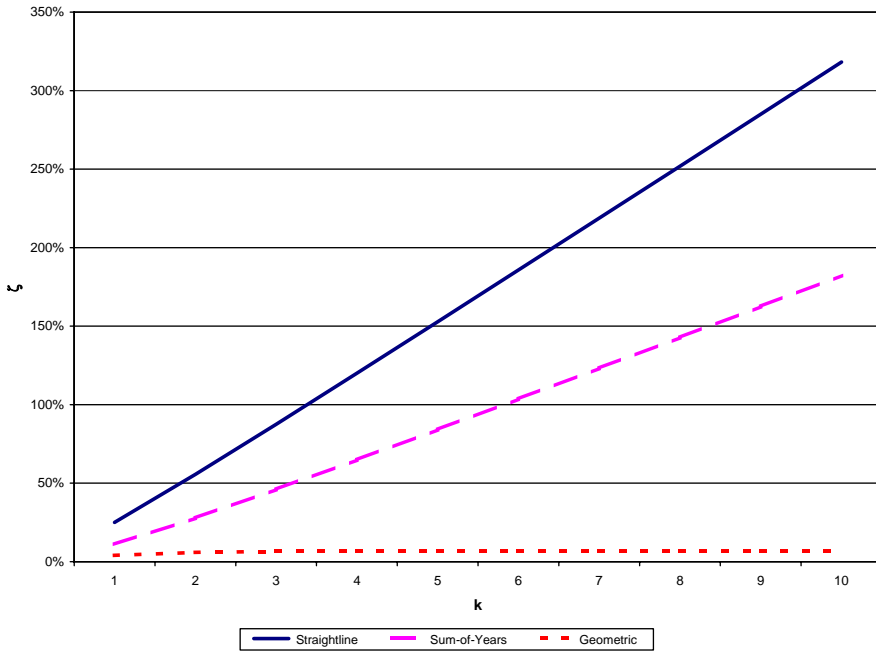
Fig. 3. Multiplicative factor ζ for straightline, sum-of-years and geometric smoothing profiles as a function of k. The variance of the discrepancy between true and observed returns, $\mathrm{Var}[\Delta(T)] = 2\sigma^2\zeta$ is proportional to ζ.

estimating the smoothing profile in Section 5 and apply these methods to the data in Section 6.

5. Estimation of smoothing profiles and Sharpe ratios

Although the smoothing profiles described in Section 4.2 can all be easily estimated from the sample moments of fund returns, e.g., means, variances, and autocorrelations, we wish to estimate more general forms of smoothing. Therefore, in this section we propose two estimation procedures (maximum likelihood and linear regression) that place fewer restrictions on a fund's smoothing profile than the three examples in Section 4.2. In Section 5.1 we review the steps for maximum likelihood estimation of an MA(k) process, slightly modified to accommodate our context and constraints, and in Section 5.2 we consider a simpler alternative based on linear regression under the assumption that true returns are generated by the linear single-factor model Eq. (20). We propose several specification checks to evaluate the robustness of our smoothing model in Section 5.3, and in Section 5.4 we show how to adjust Sharpe ratios to take smoothed returns into account.

5.1. Maximum likelihood estimation

Given the specification of the smoothing process in Eqs. (21) through (23), we can estimate the smoothing profile using maximum likelihood estimation in a fashion similar to the estimation of standard moving-average time series models (see, for example, Brockwell and Davis, 1991, Chapter 8). We begin by defining the de-meaned observed returns process X_t

$$X_t = R_t^o - \mu \tag{47}$$

and observing that Eqs. (21) through (23) imply the following properties for X_t:

$$X_t = \theta_0 \eta_t + \theta_1 \eta_{t-1} + \cdots + \theta_k \eta_{t-k}, \tag{48}$$

$$1 = \theta_0 + \theta_1 + \cdots + \theta_k, \tag{49}$$

$$\eta_k \sim \mathcal{N}(0, \sigma_\eta^2), \tag{50}$$

where, for purposes of estimation, we have added the parametric assumption Eq. (50) that η_k is normally distributed. From Eq. (48), it is apparent that X_t is a moving-average process of order k, or an MA(k). For a given set of observations $\mathbf{X} \equiv [X_1 \cdots X_T]'$, the likelihood function is well known to be

$$\mathscr{L}(\boldsymbol{\theta}, \sigma_\eta) = (2\pi)^{-T/2} (\det \boldsymbol{\Gamma})^{-1/2} \exp\left(-\tfrac{1}{2} \mathbf{X}' \boldsymbol{\Gamma}^{-1} \mathbf{X}\right), \quad \boldsymbol{\Gamma} \equiv \mathrm{E}[\mathbf{X}\mathbf{X}'], \tag{51}$$

where $\boldsymbol{\theta} \equiv [\theta_0 \cdots \theta_k]'$ and the covariance matrix $\boldsymbol{\Gamma}$ is a function of the parameters $\boldsymbol{\theta}$ and σ_η. It can be shown that for any constant κ,

$$\mathscr{L}(\kappa\boldsymbol{\theta}, \sigma_\eta/\kappa) = \mathscr{L}(\boldsymbol{\theta}, \sigma_\eta), \tag{52}$$

therefore, an additional identification condition is required.

The most common identification condition imposed in the time-series literature is the normalization $\theta_0 \equiv 1$. However, in our context, we impose the condition Eq. (49) that the MA coefficients sum to 1—an economic restriction that smoothing takes place over only the most recent $k+1$ periods—and this is sufficient to identify the parameters $\boldsymbol{\theta}$ and σ_η. The likelihood function Eq. (51) can then be evaluated and maximized via the "innovations algorithm" of Brockwell and Davis (1991, Chapter 8.3). Specifically, let $\hat{\mathbf{X}} = [\hat{X}_1 \cdots \hat{X}_T]'$ where $\hat{X}_1 = 0$ and $\hat{X}_j = \mathrm{E}[X_j|X_1, \ldots, X_{j-1}], j \geqslant 2$. Let $r_t = \mathrm{E}[(X_{t+1} - \hat{X}_{t+1})^2]/\sigma_\eta^2$. Brockwell and Davis (1991) show that Eq. (51) can be rewritten as

$$\mathscr{L}(\boldsymbol{\theta}, \sigma_\eta^2) = (2\pi\sigma_\eta^2)^{-T/2} (r_0 \cdots r_{T-1})^{-1/2} \exp\left[-\frac{1}{2}\sigma_\eta^2 \sum_{t=1}^{T} (X_t - \hat{X}_t)^2/r_{t-1}\right], \tag{53}$$

where the one-step-ahead predictors $\hat{X}_t$ and their normalized mean-squared errors r_{t-1}, $t = 1, \ldots, T$ are calculated recursively according to the formulas given in Brockwell and Davis (1991, Proposition 5.2.2). Taking the derivative of Eq. (53) with respect to σ_η^2, we see that the maximum likelihood estimator $\hat{\sigma}_\eta^2$ is given by

$$\hat{\sigma}_\eta^2 = S(\boldsymbol{\theta}) = T^{-1} \sum_{t=1}^{T} (X_t - \hat{X}_t)^2/r_{t-1} \tag{54}$$

hence we can "concentrate" the likelihood function by substituting Eq. (54) into Eq. (53) to obtain

$$\mathcal{L}_o(\boldsymbol{\theta}) = \log S(\boldsymbol{\theta}) + T^{-1} \sum_{t=1}^{T} \log r_{t-1} \tag{55}$$

which can be minimized in $\boldsymbol{\theta}$ subject to the constraint Eq. (49) using standard numerical optimization packages (we use Matlab's Optimization Toolbox in our empirical analysis). Maximum likelihood estimates obtained in this fashion need not yield an invertible MA(k) process, but it is well known that any non-invertible process can always be transformed into an invertible one simply by adjusting the parameters σ_η^2 and $\boldsymbol{\theta}$. To address this identification problem, we impose the additional restriction that the estimated MA(k) process be invertible. The properties of the estimator are given by:

Proposition 3. *Under the specification Eqs. (48) through (50), X_t is invertible on the set $\{\boldsymbol{\theta} : \theta_0 + \theta_1 + \theta_2 = 1, \theta_1 < 1/2, \theta_1 < 1 - 2\theta_2\}$, and the maximum likelihood estimator $\hat{\boldsymbol{\theta}}$ satisfies the following properties*:

$$1 = \hat{\theta}_0 + \hat{\theta}_1 + \hat{\theta}_2, \tag{56}$$

$$\sqrt{T}\left(\begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix} - \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \right) \overset{a}{\sim} \mathcal{N}(0, \mathbf{V}_\theta), \tag{57}$$

$$\mathbf{V}_\theta$$
$$= \begin{bmatrix} -(-1+\theta_1)(-1+2\theta_1)(-1+\theta_1+2\theta_2) & -\theta_2(-1+2\theta_1)(-1+\theta_1+2\theta_2) \\ -\theta_2(-1+2\theta_1)(-1+\theta_1+2\theta_2) & (-1+\theta_1-2(-1+\theta_2)\theta_2)(-1+\theta_1+2\theta_2) \end{bmatrix}. \tag{58}$$

By applying the above procedure to observed de-meaned returns, we obtain estimates of the smoothing profile $\hat{\boldsymbol{\theta}}$ for each fund. Recall from Proposition 1 that the smoothing process Eqs. (21) through (23) does not affect the expected return, i.e., the sample mean of observed returns is a consistent estimator of the true expected return. Therefore, we use $R_t^o - \hat{\mu}$ in place of X_t in the estimation process without altering any of the asymptotic properties of the maximum likelihood estimator. Because of the scaling property Eq. (52) of the MA(k) likelihood function, a simple procedure for obtaining estimates of our smoothing model with the normalization Eq. (49) is to transform estimates $(\breve{\boldsymbol{\theta}}, \breve{\sigma})$ from standard MA(k) estimation packages such as SAS or RATS by dividing each $\breve{\theta}_i$ by $1 + \breve{\theta}_1 + \cdots + \breve{\theta}_k$ and multiplying $\breve{\sigma}$ by the same factor. The likelihood function remains unchanged but the transformed smoothing coefficients will now satisfy Eq. (49).

5.2. Linear regression analysis

Although we proposed a linear single-factor model Eq. (20) in Section 4 for true returns so as to derive the implications of smoothed returns for the market beta of

observed returns, the maximum likelihood procedure outlined in Section 5.1 is designed to estimate the more general specification of IID Gaussian returns, regardless of any factor structure. However, if we are willing to impose Eq. (20), a simpler method for estimating the smoothing profile is available. By substituting Eq. (20) into Eq. (21), we can re-express observed returns as

$$R_t^o = \mu + \beta(\theta_0 \Lambda_t + \theta_1 \Lambda_{t-1} + \cdots + \theta_k \Lambda_{t-k}) + u_t \tag{59}$$

where

$$u_t = \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_k \varepsilon_{t-k}. \tag{60}$$

Suppose we estimate the following linear regression of observed returns on contemporaneous and lagged market returns:

$$R_t^o = \mu + \gamma_0 \Lambda_t + \gamma_1 \Lambda_{t-1} + \cdots + \gamma_k \Lambda_{t-k} + u_t \tag{61}$$

as in Asness et al. (2001). Using the normalization Eq. (23) from our smoothing model, we can obtain estimators for β and $\{\theta_j\}$ readily:

$$\hat{\beta} = \hat{\gamma}_0 + \hat{\gamma}_1 + \cdots + \hat{\gamma}_k, \quad \hat{\theta}_j = \hat{\gamma}_j / \hat{\beta}. \tag{62}$$

Moreover, a specification check for Eqs. (59) and (60) can be performed by testing the following set of equalities:

$$\beta = \frac{\gamma_0}{\theta_0} = \frac{\gamma_1}{\theta_1} = \cdots = \frac{\gamma_k}{\theta_k}. \tag{63}$$

Because of serial correlation in u_t, ordinary least squares estimates Eq. (62) will not be efficient and the usual standard errors are incorrect, but the estimates are still consistent and could be a useful first approximation for identifying smoothing in hedge-fund returns. To obtain efficient estimates of the smoothing coefficients, a procedure like the maximum likelihood estimator of Section 5.1 must be used.

There is yet another variation of the linear single-factor model that may help to disentangle the effects of illiquidity from return smoothing.[9] Suppose that a fund's true economic returns R_t satisfies

$$R_t = \mu + \beta \Lambda_t + \varepsilon_t, \quad \varepsilon_t \sim \text{IID}(0, \sigma_\varepsilon^2) \tag{64}$$

but instead of assuming that the common factor Λ_t is IID as in Eq. (20), let Λ_t be serially correlated. While this alternative seems a minor variation of the smoothing model Eqs. (21) through (23), the difference in interpretation is significant. A serially correlated Λ_t captures the fact that a fund's returns could be autocorrelated because of an illiquid common factor, even in the absence of any smoothing process such as Eqs. (21) through (23). Of course, this still begs the question of what the ultimate source of serial correlation in the common factor could be, but by combining Eq. (64) with the smoothing process Eqs. (21) through (23), it is possible to distinguish between systematic versus idiosyncratic smoothing, the former attributable to the asset class and the latter resulting from fund-specific characteristics.

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[9] We thank the referee for encouraging us to explore this alternative.

To see why the combination of Eq. (64) and Eqs. (21) through (23) have different implications for observed returns, suppose for the moment that there is no smoothing, i.e., $\theta_0 = 1$ and $\theta_k = 0$ for $k > 0$ in Eqs. (21) through (23). Then observed returns are simply given by

$$R_t^o = \mu + \beta \Lambda_t + \varepsilon_t, \quad \varepsilon_t \sim \text{IID}(0, \sigma_\varepsilon^2) \tag{65}$$

where R_t^o is now serially correlated solely through Λ_t. This specification implies that the ratios of observed-return autocovariances will be identical across all funds with the same common factor:

$$\frac{\text{Cov}[R_t^o, R_{t-k}^o]}{\text{Cov}[R_t^o, R_{t-l}^o]} = \frac{\beta \, \text{Cov}[\Lambda_t, \Lambda_{t-k}]}{\beta \, \text{Cov}[\Lambda_t, \Lambda_{t-l}]} = \frac{\text{Cov}[\Lambda_t, \Lambda_{t-k}]}{\text{Cov}[\Lambda_t, \Lambda_{t-l}]}. \tag{66}$$

Moreover, Eq. (64) implies that in the regression Eq. (61), the coefficients of the lagged factor returns are zero and the error term is not serially correlated.

More generally, consider the combination of a serially correlated common factor Eq. (64) and smoothed returns Eqs. (21) through (23). This more general econometric model of observed returns implies that the appropriate specification of the regression equation is

$$R_t^o = \mu + \gamma_0 \Lambda_t + \gamma_1 \Lambda_{t-1} + \cdots + \gamma_k \Lambda_{t-k} + u_t, \tag{67}$$

$$u_t = \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_k \varepsilon_{t-k}, \quad \varepsilon_t \sim \text{IID}(0, \sigma_\varepsilon^2) \tag{68}$$

where

$$1 = \theta_0 + \theta_1 + \cdots + \theta_k. \tag{69}$$

To the extent that serial correlation in R_t^o can be explained mainly by the common factor, the lagged coefficient estimates of Eq. (67) will be statistically insignificant, the residuals will be serially uncorrelated, and the ratios of autocovariance coefficients will be roughly constant across funds with the same common factor. To the extent that the smoothing process Eqs. (21) through (23) is responsible for serial correlation in R_t^o, the lagged coefficient estimates of Eq. (67) will be significant, the residuals will be serially correlated, and the ratios $\hat{\gamma}_j / \hat{\theta}_j$ will be roughly the same for all $j \geqslant 0$ and will be a consistent estimate of the factor loading or beta of the fund's true economic returns with respect to the factor Λ_t.

Perhaps the most difficult challenge in estimating Eqs. (67) through (69) is to correctly identify the common factor Λ_t. Unlike a simple market-model regression that is meant to estimate the sensitivity of a fund's returns to a broad-based market index, the ability to distinguish between the effects of systematic illiquidity and idiosyncratic return smoothing via Eq. (67) relies heavily on the correct specification of the common factor. Using a common factor in Eq. (67) that is highly serially correlated, but not exactly the right factor for a given fund, could yield misleading estimates for the degree of smoothing in that fund's observed returns. Therefore, the common factor Λ_t must be selected or constructed carefully to match the specific risk exposures of the fund, and the parameter estimates of Eq. (67) must be interpreted cautiously and with several specific alternative hypotheses at hand. In Section 6.4, we

provide an empirical example that highlights the pitfalls and opportunities of the common factor specification Eqs. (67) through (69).

5.3. Specification checks

Although the maximum likelihood estimator proposed in Section 5.1 has some attractive properties—it is consistent and asymptotically efficient under certain regularity conditions—it does not always perform well in small samples or when the underlying distribution of true returns is not normal as hypothesized. There is substantial evidence that financial asset returns are not normally distributed, but characterized instead by skewness, leptokurtosis, and other non-Gaussian properties (see, for example, Lo and MacKinlay, 1999). Given the dynamic nature of hedge-fund strategies, it would be even less plausible for their returns to be normally distributed. Moreover, even if normality is satisfied and a sufficient sample size is available, our proposed smoothing model Eqs. (21) through (23) may simply not apply to some of the funds in our sample. Therefore, it is important to have certain specification checks in mind when interpreting the empirical results.

The most obvious specification check is whether or not the maximum likelihood estimation procedure, which involves numerical optimization, converges. If not, this is one sign that our model is misspecified, either because of nonnormality or because the smoothing process is inappropriate.

A second specification check is whether or not the estimated smoothing coefficients are all positive in sign (we do not impose nonnegative restrictions in our estimation procedure, despite the fact that the specification does assume nonnegativity). Estimated coefficients that are negative and significant could be a sign that the constraint Eq. (49) is violated, which suggests that a somewhat different smoothing model applies.

A third specification check compares the smoothing-parameter estimates from the maximum likelihood approach of Section 5.1 with the linear regression approach of Section 5.2. If the linear single-factor model Eq. (20) holds, the two sets of smoothing-parameter estimates should be close. Of course, omitted factors could be a source of discrepancies between the two sets of estimates, so this specification check must be interpreted cautiously and with some auxiliary information about the economic motivation for the common factor Λ_t.

Finally, a more direct approach to testing the specification of Eqs. (21) through (23) is to impose a different identification condition than Eq. (49). Suppose that the standard deviation σ_η of true returns was observable. Then the smoothing parameters $\boldsymbol{\theta}$ are identified, and a simple check of the specification Eqs. (21) through (23) is to see whether the estimated parameters $\hat{\boldsymbol{\theta}}$ sum to 1. Of course, σ_η is not observable, but if we had an alternative estimator $\tilde{\sigma}_\eta$ for σ_η, we can achieve identification of the MA(k) process by imposing the restriction:

$$\sigma_\eta = \tilde{\sigma}_\eta \tag{70}$$

instead of Eq. (49). If, under this normalization, the smoothing parameter estimates are significantly different, this could be a sign of misspecification.

The efficacy of this specification check depends on the quality of $\tilde{\sigma}_\eta$. We propose to construct such an estimator by exploiting the fact that the discrepancy between observed and true returns becomes small for multiperiod returns as the number of periods grows. Specifically, recall from Eq. (37) that

$$
\begin{aligned}
(R_1^o + R_2^o + \cdots + R_T^o) = {} & (R_1 + R_2 + \cdots + R_T) \\
& + \sum_{j=0}^{k-1} (R_{-j} - R_{T-j}) \left(1 - \sum_{i=0}^{j} \theta_i \right),
\end{aligned}
\tag{71}
$$

$$
\frac{1}{T} \text{Var}\left[\sum_{t=1}^{T} R_t^o \right] = \sigma_\eta^2 + \frac{\sigma_\eta^2}{T} \sum_{j=0}^{k-1} \left(1 - \sum_{i=0}^{j} \theta_i \right) \left(1 - 2 \sum_{i=0}^{j} \theta_i \right)
\tag{72}
$$

and under the specification Eqs. (21) through (23), it is easy to show that the second term on the right side of Eq. (72) vanishes as T increases without bound, hence

$$
\lim_{T \to \infty} \frac{1}{T} \text{Var}\left[\sum_{t=1}^{T} R_t^o \right] = \sigma_\eta^2.
\tag{73}
$$

To estimate this normalized variance of multiperiod observed returns, we can apply Newey and West's (1987) estimator

$$
\tilde{\sigma}_\eta^2 \equiv \frac{1}{T} \sum_{1}^{T} (R_t^o - \hat{\mu})^2 + \frac{2}{T} \sum_{j=1}^{m} \left(1 - \frac{j}{m+1} \right) \left(\sum_{t=j+1}^{T} (R_t^o - \hat{\mu})(R_{t-j}^o - \hat{\mu}) \right),
\tag{74}
$$

where $\hat{\mu}$ is the sample mean of $\{R_t^o\}$ and m is a truncation lag that must increase with T but at a slower rate to ensure consistency and asymptotic normality of the estimator. By imposing the identification restriction

$$
\sigma_\eta = \tilde{\sigma}_\eta
\tag{75}
$$

in estimating the smoothing profile of observed returns, we obtain another estimator of $\boldsymbol{\theta}$ which can be compared against the first. As in the case of the normalization Eq. (49), the alternate normalization Eq. (75) can be imposed by rescaling estimates $(\check{\boldsymbol{\theta}}, \check{\sigma})$ from standard MA(k) estimation packages, in this case by dividing each $\check{\theta}_i$ by $\tilde{\sigma}_\eta/\check{\sigma}$ and multiplying $\check{\sigma}$ by the same factor.

5.4. Smoothing-adjusted Sharpe ratios

One of the main implications of smoothed returns is that Sharpe ratios are biased upward, in some cases substantially (see Proposition 1).[10] The mechanism by which this bias occurs is through the reduction in volatility because of the smoothing, but there is an additional bias that occurs when monthly Sharpe ratios are annualized by

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[10] There are a number of other concerns regarding the use and interpretation of Sharpe ratios in the context of hedge funds. Agarwal and Naik (2000a, 2002), Goetzmann et al. (2002), Lo (2001), Sharpe (1994), Spurgin (2001), and Weisman (2002) provide examples where Sharpe ratios can be misleading indicators of the true risk-adjusted performance of hedge-fund strategies and provide alternate methods of constructing optimal portfolios of hedge funds.

multiplying by $\sqrt{12}$. If monthly returns are independently and identically distributed (IID), this is the correct procedure, but Lo (2002) shows that for non-IID returns, an alternate procedure must be used, one that accounts for serial correlation in returns in a very specific manner.[11] Specifically, denote by $R_t(q)$ the following q-period return:

$$R_t(q) \equiv R_t + R_{t-1} + \cdots + R_{t-q+1}, \tag{76}$$

where we ignore the effects of compounding for computational convenience. The exact expression is, of course: $R_t(q) \equiv \prod_{j=0}^{q-1} (1 + R_{t-j}) - 1$. For most (but not all) applications, Eq. (76) is an excellent approximation. Alternatively, if R_t is defined to be the continuously compounded return, i.e., $R_t \equiv \log(P_t/P_{t-1})$ where P_t is the price or net asset value at time t, then Eq. (76) is exact. For IID returns, the variance of $R_t(q)$ is directly proportional to q, hence the Sharpe ratio (SR) satisfies the simple relation

$$SR(q) = \frac{E[R_t(q)] - R_f(q)}{\sqrt{Var[R_t(q)]}} = \frac{q(\mu - R_f)}{\sqrt{q}\sigma} = \sqrt{q}SR. \tag{77}$$

Using Hansen's (1982) Generalized Method of Moments (GMM) estimator, Lo (2002) derives the asymptotic distribution of $\widehat{SR}(q)$ as

$$\sqrt{T}\left(\widehat{SR}(q) - \sqrt{q}\,SR\right) \overset{a}{\sim} \mathcal{N}(0, V_{IID}(q)), \quad V_{IID}(q) = q\,V_{IID} = q\left(1 + \tfrac{1}{2}SR^2\right). \tag{78}$$

For non-IID returns, the relation between SR and SR(q) is somewhat more involved because the variance of $R_t(q)$ is not just the sum of the variances of component returns, but also includes all the covariances. Specifically, under the assumption that returns $\{R_t\}$ are stationary,

$$Var[R_t(q)] = \sum_{i=0}^{q-1} \sum_{j=0}^{q-1} Cov[R_{t-i}, R_{t-j}] = q\sigma^2 + 2\sigma^2 \sum_{k=1}^{q-1} (q-k)\rho_k, \tag{79}$$

where $\rho_k \equiv Cov[R_t, R_{t-k}]/Var[R_t]$. This yields the following relation between SR and SR(q):

$$SR(q) = \eta(q)SR, \quad \eta(q) \equiv \frac{q}{\sqrt{q + 2\sum_{k=1}^{q-1} (q-k)\rho_k}}. \tag{80}$$

Note that Eq. (80) reduces to Eq. (77) if the autocorrelations $\{\rho_k\}$ are zero, as in the case of IID returns. However, for non-IID returns, the adjustment factor for time-aggregated Sharpe ratios is generally not $\sqrt{q}$ but a function of the first $q-1$ autocorrelations of returns, which is readily estimated from the sample

---

[11] Jobson and Korkie (1981) were perhaps the first to derive rigorous statistical properties of performance measures such as the Sharpe ratio and the Treynor measure.

autocorrelations of returns, hence

$$\widehat{SR}(q) = \hat{\eta}(q)\widehat{SR}, \quad \hat{\eta}(q) \equiv \frac{q}{\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\hat{\rho}_k}}, \tag{81}$$

where $\hat{\rho}_k$ is the sample kth order autocorrelation coefficient.

Lo (2002) also derives the asymptotic distribution of Eq. (81) under fairly general assumptions for the returns process (stationarity and ergodicity) using generalized method of moments. However, in the context of hedge-fund returns, the usual asymptotic approximations may not be satisfactory because of the small sample sizes that characterize hedge-fund data. A five-year track record, which amounts to only 60 monthly observations, is considered quite a long history in this fast-paced industry. Therefore, we derive an alternate asymptotic distribution using the continuous-record asymptotics of Richardson and Stock (1989). Specifically, as the sample size T increases without bound, let q grow as well so that the ratio converges to some finite limit between 0 and 1:

$$\lim_{q,T \to \infty} q/T = \tau \in (0,1). \tag{82}$$

This condition is meant to provide an asymptotic approximation that is more accurate for small-sample situations, i.e., situations where q is a significant fraction of T. For example, in the case of a fund with a five-year track record, computing an annual Sharpe ratio with monthly data corresponds to a value of 0.20 for the ratio q/T.

Now as q increases without bound, $SR(q)$ also tends to infinity, hence we must renormalize it to obtain a well-defined asymptotic sampling theory. In particular, observe that

$$SR(q) = \frac{E[R_t(q)] - R_f(q)}{\sqrt{Var[R_t(q)]}} = \frac{q(\mu - R_f)}{\sqrt{Var[R_t(q)]}}, \tag{83}$$

$$SR(q)/\sqrt{q} = \frac{\mu - R_f}{\sqrt{Var[R_t(q)]/q}} \tag{84}$$

$$\lim_{q \to \infty} SR(q)/\sqrt{q} = \frac{\mu - R_f}{\bar{\sigma}}, \tag{85}$$

where $\bar{\sigma}$ can be viewed as a kind of long-run average return standard deviation, which is generally not identical to the unconditional standard deviation σ of monthly returns except in the IID case. To estimate $\bar{\sigma}$, we can either follow Lo (2002) and use sample autocorrelations as in Eq. (81), or estimate $\bar{\sigma}$ directly accordingly to Newey and West (1987):

$$\hat{\bar{\sigma}}_{NW}^2 \equiv \frac{1}{T}\sum_1^T (R_t - \hat{\mu})^2 + \frac{2}{T}\sum_{j=1}^m \left(1 - \frac{j}{m+1}\right)\sum_{t=j+1}^T (R_t - \hat{\mu})(R_{t-j} - \hat{\mu}), \tag{86}$$

where $\hat{\mu}$ is the sample mean of $\{R_t\}$. For this estimator of $\bar{\sigma}$, we have the following asymptotic result:

Proposition 4. *As m and T increase without bound so that $m/T \to \lambda \in (0,1)$, $\hat{\bar{\sigma}}^2_{\text{NW}}$ converges weakly to the following functional $f(W)$ of standard Brownian motion on $[0,1]$:*[12]

$$f(W) \equiv \frac{2\bar{\sigma}^2}{\lambda}\left(\int_0^1 W(r)[W(r) - W(\min(r+\lambda,1))]\,\mathrm{d}r\right.$$
$$\left. - W(1)\int_0^\lambda (\lambda - r)(W(1-r) - W(r))\,\mathrm{d}r + \frac{\lambda(1-\lambda^2/3)}{2}\,W^2(1)\right). \quad (87)$$

From Eq. (87), a straightforward computation yields the following expectations:

$$\mathrm{E}[\hat{\bar{\sigma}}^2_{\text{NW}}] = 1 - \lambda + \frac{\lambda^2}{3}, \quad \mathrm{E}[1/\hat{\bar{\sigma}}_{\text{NW}}] \approx \sqrt{\frac{1+\lambda}{1-\lambda+\lambda^2/3}} \quad (88)$$

hence we propose the following bias-corrected estimator for the Sharpe ratio for small samples:

$$\widehat{\text{SR}}(q) = \frac{\sqrt{q}(\hat{\mu} - R_f)}{\hat{\bar{\sigma}}_{\text{NW}}}\sqrt{\frac{1-\lambda+\lambda^3/2}{1+\lambda}} \quad (89)$$

and its asymptotic distribution is given by the following proposition:

Proposition 5. *As m, q, and T increase without bound so that $m/T \to \lambda \in (0,1)$ and $q/T \to \tau \in (0,1)$, the Sharpe ratio estimator $\widehat{\text{SR}}(q)$ converges weakly to the following random variable:*

$$\widehat{\text{SR}}(q) \Rightarrow \left(\frac{\text{SR}(q)}{f(W)} + \frac{\sqrt{\tau}W(1)}{f(W)}\right)\sqrt{\frac{1-\lambda+\lambda^3/2}{1+\lambda}}, \quad (90)$$

where $f(W)$ is given by Eq. (87), $\text{SR}(q)$ is given by Eq. (83) and $W(\cdot)$ is standard Brownian motion defined on $[0,1]$.

Monte Carlo simulations show that the second term of Eq. (90) does not account for much bias when $\tau \in \left(0, \frac{1}{2}\right]$, and that Eq. (90) is an excellent approximation to the small-sample distributions of Sharpe ratios for non-IID returns.[13]

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[12] See Billingsley (1968) for a definition of weak convergence and related results.

[13] We have tabulated the percentiles of the distribution of Eq. (90) by Monte Carlo simulation for an extensive combination of values of q, τ, and λ and would be happy to provide them to interested readers upon request.

6. Empirical analysis

For our empirical analysis, we use the TASS database of hedge funds which consists of monthly returns and accompanying information for 2,439 hedge funds (as of January 2001) from November 1977 to January 2001 (http://www.tassresearch. com). The database is divided into two parts: "Live" and "Graveyard" funds. Hedge funds that belong to the Live database are considered active as of January 1, 2001. Once a hedge fund decides not to report its performance, is liquidated, restructured, or merged with other hedge funds, the fund is transferred into the Graveyard database. A hedge fund can only be listed in the Graveyard database after being listed in the Live database, but the TASS database is subject to "backfill bias" — when a fund decides to be included in the database, TASS adds the fund to the Live database and includes available prior performance of the fund (hedge funds do not need to meet any specific requirements to be included in the TASS database). Due to reporting delays and time lags in contacting hedge funds, some Graveyard funds can be incorrectly listed in the Live database for a period of time. However, TASS has adopted a policy of transferring funds from the Live to the Graveyard database if they do not report over a six- to eight-month period.

As of January 1, 2001, the combined data set of both live and dead hedge funds contained 2,439 funds with at least one monthly net return observation. Out of these 2,439 funds, 1,512 are in the Live database and 927 are in the Graveyard database. The earliest data available for a fund in either database is November 1, 1977. The Graveyard database became active only in 1994, i.e., funds that were dropped from the Live database prior to 1994 are not included in the Graveyard database, which can yield a certain degree of survivorship bias.[14]

A majority of the 2,439 funds reported returns net of various fees on a monthly basis. TASS defines returns as the change in net asset value during the month (assuming the reinvestment of any distributions on the reinvestment date used by the fund) divided by the net asset value at the beginning of the month, net of management fees, incentive fees, and other fund expenses. Therefore, these reported returns should approximate the returns realized by investors. TASS also converts all foreign-currency denominated returns to US-dollar returns using the appropriate exchange rates. We eliminated 30 funds that reported only gross returns and/or quarterly returns (15 from each of the Live and Graveyard databases, respectively), leaving 2,409 funds in our sample. We imposed an additional filter of including only those funds with at least five years of data, leaving 651 funds in the Live database and 258 in the Graveyard database for a combined total of 909 funds. This obviously creates additional survivorship bias in our sample, but since our main objective is to estimate smoothing profiles and not to make inferences about overall performance, our filter may not be as problematic.[15]

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[14] Studies attempting to quantify the degree and impact of survivorship bias include Baquero et al. (2002), Brown et al. (1992, 1997, 1999), Carpenter and Lynch (1999), Fung and Hsieh (1997b, 2000), Horst et al. (2001), Hendricks et al. (1997), and Schneeweis and Spurgin (1996).

[15] See the references in footnote 14.

Table 5
Number of funds in the TASS Hedge Fund Live and Graveyard databases
Funds have at least five years of returns history during the period from November 1977 to January 2001.

Code	Category	Number of funds in:		
		Combined	Live	Graveyard
0	Not Categorized	111	44	67
1	US Equity Hedge	162	139	23
2	European Equity Hedge	22	19	3
3	Asian Equity Hedge	5	5	0
4	Global Equity Hedge	27	24	3
5	Dedicated Shortseller	7	6	1
6	Fixed-Income Directional	13	12	1
7	Convertible Fund (Long Only)	15	12	3
8	Event-Driven	109	97	12
9	Nondirectional/Relative Value	85	63	22
10	Global Macro	25	15	10
11	Global Opportunity	1	1	0
12	Natural Resources	3	1	2
13	Pure Leveraged Currency	26	15	11
14	Pure Managed Futures	93	28	65
15	Pure Emerging Market	72	54	18
16	Pure Property	1	1	0
17	Fund of Funds	132	115	17
	All	909	651	258

TASS also attempts to classify funds according to one of 17 different investment styles, listed in Table 5 and described in Appendix A.4; funds that TASS is not able to categorize are assigned a category code of '0'. A hedge fund can have at most two different categories (CAT1 and CAT2) in the TASS database. For all hedge funds in the TASS database, the second category (CAT2) is always 17, Fund of Funds. Table 5 also reports the number of funds in each category for the Live, Graveyard, and Combined databases, and it is apparent from these figures that the representation of investment styles is not evenly distributed, but is concentrated among six categories: US Equity Hedge (162), Event-Driven (109), Nondirectional/ Relative Value (85), Pure Managed Futures (93), Pure Emerging Market (72), and Fund of Funds (132). Together, these six categories account for 72% of the funds in the Combined database.

To develop a sense of the dynamics of the TASS database and the impact of our minimum return-history filter, in Table 6 we report annual frequency counts of the funds in the database at the start of each year, funds entering during the year, funds exiting during the year, and funds entering and exiting within the year. Panel A contains counts for the entire TASS database, and Panel B contains counts for our sample of 909 funds with at least five years of returns. Panel A shows that despite the start date of November 1977, the database is relatively sparsely populated until

Table 6
Annual frequency counts of entries into and exits out of the TASS Hedge Fund Database from November 1977 to January 2001

Annual frequency counts of the number of hedge funds entering and exiting the TASS Hedge Fund (Live and Graveyard) Database from November 1977 to January 2001.

Year	Panel A: all funds						Panel B: funds with at least 5 years' history					
	Existing funds	New entries	New exits	Intra-year entry/exit	Total funds	Attrition rate (%)	Existing funds	New entries	New exits	Intra-year entry/exit	Total funds	Attrition rate (%)
1977	0	1	0	0	1	—	0	1	0	0	1	—
1978	1	2	0	0	3	0.0	1	2	0	0	3	0.0
1979	3	1	0	0	4	0.0	3	1	0	0	4	0.0
1980	4	1	0	0	5	0.0	4	1	0	0	5	0.0
1981	5	3	0	0	8	0.0	5	3	0	0	8	0.0
1982	8	4	0	0	12	0.0	8	4	0	0	12	0.0
1983	12	6	0	0	18	0.0	12	6	0	0	18	0.0
1984	18	14	0	0	32	0.0	18	14	0	0	32	0.0
1985	32	8	0	0	40	0.0	32	8	0	0	40	0.0
1986	40	19	0	0	59	0.0	40	19	0	0	59	0.0
1987	59	32	0	0	91	0.0	59	32	0	0	91	0.0
1988	91	28	0	0	119	0.0	91	28	0	0	119	0.0
1989	119	39	0	0	158	0.0	119	39	0	0	158	0.0
1990	158	99	0	0	257	0.0	158	93	0	0	251	0.0
1991	257	87	0	0	344	0.0	251	78	0	0	329	0.0
1992	344	156	0	0	500	0.0	329	118	0	0	447	0.0
1993	500	233	0	0	733	0.0	447	135	0	0	582	0.0
1994	733	245	28	2	950	3.8	582	153	3	0	732	0.5
1995	950	256	71	1	1135	7.5	732	142	15	0	859	2.0
1996	1135	263	127	9	1271	11.2	859	32	30	0	861	3.5
1997	1271	290	105	7	1456	8.3	861	0	14	0	847	1.6
1998	1456	255	166	10	1545	11.4	847	0	51	0	796	6.0
1999	1545	222	193	7	1574	12.5	796	0	73	0	723	9.2
2000	1574	90	421	19	1243	26.7	723	0	182	0	541	25.2

the 1990s, with the largest increase in new funds in 1998 and, in the aftermath of the collapse of Long-Term Capital Management (LTCM), the largest number of funds exiting the database in 1999 and 2000. Panel B of Table 6 illustrates the impact of our five-year filter. Here, the number of funds is considerably smaller, and although the impact of survivorship bias can be ameliorated by the use of Live and Graveyard funds, our sample of 909 funds will not include any of the funds started in 1997 and later which is a substantial proportion of the TASS database.

The attrition rates reported in Table 6 are defined as the ratio of funds exiting in a given year to the number of existing funds at the start of the year. TASS began tracking the exits of funds starting only in 1994, hence attrition rates could not be computed in prior years. For the unfiltered sample of all funds, the average attrition

rate from 1994 to 1999 is 9.11%, which is very similar to the 8.54% attrition rate obtained by Liang (2001) for the same period. As observed above, the attrition rate skyrocketed in 2000 in the wake of LTCM's demise. In Panel B of Table 6, we see smaller attrition rates (the average over the 1994 to 1999 period is only 3.81%) because of our five-year minimum return history filter. Since many hedge funds fail in their first three years, our filtered sample is likely to have a much lower attrition rate by construction.

Fig. 4 contains a visual depiction of the variation in sample sizes of our 909 funds. The start and end dates of the return history for each fund are connected by a vertical line and plotted in Fig. 4 according to the primary category of the fund (Categories 0–7 in the top panel and Categories 8–17 in the bottom panel). It is apparent from the increasing density of the graphs as we move from the bottom to the top that the majority of funds in our sample are relatively new.

In Section 6.1, we present summary statistics for the sample of hedge funds included in our analysis. We implement the smoothing profile estimation procedures outlined in Section 5 for each of the funds, and in Section 6.2 we summarize the results of the maximum likelihood and linear regression estimation procedures for the entire sample of funds and for each style category. Section 6.3 reports the results of cross-sectional regressions of the estimated smoothing coefficients, and in Section 6.4 we consider idiosyncratic and systematic effects of illiquidity and smoothing by estimating a linear factor model with smoothing for all funds in one particular style category using a common factor appropriate for that style. And in Section 6.5 we summarize the properties of smoothing-adjusted Sharpe ratios for all the funds in our sample and compare them to their unadjusted counterparts.

6.1. Summary statistics

Table 7 contains basic summary statistics for the 909 funds in our combined extract from the TASS Live and Graveyard databases. Not surprisingly, there is a great deal of variation in mean returns and volatilities both across and within categories. For example, the 162 US Equity Hedge funds in our sample exhibited a mean return of 22.53%, but with a standard deviation of 10.80% in the cross section, and a mean volatility of 21.69% with a cross-sectional standard deviation of 11.63%. Average serial correlations also vary considerably across categories, but five categories stand out as having the highest averages: Fixed-Income Directional (21.6%), Convertible Fund (long only) (22.5%), Event-Driven (20.8%), Nondirec-

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Fig. 4. Length of return histories. We depict the length of return histories by vertical solid lines, for all funds in the TASS hedge fund database with at least five years of returns during the period from November 1977 to January 2001, ordered by categories 0–7 in the top panel and categories 8–17 in the bottom panel. Each fund is represented by a single solid vertical line that spans the start and end dates of the fund's return history.
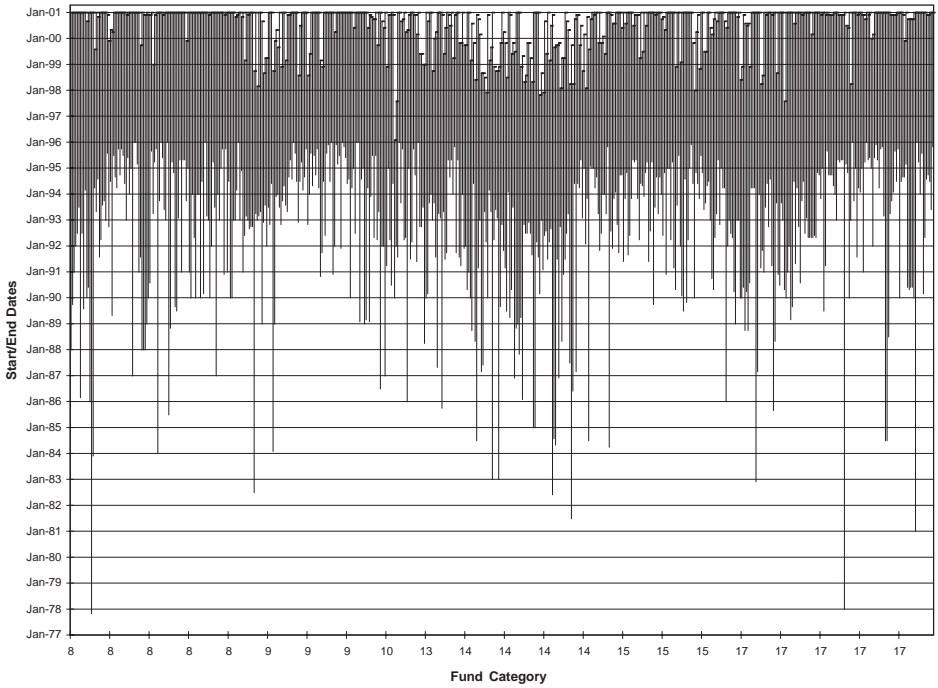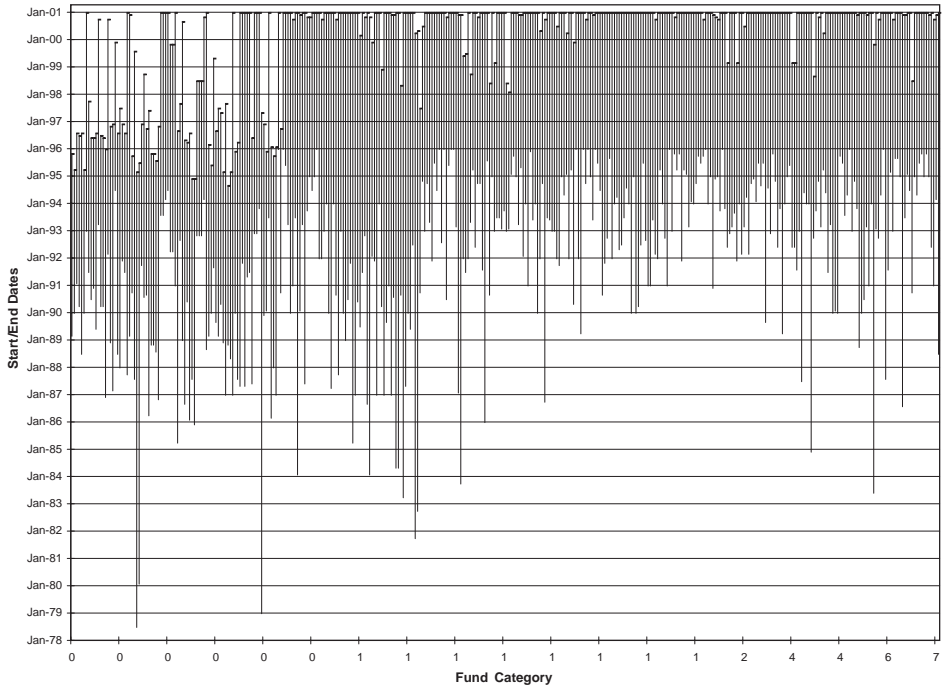
Table 7

Means and standard deviations of basic summary statistics for 909 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database

The 909 funds have at least five years of returns history during the period from November 1977 to January 2001. The columns 'p-Value(Q)' contain means and standard deviations of p-values for the Box-Pierce Q-statistic for each fund using the first six autocorrelations of returns.

Category	N	Annual mean		Annual SD		Skewness		Kurtosis		$\hat{\rho}_1(\%)$		$\hat{\rho}_2(\%)$		$\hat{\rho}_3(\%)$		p-Value(Q)	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	111	10.78	6.99	15.60	11.25	0.12	1.06	3.67	4.94	5.5	17.0	0.9	14.7	0.5	11.4	36.1	30.4
US Equity Edge	162	22.53	10.80	21.69	11.63	0.17	0.96	3.82	5.02	7.8	13.5	0.6	11.2	−4.2	11.0	33.9	28.8
European Equity Hedge	22	18.01	7.15	14.71	6.41	0.39	0.81	2.97	2.73	12.7	10.9	11.2	11.3	−2.0	8.9	21.2	23.0
Asian Equity Hedge	5	9.16	7.96	21.13	6.64	0.50	0.85	2.32	0.73	11.7	13.0	5.0	5.6	−5.4	13.4	40.2	21.1
Global Equity Hedge	27	13.71	8.41	17.41	6.61	−0.24	1.06	3.96	5.60	12.4	11.3	−0.1	9.6	−2.7	7.2	41.9	27.2
Dedicated Shortseller	7	0.33	10.26	21.55	13.80	0.49	0.20	2.28	2.40	4.4	8.4	−2.7	9.5	−6.5	6.9	31.8	27.7
Fixed-Income Directional	13	9.82	3.49	9.55	8.74	−0.94	1.29	5.37	7.32	21.6	15.7	14.1	13.2	1.9	13.6	18.1	21.0
Convertible Fund (Long Only)	15	14.95	5.33	11.03	6.76	−0.19	1.68	6.14	9.97	22.5	12.3	6.2	13.4	−2.1	12.5	17.2	25.2
Event-Driven	109	15.35	6.70	9.51	9.65	−0.58	1.82	8.26	11.68	20.8	16.8	6.4	13.4	−0.1	12.3	25.7	29.6
Nondirectional/Relative Value	85	12.54	6.65	8.19	5.05	−0.76	1.71	6.44	9.50	18.2	23.5	12.2	15.4	5.0	14.5	17.1	25.0
Global Macro	25	16.75	7.94	18.92	9.53	0.53	1.03	4.66	6.82	7.2	15.0	−0.1	10.8	−0.9	8.1	33.4	22.9
Global Opportunity	1	−17.39	—	31.03	—	−0.48	—	2.80	—	23.1	—	14.1	—	0.0	—	20.8	—
Natural Resources	3	11.39	4.89	18.91	1.06	0.48	0.44	1.25	1.49	5.0	14.5	8.5	12.1	1.9	7.0	47.6	43.7
Pure Leveraged Currency	26	9.29	7.11	17.18	8.93	0.44	1.08	3.50	4.92	5.2	8.7	−6.2	9.5	−4.4	8.1	35.5	26.6
Pure Managed Futures	93	9.60	9.28	21.74	17.06	0.22	1.35	4.93	8.31	−0.1	12.9	−3.8	10.0	−3.5	10.7	35.4	29.1
Pure Emerging Market	72	9.57	10.94	26.60	14.44	−0.69	1.49	6.17	8.05	18.8	11.8	4.4	11.6	−1.4	9.0	32.6	30.6
Pure Property	1	3.96	—	9.41	—	−1.33	—	5.43	—	−23.1	—	2.3	—	7.3	—	18.9	—
Fund of Funds	132	11.04	5.16	10.07	5.63	−0.26	1.30	4.78	6.16	17.6	14.1	5.7	11.5	−0.7	9.3	26.3	27.5
All	909	13.72	9.46	16.06	12.21	−0.15	1.38	5.05	7.56	12.1	16.8	3.4	13.1	−1.2	11.2	30.1	28.7

tional/Relative Value (18.2%), and Pure Emerging Market (18.8%).[16] Given the descriptions of these five categories provided by TASS (see Appendix A.4) and common wisdom about the nature of the strategies involved—these categories include some of the most illiquid securities traded—serial correlation is a reasonable proxy for illiquidity and smoothed returns. Alternatively, equities and futures are among the most liquid securities in which hedge funds invest, and not surprisingly, the average first-order serial correlation for US Equity Hedge funds and Pure Managed Futures is 7.8% and −0.1%, respectively. In fact, all of the equity funds have average serial correlations that are much smaller than those of the top five categories. Dedicated Shortseller funds also have a low average first-order autocorrelation, 4.4%, which is consistent with the high degree of liquidity that often characterizes shortsellers (since, by definition, the ability to short a security implies a certain degree of liquidity).

These summary statistics suggest that illiquidity and smoothed returns are important attributes for hedge-fund returns which can be captured to some degree by serial correlation and our time-series model of smoothing.

6.2. Smoothing profile estimates

Using the methods outlined in Section 5, we estimate the smoothing model Eqs. (21) through (23) and summarize the results in Tables 8 and 9. Our maximum likelihood procedure—programmed in Matlab using the *Optimization Toolbox* and replicated in Stata using its MA(k) estimation routine—converged without difficulty for all but one of the 909 funds: fund 1055, a Global Macro fund with returns from June 1994 to January 2001 for which the maximum likelihood estimation procedure yielded the following parameter estimates: $\hat{\theta}_0 = 490.47$, $\hat{\theta}_1 = -352.63$, and $\hat{\theta}_2 = -136.83$, which suggests that our MA(2) model is severely misspecified for this fund.[17] Therefore, we drop fund 1055 from our sample and for the remainder of our analysis, we focus on the smoothing profile estimates for the remaining 908 funds in our sample. We also constrain our maximum likelihood estimators to yield invertible MA(2) processes, and this constraint was binding for only two funds: 1711 and 4298.

Table 8 contains summary statistics for maximum likelihood estimate of the smoothing parameters $(\theta_0, \theta_1, \theta_2)$ and smoothing index ξ; Table 12 reports comparable statistics for the regression estimates of the smoothing parameters

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[16] At 23.1% and −23.1%, respectively, Global Opportunity and Pure Property have higher first-order autocorrelation coefficients in absolute value than the other categories, but since these two categories contain only a single fund each, we set them aside in our discussions.

[17] The apparent source of the problem in this case is two consecutive outliers, 39.4% in December 1999 followed by −27.6% in January 2000 (these are *monthly* returns, not annualized). The effect of two outliers on the parameter estimates of the MA(2) model Eqs. (21) through (23) is to pull the values of the coefficients in opposite directions so as to fit the extreme reversals. We contacted TASS to investigate these outliers and were informed that they were data errors. We also checked the remaining 908 funds in our sample for similar outliers, i.e., consecutive extreme returns of opposite sign, and found none. We also computed the maximum and minimum monthly returns for each fund in our sample, ranked the 908 funds according to these maxima and minima, and checked the parameter estimates of the top and bottom ten funds, and none exhibited the extreme behavior of fund 1055's parameter estimates.

Table 8

Means and standard deviations of maximum likelihood estimates

This table includes the means and standard deviations of maximum likelihood estimates of MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$, for 908 hedge funds in the TASS combined database with at least five years of returns history during the period from November 1977 to January 2001.

Category	N	Panel A: MA(2) with constrained sum								Panel B: MA(2) with constrained σ_n							
		$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$		$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	111	1.04	0.33	0.01	0.18	−0.05	0.23	1.27	1.06	0.97	0.27	0.01	0.16	−0.04	0.19	1.08	0.60
US Equity Hedge	162	0.95	0.21	0.06	0.15	−0.01	0.15	1.00	0.54	0.98	0.20	0.07	0.14	−0.01	0.15	1.04	0.42
European Equity Hedge	22	0.82	0.15	0.08	0.09	0.11	0.11	0.72	0.28	0.90	0.17	0.09	0.10	0.13	0.12	0.88	0.32
Asian Equity Hedge	5	0.94	0.30	0.05	0.19	0.01	0.11	1.00	0.71	0.90	0.20	0.06	0.17	0.01	0.10	0.88	0.40
Global Equity Hedge	27	0.91	0.14	0.11	0.10	−0.02	0.09	0.88	0.27	0.96	0.20	0.11	0.10	−0.02	0.09	0.99	0.37
Dedicated Shortseller	7	1.03	0.20	0.03	0.12	−0.06	0.13	1.12	0.48	1.13	0.14	0.05	0.12	−0.05	0.14	1.33	0.34
Fixed-Income Directional	13	0.76	0.17	0.15	0.10	0.08	0.12	0.67	0.27	0.75	0.21	0.15	0.10	0.09	0.12	0.66	0.30
Convertible Fund (Long Only)	15	0.84	0.35	0.18	0.08	−0.02	0.35	0.98	1.36	0.89	0.27	0.20	0.10	0.00	0.24	0.97	0.58
Event-Driven	109	0.81	0.17	0.15	0.13	0.04	0.11	0.74	0.28	0.84	0.24	0.16	0.12	0.04	0.11	0.82	0.44
Nondirectional/Relative Value	85	0.82	0.24	0.09	0.25	0.09	0.14	0.82	0.62	0.77	0.27	0.09	0.24	0.08	0.13	0.75	0.70
Global Macro	24	0.95	0.18	0.08	0.11	−0.03	0.15	0.99	0.40	0.97	0.23	0.08	0.10	−0.03	0.17	1.03	0.51
Global Opportunity	1	0.74	—	0.16	—	0.10	—	0.58	—	0.63	—	0.14	—	0.09	—	0.43	—
Natural Resources	3	0.91	0.19	0.02	0.11	0.07	0.09	0.87	0.34	0.85	0.04	0.03	0.10	0.08	0.10	0.74	0.04
Pure Leveraged Currency	26	1.10	0.34	0.03	0.13	−0.13	0.25	1.41	1.38	1.00	0.16	0.04	0.10	−0.10	0.16	1.07	0.37
Pure Managed Futures	93	1.13	0.32	−0.05	0.22	−0.08	0.18	1.47	1.19	1.10	0.25	−0.03	0.18	−0.07	0.16	1.33	0.63
Pure Emerging Market	72	0.83	0.16	0.15	0.09	0.02	0.13	0.76	0.31	0.79	0.17	0.14	0.08	0.02	0.13	0.70	0.30
Pure Property	1	1.23	—	−0.31	—	0.07	—	1.62	—	0.95	—	−0.24	—	0.06	—	0.96	—
Fund of Funds	132	0.85	0.21	0.13	0.13	0.03	0.13	0.81	0.50	0.85	0.23	0.13	0.12	0.03	0.13	0.83	0.45
All	908	0.92	0.26	0.08	0.17	0.00	0.17	0.98	0.76	0.91	0.25	0.08	0.16	0.01	0.15	0.95	0.53

Table 9
First 50 funds of ranked list of 908 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database

These 908 funds have at least five years of returns history during the period from November 1977 to January 2001, ranked in increasing order of the estimated smoothing parameter $\hat{\theta}_0$ of the MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, subject to the normalization $1 = \theta_0 + \theta_1 + \theta_2$, and estimated via maximum likelihood.

Code	Category	Period	T	Status	$\hat{\theta}_0$	SE($\hat{\theta}_0$)	$\hat{\theta}_1$	SE($\hat{\theta}_1$)	$\hat{\theta}_2$	SE($\hat{\theta}_2$)	$\hat{\xi}$
1201	Nondirectional/Relative Value	199409–200101	77	1	0.464	0.040	0.365	0.025	0.171	0.047	0.378
4346	Event-Driven	199412–200011	71	1	0.471	0.041	0.335	0.029	0.195	0.048	0.371
180	Not Categorized	198906–199608	87	0	0.485	0.041	0.342	0.027	0.173	0.046	0.382
1204	Nondirectional/Relative Value	199510–200101	64	1	0.492	0.049	0.339	0.033	0.169	0.055	0.386
1584	Fund of Funds	199601–200101	61	1	0.504	0.046	0.245	0.040	0.251	0.051	0.377
518	Nondirectional/Relative Value	199312–200005	78	0	0.514	0.034	0.142	0.037	0.343	0.037	0.403
971	Nondirectional/Relative Value	199409–200012	76	1	0.515	0.038	0.176	0.038	0.309	0.041	0.392
1234	Fund of Funds	199410–200012	75	1	0.527	0.061	0.446	0.020	0.027	0.061	0.477
2185	Fixed Income Directional	199108–200101	114	1	0.532	0.040	0.265	0.032	0.202	0.043	0.395
26	Nondirectional/Relative Value	199303–200101	95	1	0.533	0.047	0.305	0.033	0.162	0.050	0.403
171	Nondirectional/Relative Value	199304–200101	94	1	0.536	0.040	0.193	0.037	0.271	0.043	0.398
1696	Fund of Funds	199501–200001	61	0	0.536	0.067	0.406	0.030	0.058	0.068	0.456
120	Nondirectional/Relative Value	198207–199810	196	0	0.546	0.032	0.238	0.026	0.215	0.034	0.402
1396	Nondirectional/Relative Value	199507–200101	67	1	0.548	0.056	0.255	0.044	0.197	0.059	0.404
2774	Nondirectional/Relative Value	199501–200006	66	0	0.553	0.058	0.262	0.044	0.185	0.061	0.409
1397	Nondirectional/Relative Value	199410–200101	76	1	0.556	0.055	0.260	0.042	0.184	0.057	0.410
57	Nondirectional/Relative Value	199210–200101	100	1	0.561	0.052	0.311	0.034	0.127	0.053	0.428
4158	Event-Driven	199510–200101	64	1	0.565	0.062	0.261	0.046	0.174	0.064	0.417
1773	Fund of Funds	199506–200101	68	1	0.569	0.056	0.194	0.049	0.238	0.058	0.417
415	Not Categorized	198807–199608	98	0	0.570	0.054	0.306	0.035	0.124	0.055	0.434
1713	Fixed Income Directional	199601–200101	61	1	0.573	0.067	0.269	0.048	0.158	0.068	0.426
1576	Fund of Funds	199504–200010	67	1	0.575	0.066	0.293	0.044	0.132	0.067	0.434
1633	Event-Driven	199304–199901	70	0	0.576	0.066	0.309	0.042	0.115	0.066	0.440

Table 9 (*continued*)

Code	Category	Period	T	Status	$\hat{\theta}_0$	SE($\hat{\theta}_0$)	$\hat{\theta}_1$	SE($\hat{\theta}_1$)	$\hat{\theta}_2$	SE($\hat{\theta}_2$)	$\hat{\xi}$
1883	Event-Driven	199306–200101	92	1	0.578	0.053	0.236	0.041	0.187	0.054	0.424
1779	Non-Directional/Relative Value	199512–200101	62	1	0.584	0.067	0.241	0.051	0.175	0.068	0.429
3860	Fund of Funds	199511–200101	63	1	0.584	0.064	0.207	0.053	0.210	0.065	0.428
482	Non-Directional/Relative Value	199406–200101	80	1	0.584	0.060	0.253	0.044	0.163	0.060	0.432
2755	US Equity Hedge	199203–200010	104	1	0.584	0.057	0.321	0.034	0.095	0.056	0.453
1968	Fund of Funds	199402–200012	83	1	0.587	0.060	0.253	0.044	0.160	0.060	0.434
1240	Fund of Funds	199405–200101	81	1	0.592	0.055	0.168	0.049	0.240	0.056	0.436
1884	Event-Driven	199301–200101	97	1	0.592	0.059	0.298	0.038	0.110	0.058	0.451
2864	Nondirectional/Relative Value	199408–200101	78	1	0.598	0.064	0.255	0.046	0.147	0.064	0.444
1	Nondirectional/Relative Value	199101–200101	121	1	0.600	0.051	0.241	0.038	0.159	0.051	0.444
412	Pure Emerging Market	199211–200101	99	1	0.600	0.061	0.318	0.036	0.082	0.059	0.468
1046	Pure Emerging Market	199406–200101	80	1	0.603	0.058	0.169	0.051	0.228	0.058	0.444
2570	Event-Driven	199001–200101	133	1	0.604	0.046	0.171	0.039	0.225	0.046	0.445
945	Fund of Funds	199210–200101	100	1	0.605	0.060	0.273	0.040	0.122	0.058	0.456
1994	Nondirectional/Relative Value	199507–200101	67	1	0.606	0.069	0.217	0.053	0.177	0.068	0.446
1691	Event-Driven	199310–200101	88	0	0.610	0.062	0.232	0.046	0.158	0.061	0.451
34	Nondirectional/Relative Value	199209–200101	101	1	0.612	0.052	0.143	0.047	0.245	0.052	0.455
2630	Nondirectional/Relative Value	199001–200101	133	1	0.612	0.052	0.255	0.036	0.133	0.051	0.457
2685	US Equity Hedge	199207–200101	103	1	0.613	0.060	0.271	0.040	0.116	0.058	0.463
2549	Nondirectional/Relative Value	199408–200101	78	1	0.614	0.079	0.412	0.029	−0.026	0.069	0.547
490	Event-Driven	199404–199908	65	0	0.615	0.074	0.237	0.054	0.148	0.072	0.456
3099	Convertible Fund (Long Only)	199501–200101	73	1	0.615	0.060	0.121	0.057	0.263	0.060	0.463
1326	US Equity Hedge	199601–200101	61	1	0.619	0.069	0.145	0.062	0.236	0.068	0.460
1418	Fund of Funds	199301–199804	64	0	0.621	0.085	0.363	0.041	0.016	0.078	0.518
3712	Not Categorized	199304–200101	94	1	0.625	0.062	0.211	0.047	0.164	0.060	0.462
1534	Fund of Funds	199402–200101	84	1	0.625	0.068	0.246	0.048	0.129	0.065	0.468
167	Event-Driven	199207–200101	103	1	0.627	0.061	0.239	0.044	0.134	0.059	0.468

under the assumption of a linear one-factor model for true returns, and Table 9 presents the maximum likelihood estimates of the smoothing model for the 50 most illiquid funds of the 908 funds, as ranked by $\hat{\theta}_0$.

Panel A of Table 8 reports summary statistics for the maximum likelihood estimates under the normalization Eq. (49) where the smoothing coefficients sum to 1, and Panel B reports the same statistics for the maximum likelihood estimates under the normalization Eq. (75) where the variance σ_η^2 is set equal to the nonparametric estimate $\tilde{\sigma}_\eta^2$ given by Eq. (74). A comparison of panels A and B reveals roughly similar characteristics, indicating the general equivalence of these two normalization methods and the fact that the smoothing model Eqs. (21) through (23) is a reasonable specification for hedge-fund returns. However, Table 8 contains only summary statistics, not the maximum likelihood estimators of individual funds, hence differences in the two normalizations for given funds are not apparent from this table. In particular, side-by-side comparisons of maximum likelihood estimates for an individual fund under these two normalizations could still be a useful specification check despite the broad similarities that these two approaches seem to exhibit in Table 8.

Table 8 shows that seven categories seem to exhibit smaller average values of $\hat{\theta}_0$ than the rest: European Equity Hedge (0.82), Fixed-Income Directional (0.76), Convertible Fund (Long Only) (0.84), Event-Driven (0.81), Nondirectional/Relative Value (0.82), Pure Emerging Market (0.83), and Fund of Funds (0.85). We omit the Global Opportunity category from our discussions because it consists of only a single fund. Consider, in particular, the Fixed-Income Directional category, which has a mean of 0.76 for $\hat{\theta}_0$. This is, of course, the average across all 13 funds in this category, but if it were the point estimate of a given fund, it would imply that only 76% of that fund's true current monthly return would be reported, with the remaining 24% distributed over the next two months (recall the constraint that $\hat{\theta}_0 + \hat{\theta}_1 + \hat{\theta}_2 = 1$). The estimates 0.15 and 0.08 for $\hat{\theta}_1$ and $\hat{\theta}_2$ imply that on average, the current reported return also includes 15% of last month's true return and 8% of the true return two months ago.[18] These averages suggest a significant amount of smoothing and illiquidity in this category, and are approximated by the geometric smoothing model of Section 4.2 with $\delta = 0.25$ (see Table 4). Recall from Table 4 that in this case, with $k = 2$, the impact of geometric smoothing was a 24% decrease in the market beta and a 27% increase in the Sharpe ratio of observed returns. Overall, the summary statistics in Table 8 are broadly consistent with common intuition about the nature of the strategies and securities involved in these fund categories, which contain the most illiquid securities and, therefore, have the most potential for smoothed returns.

Table 9 contains the smoothing parameter estimates for the top 50 funds ranked by $\hat{\theta}_0$, which provides a more direct view of illiquidity and smoothed returns. In contrast to the averages of Table 8, the parameter estimates of θ_0 among these 50 funds range from 0.464 to 0.627, implying that observed returns reflect only one-half to two-thirds of the current month's true returns. The asymptotic standard errors are

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[18] The averages do not sum to one exactly because of rounding errors.

generally quite small, ranging from 0.032 to 0.085, hence the smoothing parameters are estimated reasonably precisely.

The funds in Table 9 fall mainly into three categories including Nondirectional/ Relative Value, Event-Driven, and Fund of Funds. Together, these three categories account for 40 of the 50 funds in Table 9. A more complete summary of the distribution of smoothing parameter estimates across the different fund categories is provided in Figs. 5 and 6. Fig. 5 contains a graph of the smoothing coefficients $\hat{\theta}_0$ for all 908 funds by category, and Fig. 6 contains a similar graph for the smoothing index $\hat{\xi}$. These figures show that although there is considerable variation within each category, nevertheless, some differences emerge between categories. For example, categories 6 through 9, 15, and 17 (Fixed-Income Directional, Convertible Fund (Long Only), Event-Driven, Nondirectional/Relative Value, Pure Emerging Market, and Fund of Funds, respectively), have clearly discernible concentrations that are lower than the other categories, suggesting more illiquid funds and more smoothed returns. On the other hand, categories 1, 4, and 14 (US Equity Hedge, Global Equity Hedge, and Pure Managed Futures, respectively) have concentrations that are at the upper end, suggesting just the opposite, that is, more liquidity and less return-smoothing. The smoothing index estimates $\hat{\xi}$ plotted in Fig. 6 show similar patterns of concentration and dispersion within and between the categories.

To develop further intuition for the smoothing model Eqs. (21) through (23) and the possible interpretations of the smoothing parameter estimates, we apply the same estimation procedure to the returns of the Ibbotson stock and bond indexes, the Merrill Lynch Convertible Securities Index, the CSFB/Tremont hedge-fund indexes, and two mutual funds, including the highly liquid Vanguard 500 Index Fund and the considerably less liquid American Express Extra Income Fund. Table 10 contains summary statistics, market betas, contemporaneous and lagged market betas as in Asness et al. (2001) and smoothing-coefficient estimates for these index and mutual-fund returns. Market betas are obtained by regressing returns on a constant and the total return of the S&P 500, and contemporaneous and lagged market betas are obtained by regressing returns on a constant, the contemporaneous total return of the S&P 500, and the first two lags.

Consistent with our interpretation of $\hat{\theta}_0$ as an indicator of liquidity, the returns of the most liquid portfolios in Panel A of Table 10—the Ibbotson Large Company Index, the Vanguard 500 Index Fund (which is virtually identical to the Ibbotson Large Company Index except for sample period and tracking error), and the Ibbotson Long-Term Government Bond Index—have smoothing parameter estimates near unity: 0.92 for the Ibbotson Large Company Index, 1.12 for the Vanguard 500 Index Fund, and 0.92 for the Ibbotson Long-Term Government Bond Index. The first-order autocorrelation coefficients and lagged market betas also confirm their lack of serial correlation; 9.8% first-order autocorrelation for the Ibbotson Large Company Index, −2.3% for the Vanguard 500 Index Fund, and 6.7% for the Ibbotson Long-Term Government Bond Index, and lagged market betas that are statistically indistinguishable from 0. However, the values of $\hat{\theta}_0$ of the less liquid portfolios are less than 1.00: 0.82 for the Ibbotson Small Company Index, 0.84 for the Ibbotson Long-Term Corporate Bond Index, 0.82 for the Merrill Lynch
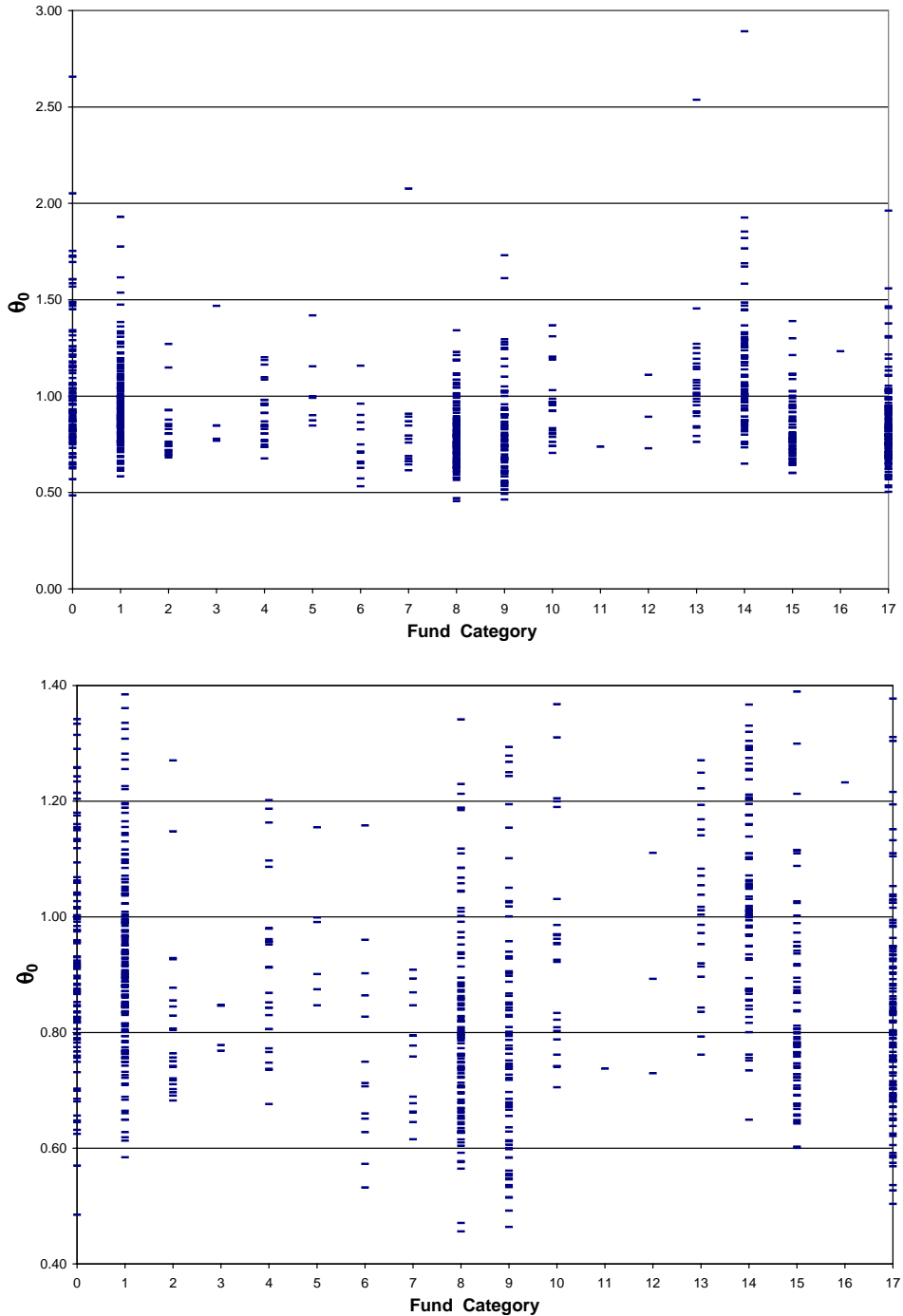
Fig. 5. Estimated smoothing coefficients $\hat{\theta}_0$ for all funds in the TASS Hedge Fund database. Funds include at least five years of returns during the period from November 1977 to January 2001, ordered by categories 0–17. The two panels differ only in the range of the vertical axis, which is smaller for the lower panel so as to provide finer visual resolution of the distribution of estimated coefficients in the sample.

Fig. 6. Estimated smoothing index $\hat{\xi}$ for all funds in the TASS Hedge Fund database. These funds include at least five years of returns during the period from November 1977 to January 2001, ordered by categories 0–17. The two panels differ only in the range of the vertical axis, which is smaller for the lower panel so as to provide finer visual resolution of the distribution of estimated smoothing indexes in the sample.
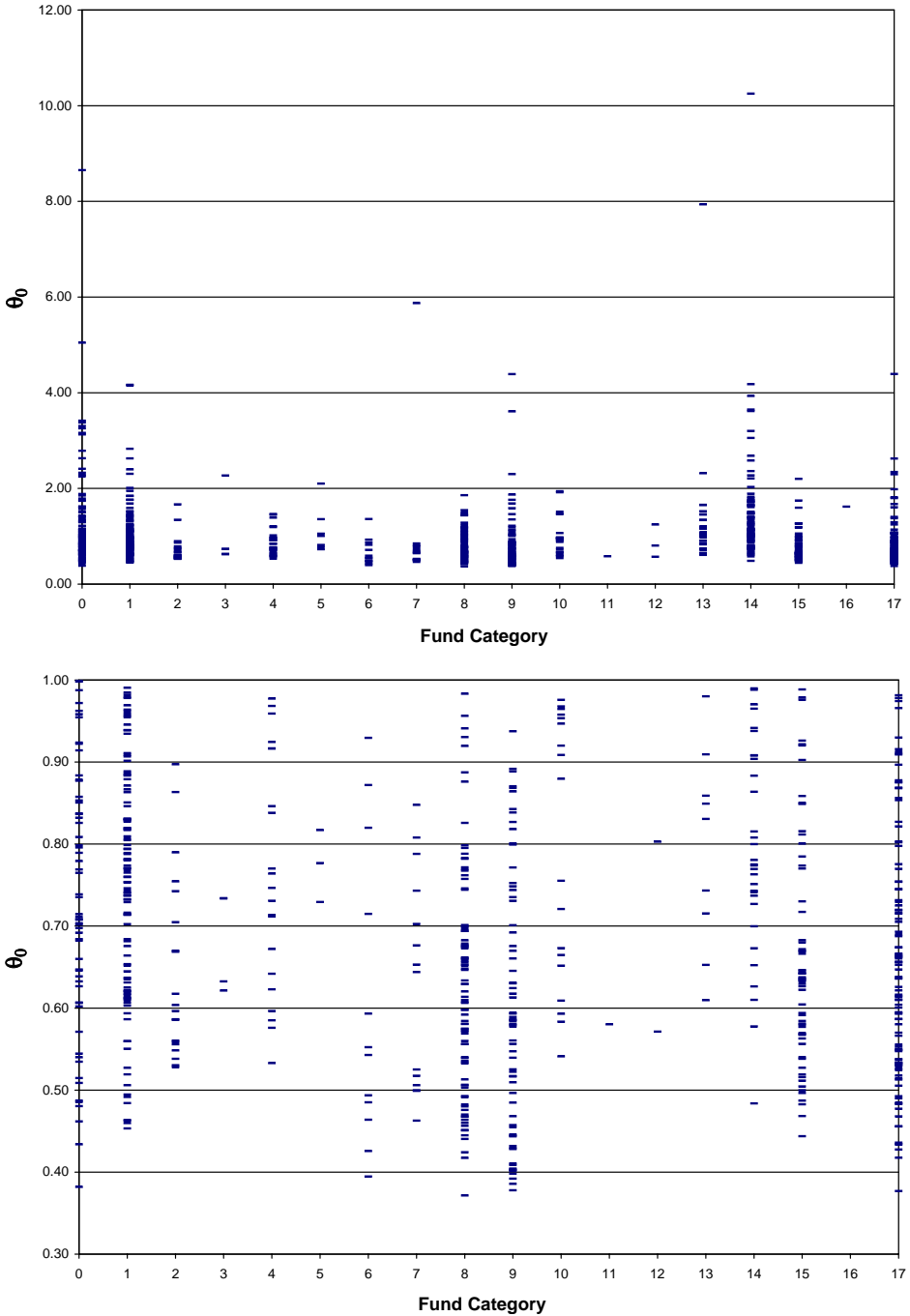
Table 10

Summary statistics and smoothing parameters for various indexes and two mutual funds

Summary statistics and maximum likelihood estimates of MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$, subject to the normalization $1 = \theta_0 + \theta_1 + \theta_2$, for the returns of various indexes and two mutual funds, the Vanguard 500 Index Trust (which tracks the S&P 500 index), and the AXP Extra Income Fund (which focuses on high current income and invests in long-term high-yielding lower-rated corporate bonds). Total returns of the S&P 500 index are used for both market models.

Series	Period	T	Mean (%)	SD (%)	$\hat{\rho}_1$ (%)	$\hat{\rho}_2$ (%)	$\hat{\rho}_3$ (%)	Market model			Contemporaneous and lagged market model						
								$\hat{\beta}$	SE($\hat{\beta}$)	R^2 (%)	$\hat{\beta}_0$	SE($\hat{\beta}_0$)	$\hat{\beta}_1$	SE($\hat{\beta}_1$)	$\hat{\beta}_2$	SE($\hat{\beta}_2$)	R^2 (%)
Panel A																	
Ibbotson Small Company	192601–200112	912	1.35	8.63	15.6	1.7	−10.6	1.27	0.03	66.9	1.25	0.03	0.16	0.03	0.03	0.03	68.0
Ibbotson Long-Term Government Bonds	192601–200112	912	0.46	2.22	6.7	0.3	−8.3	0.07	0.01	2.8	0.07	0.01	−0.03	0.01	−0.02	0.01	3.6
Ibbotson Long-Term Corporate Bonds	192601–200112	912	0.49	1.96	15.6	0.3	−6.0	0.08	0.01	5.2	0.08	0.01	−0.01	0.01	−0.01	0.01	5.3
Ibbotson Large Company	192601–200112	912	1.03	5.57	9.8	−3.2	−10.7	1.00	0.00	100.0	1.00	0.00	0.00	0.00	0.00	0.00	100.0
Merrill Lynch Convertibles Index	199401–200210	168	0.99	3.43	6.4	12.0	5.1	0.59	0.05	48.6	0.60	0.05	0.15	0.05	0.07	0.04	52.2
AXP Extra Income Fund (INEAX)	198401–200112	216	0.67	2.04	35.4	13.1	2.5	0.21	0.03	20.7	0.21	0.03	0.12	0.03	0.04	0.03	28.7
Vanguard 500 Index Trust (VFINX)	197609–200112	304	1.16	4.36	−2.3	−6.8	−3.2	1.00	0.00	100.0	1.00	0.00	0.00	0.00	0.00	0.00	100.0
CSFB/Tremont Indices:																	
Aggregate Hedge Fund Index	199401–200210	106	0.87	2.58	11.2	4.1	−0.4	0.31	0.05	24.9	0.32	0.05	0.06	0.05	0.16	0.05	32.1
Convertible Arbitrage	199401–200210	106	0.81	1.40	56.6	42.6	15.6	0.03	0.03	1.1	0.04	0.03	0.09	0.03	0.06	0.03	12.0
Dedicated Short Bias	199401–200210	106	0.22	5.29	7.8	−6.3	−5.0	−0.94	0.08	58.6	−0.93	0.08	−0.06	0.08	0.08	0.08	59.3
Emerging Markets	199401–200210	106	0.54	5.38	29.4	1.2	−2.1	0.62	0.11	24.0	0.63	0.11	0.19	0.11	0.03	0.12	26.2
Equity Market Neutral	199401–200210	106	0.89	0.92	29.4	18.1	8.4	0.10	0.02	21.1	0.10	0.02	0.02	0.02	0.00	0.02	22.1
Event-Driven	199401–200210	106	0.83	1.81	34.8	14.7	3.8	0.23	0.04	30.2	0.23	0.03	0.11	0.03	0.04	0.03	38.2
Fixed-Income Arbitrage	199401–200210	106	0.55	1.18	39.6	10.8	5.4	0.02	0.03	0.7	0.03	0.03	0.05	0.03	0.09	0.03	12.9

Table 10 (*continued*)

Series	Period	T	Mean (%)	SD (%)	$\hat{\rho}_1$ (%)	$\hat{\rho}_2$ (%)	$\hat{\rho}_3$ (%)	Market model			Contemporaneous and lagged market model						
								$\hat{\beta}$	SE($\hat{\beta}$)	R^2 (%)	$\hat{\beta}_0$	SE($\hat{\beta}_0$)	$\hat{\beta}_1$	SE($\hat{\beta}_1$)	$\hat{\beta}_2$	SE($\hat{\beta}_2$)	R^2 (%)
Global Macro	199401–200210	106	1.17	3.69	5.6	4.6	8.3	0.24	0.09	7.5	0.26	0.09	−0.01	0.09	0.23	0.09	14.1
Long/Short	199401–200210	106	0.98	3.34	15.9	5.9	−4.6	0.48	0.06	36.7	0.49	0.06	0.06	0.06	0.15	0.06	40.7
Managed Futures	199401–200210	106	0.55	3.44	3.2	−6.3	0.7	−0.12	0.08	2.5	−0.13	0.08	−0.17	0.08	0.02	0.08	7.8

Series	Period	T	$\hat{\theta}_0$	SE($\hat{\theta}_0$)	$\hat{\theta}_1$	SE($\hat{\theta}_1$)	$\hat{\theta}_2$	SE($\hat{\theta}_2$)	$\hat{\xi}$
Panel B									
Ibbotson Small Company	192601–200112	912	0.82	0.03	0.13	0.02	0.04	0.03	0.69
Ibbotson Long-Term Government Bonds	192601–200112	912	0.92	0.05	0.06	0.03	0.01	0.03	0.86
Ibbotson Long-Term Corporate Bonds	192601–200112	912	0.84	0.04	0.14	0.02	0.02	0.03	0.73
Ibbotson Large Company	192601–200112	912	0.92	0.05	0.09	0.03	−0.01	0.03	0.85
Merrill Lynch Convertibles Index	199401–200210	168	0.82	0.07	0.02	0.06	0.16	0.05	0.70
AXP Extra Income Fund (INEAX)	198401–200112	216	0.67	0.03	0.24	0.03	0.09	0.04	0.51
Vanguard 500 Index Trust (VFINX)	197609–200112	304	1.12	0.17	−0.03	0.07	−0.09	0.07	1.26
CSFB/Tremont Indices:									
Aggregate Hedge Fund Index	199401–200210	106	0.86	0.12	0.09	0.08	0.04	0.08	0.76
Convertible Arbitrage	199401–200210	106	0.49	0.01	0.26	0.03	0.25	0.03	0.37
Dedicated Short Bias	199401–200210	106	0.99	0.20	0.08	0.09	−0.07	0.10	0.99
Emerging Markets	199401–200210	106	0.75	0.08	0.24	0.05	0.01	0.07	0.62
Equity Market Neutral	199401–200210	106	0.71	0.06	0.18	0.05	0.12	0.06	0.54
Event-Driven	199401–200210	106	0.68	0.05	0.23	0.05	0.09	0.06	0.52
Fixed-Income Arbitrage	199401–200210	106	0.63	0.04	0.28	0.04	0.08	0.05	0.49
Global Macro	199401–200210	106	0.91	0.14	0.04	0.08	0.05	0.08	0.84
Long/Short	199401–200210	106	0.82	0.10	0.13	0.07	0.06	0.07	0.68
Managed Futures	199401–200210	106	1.04	0.23	0.04	0.10	−0.08	0.11	1.08

Convertible Securities Index, and 0.67 for the American Express Extra Income Fund, and their first-order serial correlation coefficients are 15.6%, 15.6%, 6.4%, and 35.4%, respectively, which, with the exception of the Merrill Lynch Convertible Securities Index, are considerably higher than those of the more liquid portfolios. However, note that the second-order autocorrelation of the Merrill Lynch Convertible Securities Index is 12.0% which is second only to the AXP Extra Income Fund in absolute magnitude, two orders of magnitude larger than the second-order autocorrelation of the Ibbotson Bond Indexes, and one order of magnitude larger than the Ibbotson Stock Indexes. Also, the lagged market betas are statistically significant at the 5% level for the Ibbotson Small Company Index (a t-statistic for $\hat{\beta}_1$: 5.41), the Ibbotson Long-Term Government Bond Index (t-statistic for $\hat{\beta}_1$: -2.30), the Merrill Lynch Convertible Securities Index (t-statistic for $\hat{\beta}_1$: 3.33), and the AXP Extra Income Fund (t-statistic for $\hat{\beta}_1$: 4.64).

The results for the CSFB Hedge Fund Indexes in Panel B of Table 10 are also consistent with the empirical results in Tables 8 and 9. Indexes corresponding to hedge-fund strategies involving less liquid securities tend to have lower $\hat{\theta}_0$s. For example, the smoothing-parameter estimates $\hat{\theta}_0$ of the Convertible Arbitrage, Emerging Markets, and Fixed-Income Arbitrage indexes are 0.49, 0.75, and 0.63, respectively, and first-order serial correlation coefficients of 56.6%, 29.4%, and 39.6%, respectively. In contrast, the smoothing-parameter estimates of the more liquid hedge-fund strategies such as Dedicated Short Bias and Managed Futures are 0.99 and 1.04, respectively, with first-order serial correlation coefficients of 7.8% and 3.2%, respectively. While these findings are generally consistent with the results in Tables 8 and 9, the process of aggregation can change the statistical behavior of any time series. For example, Granger (1980, 1988) observes that the aggregation of a large number of stationary autoregressive processes can yield a time series that exhibits long-term memory, characterized by serial correlation coefficients that decay very slowly—hyperbolically, as opposed to geometrically as in the case of a stationary autoregressive moving average (ARMA) process. Therefore, while it is true that the aggregation of a collection of illiquid funds will generally yield an index with smoothed returns,[19] the reverse need not be true—smoothed index returns need not imply that all of the funds comprising the index are illiquid. The latter inference can only be made with the benefit of additional information (essentially identification restrictions) about the statistical relations among the funds in the index, i.e., covariances and possibly other higher-order co-moments, or the existence of common factors driving fund returns.

It is interesting to note that the first lagged market beta, $\hat{\beta}_1$, for the Credit Suisse/First Boston (CSFB)/Tremont Indexes is statistically significant at the 5% level in only three cases (Convertible Arbitrage, Event-Driven, and Managed Futures), but the second lagged beta, $\hat{\beta}_2$, is significant in five cases (the Aggregate Index, Convertible Arbitrage, Fixed-Income Arbitrage, Global Macro, and Long/Short).

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[19] It is, of course, possible that the smoothing coefficients of some funds could exactly offset those of other funds so as to reduce the degree of smoothing in an aggregate index. However, such a possibility is extremely remote and pathological if each of the component funds exhibits a high degree of smoothing.

Of course, the S&P 500 Index is likely to be inappropriate factor for certain styles, e.g., Emerging Markets, and these somewhat inconsistent results suggest that using a lagged market-beta adjustment may not completely account for the impact of illiquidity and smoothed returns.

Overall, the patterns in Table 10 confirm our interpretation of smoothing coefficients and serial correlation as proxies for liquidity, and suggest broader applications of our model of smoothed returns to other investment strategies and asset classes.

6.3. Cross-sectional regressions

A more quantitative summary of the cross-sectional properties of the smoothing parameter estimates for the 908 funds is given in Table 11, which contains the results of cross-sectional regressions of the smoothing parameter $\hat{\theta}_0$ and the smoothing index $\hat{\xi}$ on a number of 0/1 indicator variables.[20] In the first two regressions, $\hat{\theta}_0$ and $\hat{\xi}$ are the dependent variables, respectively, and the regressors include a constant term, 17 indicator variables corresponding to the 17 hedge-fund categories defined by TASS (see Appendix A.4), and an indicator variable that takes on the value 1 if the fund is open and 0 if it is closed to new investors. The third and fourth regressions have the same dependent variables—$\hat{\theta}_0$ and $\hat{\xi}$, respectively—and include the same regressors as the first two regressions but also include 0/1 indicator variables that indicate whether the fund is US-based (USBASED), and whether the geographical focus of the fund is global (GF-GLB), US (GF-USA), Asia/Pacific (GF-APC), Western Europe (GF-WEU), Eastern Europe (GF-EEU), and Africa (GF-AFR).

The results of the first regression are consistent with the general intuition gleaned from Figs. 5 and 6. The category indicator variables with the most negative coefficients that are statistically significant at the 5% level are European Equity Hedge (-0.212), Fixed-Income Directional (-0.262), Event-Driven (-0.218), Nondirectional/Relative Value (-0.211), Pure Emerging Market (-0.195), Fund of Funds (-0.178), implying that on average, funds in these categories have smaller smoothing coefficients $\hat{\theta}_0$, i.e., less liquidity or smoother returns. These point estimates can be used to approximate the marginal impact that a given investment style has on the smoothing profile of the fund's monthly returns. For example, from a no-smoothing baseline of 1, conditioning on belonging to the Fixed-Income Directional category yields an expected smoothing parameter $\hat{\theta}_0$ of $1 - 0.262 = 0.738$ and an expected smoothing index of $\hat{\xi}$ of $1 - 0.583 = 0.417$, other things equal (and assuming that the remaining indicator variables in the two regression equations are zero).

In contrast, the coefficients for Dedicated Shortseller and Pure Leveraged Currency indicators—0.001 and 0.069, respectively, with t-statistics of 0.01 and 0.11, respectively—are positive and statistically insignificant at the 5% level, which is

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[20] To conserve space, we report regression results only for the maximum likelihood estimates under the constraint Eq. (49). Table A.8 in the appendix reports corresponding results for the maximum likelihood estimates under the alternate constraint Eq. (75).

consistent with common intuition about the liquidity of these types of funds. Moreover, the coefficient for the Pure Managed Futures indicator is both positive and significant at the 5% level—0.101 with a *t*-statistic of 3.00—which is also consistent with the intuition that managed futures involve relatively liquid securities with well-established, not easily manipulated marks.

The last indicator variable included in the first two regressions takes on the value one if the fund is open to new investors and zero if closed. If return-smoothing is actively being pursued, we expect it to be more important for funds that are open since such funds are still attempting to attract new investors. This implies that the coefficient for this indicator variable should be negative, i.e., open funds should be more prone to smoothing than closed funds. Table 11 confirms this hypothesis. The estimated coefficient for OPEN is -0.040 with a *t*-statistic of 2.03, implying that funds open to new investors have a smoothing coefficient $\hat{\theta}_0$ that is lower by 0.040 on average than funds that are closed. An alternate interpretation is that funds that are still open to new investors are typically those with smaller assets under management, and as a result, are less likely to be able to afford costly third-party valuations of illiquid securities in their portfolios. Unfortunately, many funds in the TASS database do not report assets under management so we were unable to investigate this alternative.

The third and fourth regressions in Table 11 include additional indicator variables that capture the fund's geographical base as well as the geographical focus of its investments, and we see that being in the US has a positive marginal impact on the conditional mean of $\hat{\theta}_0$, but being US-focused in its investments has a negative marginal impact. The latter result is somewhat counterintuitive but becomes less puzzling in light of the fact that approximately 46% of the funds are US-focused, hence many of the most illiquid funds are included in this category. Apart from this indicator, the geographical aspects of our sample of funds have little impact on the cross-sectional variability in smoothing parameter estimates.

With R^2's ranging from 9.0% to 17.7%, the regressions in Table 11 leave considerable cross-sectional variation unexplained, but this is no surprise given the noise inherent in the category assignments and the heterogeneity of investment styles even within each category. However, the general pattern of coefficients and *t*-statistics do suggest that the smoothing coefficients are capturing significant features of the cross section of hedge fund returns in our sample.

The final set of empirical estimates of the smoothing process Eqs. (21) through (23) is for the linear regression model of Section 5.2 and is summarized in Table 12. Recall from Section 5.2 that the linear regression estimates of $(\theta_0, \theta_1, \theta_2)$ are based on the assumption that true returns are given by the linear single-factor model Eq. (20) where the factor is the return on the S&P 500 index. To the extent that this assumption is a poor approximation to the true return-generating process, the corresponding smoothing parameter estimates will be flawed as well.

Table 12 reports the means and standard deviations of the estimates $(\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2)$ and $\hat{\xi}$ for each of the categories, as well as the Durbin-Watson statistic and the regression R^2. In contrast to the maximum likelihood estimates of Table 8, the regression estimates are considerably more noisy, with cross-sectional standard deviations for

Table 11
Regressions of maximum likelihood estimated smoothing coefficient $\hat{\theta}_0$ and smoothing index $\hat{\xi}$ on indicator variables for 908 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database

These funds have at least five years of returns history during the period from November 1977 to January 2001, where the maximum likelihood estimators of the MA coefficients $(\theta_0, \theta_1, \theta_2)$ are constrained to sum to one. Absolute values of t-statistics are given in parentheses. The indicator variables are OPEN (one if the fund is open, zero otherwise); the fund categories (one if the fund belongs to the category, zero otherwise); USBASED (one if the fund is based in the US, zero otherwise); and geographical focus categories (one if the geographical focus of the fund is in a given region, zero otherwise, where the regions are USA, Asia Pacific, Western Europe, Eastern Europe, and Africa, respectively).

Regressor	$\hat{\theta}_0$	$\hat{\xi}$	$\hat{\theta}_0$	$\hat{\xi}$
Constant	1.059	1.314	1.086	1.407
	(42.05)	(17.26)	(35.62)	(15.21)
US Equity Hedge	−0.077	−0.255	−0.076	−0.257
	(2.56)	(2.82)	(2.36)	(2.62)
European Equity Hedge	−0.212	−0.531	−0.260	−0.676
	(3.79)	(3.14)	(4.06)	(3.48)
Asian Equity Hedge	−0.086	−0.254	−0.076	−0.248
	(0.78)	(0.77)	(0.65)	(0.71)
Global Equity Hedge	−0.113	−0.366	−0.084	−0.294
	(2.19)	(2.35)	(1.55)	(1.80)
Dedicated Shortseller	0.001	−0.128	0.001	−0.130
	(0.01)	(0.45)	(0.01)	(0.46)
Fixed-Income Directional	−0.262	−0.583	−0.258	−0.575
	(3.73)	(2.75)	(3.64)	(2.67)
Convertible Fund (Long Only)	−0.180	−0.264	−0.188	−0.289
	(2.73)	(1.32)	(2.83)	(1.44)
Event-Driven	−0.218	−0.514	−0.223	−0.527
	(6.69)	(5.23)	(6.57)	(5.12)
Nondirectional/Relative Value	−0.211	−0.432	−0.194	−0.399
	(6.07)	(4.12)	(5.37)	(3.64)
Global Macro	−0.075	−0.271	−0.049	−0.230
	(1.38)	(1.66)	(0.86)	(1.33)
Global Opportunity	−0.282	−0.656	−0.275	−0.647
	(1.17)	(0.9)	(1.14)	(0.89)
Natural Resources	−0.109	−0.363	−0.112	−0.372
	(0.78)	(0.86)	(0.8)	(0.87)
Pure Leveraged Currency	0.069	0.152	0.073	0.158
	(1.32)	(0.96)	(1.37)	(0.98)
Pure Managed Futures	0.101	0.210	0.116	0.244
	(3.00)	(2.06)	(3.29)	(2.29)
Pure Emerging Market	−0.195	−0.486	−0.189	−0.481
	(5.34)	(4.40)	(4.33)	(3.63)
Pure Property	0.173	0.304	0.195	0.339
	(0.72)	(0.42)	(0.80)	(0.46)
Fund of Funds	−0.178	−0.433	−0.170	−0.419
	(5.66)	(4.57)	(5.28)	(4.29)
OPEN	−0.040	−0.077	−0.024	−0.041
	(2.03)	(1.30)	(1.17)	(0.65)
USBASED			0.018	0.030
			(1.00)	(0.54)

Table 11 (*continued*)

Regressor	$\hat{\theta}_0$	$\hat{\xi}$	$\hat{\theta}_0$	$\hat{\xi}$
GF-GLB			−0.050	−0.139
			(2.37)	(2.19)
GF-USA			−0.053	−0.144
			(2.45)	(2.19)
GF-APC			−0.049	−0.128
			(1.59)	(1.37)
GF-WEU			0.018	0.052
			(0.61)	(0.58)
GF-EEU			−0.011	−0.045
			(0.26)	(0.35)
GF-AFR			0.016	0.054
			(0.28)	(0.31)
Sample size	908	908	891	891
Adjusted R^2 (%)	16.5	9.0	17.7	9.9

the coefficients that are often an order of magnitude larger than the means, and in almost every case larger than the standard deviations of Table 8. For example, the average $\hat{\theta}_0$ for the Not Categorized category is 0.659, but the standard deviation is 8.696. The mean of $\hat{\theta}_0$ for Fixed-Income Directional funds is −1.437 and the standard deviation is 6.398. These results are not unexpected given the role that the linear single-factor model plays in the estimation process. If true returns contain additional common factors, then the linear regression approach Eq. (62) will yield biased and inconsistent estimators for the smoothing parameters in Eqs. (21) through (23).

The R^2 statistics in Table 12 yield some indication of the likelihood of omitted factors among the different categories. The highest mean R^2s are for the US Equity Hedge, Dedicated Shortseller, and Convertible Fund (Long Only) categories, with values of 26.1%, 43.0%, and 25.0%, respectively, which is consistent with the fact that our single factor is the S&P 500.[21] However, several categories have mean R^2's below 10%, implying relatively poor explanatory power for the single-factor model and, therefore, noisy and unreliable estimates of the smoothing process.

Overall, the results in Table 12 suggest that the linear regression approach is dominated by the maximum likelihood procedure, and that while the regression coefficients of lagged market returns provide some insight into the net market exposure of some funds, they are considerably less useful for making inferences about illiquidity and smoothed returns.

6.4. Illiquidity vs. smoothing

To address the issue of systematic versus idiosyncratic effects of illiquidity and return-smoothing, we estimate the more general linear factor model of smoothing

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[21] We have omitted the Global Opportunity category from this comparison despite its R^2 of 30.9% because it contains only a single fund.

Table 12

Means and standard deviations for linear regression estimates of an MA(2) smoothing process for individual hedge fund returns

Means and standard deviations of linear regression estimates of MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$ under the assumption of a linear single-factor model for R_t where the factor is the total return of the S&P 500 Index, for 908 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database with at least five years of returns history during the period from November 1977 to January 2001.

Category	N	$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$		D.W.		R^2 (%)	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	111	0.659	8.696	0.459	9.363	−0.118	2.260	167.540	1432.164	1.86	0.33	13.6	16.8
US Equity Hedge	162	0.695	1.502	0.094	0.415	0.211	1.220	4.427	38.503	1.80	0.28	26.1	17.5
European Equity Hedge	22	0.561	0.155	0.340	0.683	0.099	0.708	1.386	4.287	1.76	0.19	18.0	10.8
Asian Equity Hedge	5	0.405	0.252	0.323	0.179	0.272	0.088	0.425	0.086	1.82	0.24	7.4	4.5
Global Equity Hedge	27	0.846	0.400	0.057	0.331	0.097	0.364	1.115	0.913	1.65	0.22	22.1	19.8
Dedicated Shortseller	7	0.908	0.182	0.115	0.101	−0.023	0.203	0.911	0.360	1.94	0.16	43.0	14.7
Fixed-Income Directional	13	−1.437	6.398	4.385	14.552	−1.948	8.179	320.099	1151.235	1.62	0.29	13.1	9.9
Convertible Fund (Long Only)	15	0.538	0.262	0.343	0.152	0.120	0.178	0.536	0.219	1.53	0.24	25.0	15.0
Event-Driven	109	0.582	0.479	0.293	0.493	0.125	0.225	0.958	3.568	1.63	0.36	18.0	11.3
Nondirectional/Relative Value	85	0.643	4.057	0.895	2.321	−0.538	4.761	45.493	244.844	1.62	0.46	12.2	9.2
Global Macro	24	0.558	0.349	0.194	0.341	0.247	0.190	0.674	0.409	1.76	0.28	13.5	8.0
Global Opportunity	1	0.619	—	0.191	—	0.191	—	0.455	—	1.55	—	30.9	—
Natural Resources	3	0.691	0.138	0.106	0.087	0.203	0.068	0.551	0.163	1.88	0.31	2.6	0.9
Pure Leveraged Currency	26	−0.080	1.376	0.360	1.066	0.720	1.987	7.363	24.457	1.89	0.16	4.4	6.0
Pure Managed Futures	93	−0.032	1.588	0.696	1.649	0.336	1.611	8.350	26.079	1.99	0.26	5.9	5.9
Pure Emerging Market	72	0.899	0.801	0.157	0.465	−0.056	0.452	1.883	6.486	1.51	0.29	22.8	10.6
Pure Property	1	1.319	—	−0.024	—	−0.295	—	1.828	—	2.31	—	15.4	—
Fund of Funds	132	0.674	1.516	−0.049	2.566	0.375	3.059	18.694	171.624	1.63	0.30	21.6	15.0
All	908	0.554	3.534	0.366	3.950	0.081	2.401	34.268	529.561	1.73	0.34	17.8	15.1

Eqs. (67) through (69) with $k = 3$ for the subset of Convertible Funds (Long Only), which consists of 15 funds in our sample of 908 funds. We take as our common factor Λ_t the Merrill Lynch Convertible Securities Index and estimate the linear regression equation via maximum likelihood and then renormalize the MA coefficients according to Eq. (69) and recompute the standard errors accordingly. Table 13 contains the regression coefficients as well as the smoothing coefficients, and t-statistics are reported instead of standard errors because we have specific null hypotheses to test as described in Section 5.2.

The estimates in Table 13 show that including a common factor can have a significant impact on the smoothing parameter estimates. For example, the value of $\hat{\theta}_0$ for Fund 1146 under the smoothing process Eqs. (21) through (23) is 0.689, and its t-statistic under the null hypothesis that $\theta_0 = 1$ is -6.01. However, under the linear factor specification Eqs. (67) through (69), the smoothing coefficient estimate becomes 1.172 with a t-statistic of 0.96. Nevertheless, for other funds in our sample of 15, the smoothing parameter estimates are virtually unchanged by including the contemporaneous and lagged common factors. For example, the value of $\hat{\theta}_0$ for Fund 4243 under the smoothing process Eqs. (21) through (23) and the linear factor model Eqs. (67) through (69) is 0.645 and 0.665, respectively, with t-statistics of -8.18 and -5.36, respectively.

We see from Table 13 that the Convertible Securities Index is statistically significant at the 5% level for most, but not all, of the 15 funds, and that its first lag is significant for only four funds (818, 2245, 4204, and 4326), and its second lag is significant for only two funds (1908 and 4216). For five of these six funds, the lagged-index coefficients are positive in sign, which is consistent with the smoothing model Eqs. (68) through (69) (assuming that the funds' contemporaneous factor loadings and smoothing parameters are positive). For Fund 4216, the smoothing parameter estimate $\hat{\theta}_0$ is still significantly different from one even after accounting for the common factor, but for Fund 4204, it is not.

It is tempting to conclude from these results that the linear factor model Eqs. (67) through (69) is capable of distinguishing between systematic illiquidity and idiosyncratic return-smoothing behavior. For example, we could argue that those funds which continue to exhibit significant smoothing parameters $\hat{\theta}_0$ even after accounting for common factors must be engaged in return-smoothing behavior. However, several caveats must be kept in mind before reaching such conclusions. First, we cannot be certain that the Merrill Lynch Convertible Securities Index is the appropriate common factor for these funds, despite the TASS classification. Some funds may be involved in complex trading strategies involving convertible securities while others are engaged in simpler buy-and-hold strategies.[22] Regressing a fund's returns on a highly serially correlated common factor that is not directly relevant to

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[22] In particular, of the 15 Convertible Fund (Long Only) funds in our sample, 12 funds involve long-only positions in convertible bonds, possibly with a limited degree of leverage, but with no equity or credit protection, and the remaining three funds (4204, 4145, and 4326) are convertible arbitrage funds that involve long positions in convertible bonds and short positions in the corresponding stocks. We thank Stephen Jupp of TASS for providing us with this information.

Table 13

Maximum likelihood estimates of linear regression smoothing process with MA(2) errors for 15 Convertible (Long Only) Hedge Funds

Maximum likelihood estimates of linear regression smoothing process with MA(2) errors, $R_t^o = \mu + \gamma_0 \Lambda_t + \gamma_1 \Lambda_{t-1} + \gamma_2 \Lambda_{t-2} + u_t$, $u_t = \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$, subject to the normalization $1 = \theta_0 + \theta_1 + \theta_2$, for all 15 Convertible Funds (Long Only) among the sample of 908 hedge funds in the TASS hedge fund combined (live and graveyard) database with at least five years of returns history during the period from November 1977 to January 2001. The factor Λ_t is taken to be the Merrill Lynch Convertible Securities Index. For comparison, maximum likelihood estimates of θ_0, θ_1, θ_2, and t-statistics from the MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, subject to the normalization $1 = \theta_0 + \theta_1 + \theta_2$, are provided below each of the main rows. t-statistics are computed with respect to the null hypothesis that the coefficient is zero, except for $\hat{\theta}_0$, for which the null hypothesis $\theta_0 = 1$ is used, hence the t-statistic is computed as $(\hat{\theta}_0 - 1)/\text{SE}(\hat{\theta}_0)$.

Code	T	Period	$\hat{\mu}$ (t-stat)	$\hat{\gamma}_0$ (t-stat)	$\hat{\gamma}_1$ (t-stat)	$\hat{\gamma}_2$ (t-stat)	Maximum likelihood smoothing parameter estimates						
							$\hat{\theta}_0$	$t(\hat{\theta}_0)$	$\hat{\theta}_1$	$t(\hat{\theta}_1)$	$\hat{\theta}_2$	$t(\hat{\theta}_2)$	$\hat{\xi}$
31	90	199307–200012	−1.159 (−2.28)	1.171 (11.51)	0.036 (0.36)	−0.110 (−0.92)	0.903	−0.55	−0.296	−2.39	0.393	6.42	1.06
							0.794	−3.24	0.083	1.27	0.123	1.94	0.65
818	72	199502–200101	−0.754 (−1.66)	0.387 (5.84)	0.168 (1.98)	0.014 (0.15)	0.986	−0.10	0.139	1.31	−0.124	−0.96	1.01
							0.759	−3.46	0.261	4.71	−0.019	−0.28	0.64
843	79	199407–200101	−0.716 (−1.86)	0.340 (4.50)	0.144 (1.61)	0.106 (0.97)	0.971	−0.18	0.094	0.93	−0.065	−0.51	0.96
							0.847	−2.02	0.159	2.34	−0.006	−0.08	0.74
1146	94	199010–199807	−1.503 (−7.59)	1.043 (13.15)	0.138 (1.75)	−0.029 (−0.39)	1.172	0.96	−0.044	−0.30	−0.128	−0.98	1.39
							0.689	−6.01	0.187	3.79	0.124	2.40	0.53
1148	73	199501–200101	−0.966 (−3.07)	0.617 (10.49)	0.106 (1.25)	−0.009 (−0.11)	1.125	0.51	0.312	3.16	−0.437	−1.72	1.55
							0.795	−2.81	0.211	3.39	−0.006	−0.08	0.68
1908	81	199405–200101	−1.191 (−1.37)	1.325 (10.51)	0.121 (0.82)	−0.425 (−2.09)	0.989	−0.06	0.052	0.42	−0.041	−0.37	0.98
							0.909	−1.12	0.140	1.93	−0.049	−0.60	0.85
2245	67	199507–200101	−0.607 (−1.27)	0.287 (3.48)	0.174 (2.37)	0.034 (0.31)	0.938	−0.36	0.087	0.70	−0.025	−0.22	0.89
							0.870	−1.53	0.173	2.34	−0.043	−0.50	0.79
2564	63	199511–200101	−0.621 (−1.10)	0.291 (3.22)	0.108 (1.39)	0.123 (1.36)	0.866	−0.67	0.279	2.17	−0.145	−0.90	0.85
							0.778	−2.86	0.303	5.45	−0.080	−1.03	0.70
3099	73	199501–200101	0.093 (0.41)	−0.040 (−0.96)	−0.047 (−1.32)	0.018 (0.39)	0.632	−4.10	0.104	1.71	0.264	5.49	0.48
							0.615	−8.31	0.121	2.37	0.263	5.68	0.46
4145	63	199511–200101	−0.487 (−0.73)	0.368 (2.14)	−0.094 (−0.61)	−0.031 (−0.14)	2.042	4.82	0.015	0.03	−1.058	−1.08	5.29
							2.077	4.41	0.161	1.16	−1.238	−5.07	5.87
4204	72	199501–200012	−0.876 (−2.88)	0.303 (4.52)	0.304 (6.37)	0.069 (0.95)	0.769	−1.26	0.122	0.85	0.109	1.46	0.62
							0.678	−5.53	0.217	4.08	0.105	1.80	0.52
4216	104	199206–200101	−0.115 (−0.61)	0.012 (0.56)	0.012 (0.50)	0.079 (2.09)	0.650	−4.60	0.151	3.47	0.200	4.60	0.48
							0.663	−7.48	0.134	2.84	0.202	4.49	0.50
4243	118	199101–200010	−0.322 (−1.07)	0.036 (0.59)	0.110 (1.70)	0.081 (1.15)	0.665	−5.36	0.270	6.24	0.066	0.89	0.52
							0.645	−8.18	0.279	7.76	0.076	1.76	0.50
4295	82	199403–200012	−0.014 (−0.07)	0.039 (0.51)	−0.032 (−0.72)	0.014 (0.21)	0.894	−0.61	−0.031	−0.22	0.136	1.63	0.82
							0.893	−1.37	0.009	0.10	0.098	1.26	0.81
4326	149	198807–200101	−0.154 (−2.10)	0.082 (6.02)	0.039 (2.60)	0.021 (1.24)	0.700	−4.71	0.229	6.40	0.071	1.40	0.55
							0.662	−8.62	0.243	7.02	0.095	2.43	0.51

that fund's investment process will, nevertheless, have an effect on the smoothing-parameter estimates $\hat{\theta}_k$, and the effect could be in either direction depending on the relation between the common factor and the fund's observed returns. Second, even if a common factor can account for much of the serial correlation in a fund's observed returns, an explanation for the source of the factor's serial correlation is still required. If the fund is a buy-and-hold version of the common factor, e.g., a fund-of-funds designed to replicate the CSFB/Tremont Convertible Arbitrage Index, then it is of small comfort to investors in such a fund-of-funds that there is not much smoothing in observed returns beyond what is already present in the common factor. And finally, no econometric model can fully capture the many qualitative and often subjective characteristics of a fund's investment process, and such information is likely to be of particular relevance in distinguishing between illiquidity and smoothed returns at the fund level.

These caveats suggest that a more comprehensive econometric analysis of hedge-fund returns is worthwhile, with particular emphasis on constructing common factors for hedge funds with similar investment mandates and processes. By developing a better understanding for the common risk exposures that certain hedge funds share, researchers can differentiate between systematic and idiosyncratic illiquidity and provide investors and managers with a more refined set of tools with which to optimize their investment plans.

6.5. Smoothing-adjusted Sharpe ratio estimates

For each of the 908 funds in our sample, we compute annual Sharpe ratios in three ways relative to a benchmark return of zero: the standard method ($\sqrt{12}$ times the ratio of the mean monthly return to the monthly return standard deviation); the serial-correlation-adjusted method in Lo (2002); and the small-sample method described in Section 5.4. The results are summarized in Table 14.

The largest differences between standard and smoothing-adjusted Sharpe ratios are found in the same categories that the smoothing-process estimates of Section 6.2 identified as the most illiquid: Fixed-Income Directional (20.3% higher average Sharpe ratio relative to SR^{**}), Convertible Fund (Long Only) (17.8%), Nondirectional/Relative Value (16.0%), Pure Emerging Market (16.3%), and Fund of Funds (17.8%). For the two categories Dedicated Shortseller and Managed Futures, the bias is reversed, a result of negative serial correlation in their returns. For the other categories, Table 14 shows that the smoothing-adjusted Sharpe ratios are similar in magnitude to the usual estimates. These differences across categories suggest the importance of taking illiquidity and smoothed returns into account in evaluating the performance of hedge funds.

7. Conclusions

Although there are several potential explanations for serial correlation in asset returns, we argue in this paper that the serial correlation present in the returns of

Table 14

Means and standard deviations of Sharpe ratios of 908 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database

These funds have at least five years of returns history during the period from November 1977 to January 2001. SR is the standard Sharpe ratio, SR* is the smoothing-adjusted Sharpe ratio of Lo (2002), and SR** is the smoothing-adjusted Sharpe ratio using $\hat{\sigma}_{NW}$. All Sharpe ratios are computed with respect to a zero benchmark.

Category		Sharpe ratios for combined sample					
	N	SR		SR*		SR**	
		Mean	SD	Mean	SD	Mean	SD
Not Categorized	111	1.12	1.09	1.06	0.87	1.00	0.82
US Equity Hedge	162	1.26	0.75	1.31	0.75	1.23	0.69
European Equity Hedge	22	1.43	0.74	1.43	0.80	1.33	0.74
Asian Equity Hedge	5	0.50	0.39	0.52	0.39	0.49	0.37
Global Equity Hedge	27	0.90	0.61	0.95	0.66	0.89	0.61
Dedicated Shortseller	7	0.28	0.59	0.32	0.64	0.30	0.61
Fixed-Income Directional	13	2.02	2.35	1.80	2.23	1.68	2.06
Convertible Fund (Long Only)	15	1.83	1.20	1.66	0.85	1.55	0.80
Event-Driven	109	2.36	1.45	2.21	1.57	2.08	1.47
Nondirectional/Relative Value	85	2.20	1.86	2.03	2.39	1.89	2.22
Global Macro	24	1.08	0.67	1.14	0.73	1.07	0.70
Global Opportunity	1	−0.56	—	−0.39	—	−0.37	—
Natural Resources	3	0.60	0.25	0.56	0.23	0.52	0.21
Pure Leveraged Currency	26	0.63	0.49	0.65	0.50	0.61	0.47
Pure Managed Futures	93	0.54	0.55	0.63	0.60	0.60	0.56
Pure Emerging Market	72	0.39	0.45	0.36	0.44	0.34	0.40
Pure Property	1	0.42	—	0.45	—	0.41	—
Fund of Funds	132	1.44	1.01	1.30	0.88	1.22	0.82
All	908	1.32	1.24	1.27	1.27	1.19	1.18

hedge funds is due primarily to illiquidity and smoothed returns. Using a simple econometric model in which observed returns are a finite moving-average of unobserved economic returns, we generate empirically realistic levels of serial correlation for historical hedge-fund returns while, at the same time, explaining the findings of Asness et al. (2001) regarding the significance of lagged market returns in market-model regressions for hedge funds. Although our moving-average specification is similar to some of the early models of nonsynchronous trading, our motivation is quite different and is meant to cover a broader set of factors that give rise to serial correlation and smoothed returns, even in the presence of synchronously recorded prices.

Maximum likelihood estimates of our smoothing model for the returns of 908 hedge funds in the TASS Hedge Fund database yield empirically plausible estimates of smoothing coefficients and suggest that simple time-series measures such as our smoothing index serve as useful proxies for a hedge fund's illiquidity risk exposure.

In some cases, our econometric model may also be useful for flagging possible cases of deliberate performance-smoothing behavior, although additional information will need to be gathered before any firm conclusions regarding such behavior can be made. Regardless of the sources of serial correlation, illiquidity exposure is the main implication and this has potentially important consequences for both managers and investors. Therefore, we also develop a set of tools for quantifying the degree of smoothing in the data and adjusting for smoothed returns in computing performance statistics such as means, variances, market betas, and Sharpe ratios, and derive their asymptotic distributions using continuous-record asymptotics that can better accommodate the small sample sizes of most hedge-fund datasets.

Our empirical results suggest several applications for our econometric model of illiquidity and smoothed returns. Despite the general consistency of our empirical results with common intuition regarding the levels of illiquidity among the various hedge-fund investment styles, the variation in estimated smoothing coefficients within each category indicates that there are better ways of categorizing hedge funds. Given the importance of liquidity for the typical hedge-fund investor, it may be appropriate to subdivide each style category into "liquidity tranches" defined by our smoothing index. This may prove to be especially useful in identifying and avoiding the potential wealth transfers between new and existing investors that can occur from the opportunistic timing of hedge-fund investments and redemptions. Alternatively, our smoothing parameter estimates may be used to compute illiquidity exposure measures for portfolios of hedge funds or fund of funds, which may serve as the basis for a more systematic approach to managing portfolios that include alternative investments.

Although we focus on hedge funds in this paper, our analysis may be applied to other investments and asset classes, e.g., real estate, venture capital, private equity, art and other collectibles, and other assets for which illiquidity and smoothed returns are even more problematic, and where the estimation of smoothing profiles can be particularly useful for providing investors with risk transparency. More generally, our econometric model may be applied to a number of other contexts in which there is a gap between reported results and economic realities. For example, recent events surrounding the collapse of Enron and other cases of corporate accounting irregularities have created renewed concerns about "earnings management", in which certain corporations are alleged to have abused accounting conventions so as to smooth earnings, presumably to give the appearance of stability and consistent growth. Beneish (2001) and Healy and Wahlen (1999) provide reviews of the extensive literature on earnings management. The impact of such smoothing can sometimes be "undone" using an econometric model such as ours.

There are a number of outstanding issues regarding our analysis of illiquidity and smoothed returns that warrant further study. Perhaps the most pressing issue is whether the proximate source of smoothing is inadvertent or deliberate. Our linear regression model with contemporaneous and lagged common factors can serve as the starting point for distinguishing between systematic illiquidity versus idiosyncratic smoothing behavior. However, this issue is likely to require additional information about each fund along the lines of Chandar and Bricker's (2002) study, e.g., the size

of the fund, the types of the securities in which the fund invests, the accounting conventions used to mark the portfolio, the organization's compensation structure, and other operational aspects of the fund. With these additional pieces of information, we can construct more relevant common factors for our linear-regression framework, or relate the cross-sectional variation in smoothing coefficients to assets under management, security type, fee structure, and other characteristics, yielding a more complete picture of the sources of smoothed returns.

It may also be fruitful to view the hedge-fund industry from a broader perspective, one that acknowledges the inherent capacity constraints of certain types of strategies as well as the time lags involved in shifting assets from one type to another. Because of the inevitable cross-sectional differences in the performance of hedge-fund styles, assets often flow in loosely coordinated fashion from one style to another, albeit under various institutional restrictions such as calendar-specific periods of liquidity, tiered redemption schedules, redemption fees, and other frictions. The interactions between asset flows and institutional rigidities (especially over time) sometimes cause statistical side-effects that include periodicities in performance and volatility, time-varying correlations, structural breaks, and under certain conditions, serial correlation. The dynamics of the hedge-fund industry are likely to be quite different than that of more traditional investment products, hence the "ecological framework" of Niederhoffer (1998, Chapter 15), Farmer (1998), and Farmer and Lo (1999), or the system dynamics approach of Getmansky and Lo (2004) might be more conducive paradigms for addressing these issues.

Finally, from the hedge-fund investor's perspective, a natural extension of our analysis is to model illiquidity directly and quantify the illiquidity premium associated with each hedge-fund investment style, perhaps in a linear-factor framework such as Chordia et al. (2000, 2001, 2002) and Pastor and Stambaugh (2003). Whether such factor models can forecast liquidity crises like August 1998, or show that there are systematic illiquidity factors that are common to categories of hedge funds, are open questions that are particularly important in the context of hedge-fund investments. We plan to address these and other related questions in our ongoing research.

Appendix A

Proofs of Propositions 3–5 are provided in Sections A.1–A.3, respectively. Section A.4 provides definitions of the 17 categories from TASS, and Section A.5 contains additional empirical results.

A.1. Proof of Proposition 3

The constraint Eq. (49) can be used to substitute out θ_0, hence we need only consider (θ_1, θ_2). Now it is well known that in the standard MA(2) specification

where the usual identification condition is used in place of Eq. (49), i.e.,

$$X_t = \varepsilon_t + a\varepsilon_{t-1} + b\varepsilon_{t-2}, \tag{A.1}$$

the asymptotic distribution of the maximum likelihood estimators $(\hat{a}, \hat{b})$ is given by

$$\sqrt{T}\left(\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} - \begin{bmatrix} a \\ b \end{bmatrix}\right) \overset{a}{\sim} \mathcal{N}(0, \mathbf{V}), \tag{A.2}$$

where

$$\mathbf{V} \equiv \begin{bmatrix} 1 - b^2 & a(1-b) \\ a(1-b) & 1-b^2 \end{bmatrix}. \tag{A.3}$$

But under our normalization Eq. (49), there is a simple functional relation between $(\hat{a}, \hat{b})$ and $(\hat{\theta}_1, \hat{\theta}_2)$:

$$\hat{\theta}_1 = \frac{\hat{a}}{1 + \hat{a} + \hat{b}}, \quad \hat{\theta}_2 = \frac{\hat{b}}{1 + \hat{a} + \hat{b}}. \tag{A.4}$$

Therefore, we can apply the delta method to obtain the asymptotic distribution of $(\hat{\theta}_1, \hat{\theta}_2)$ as

$$\sqrt{T}\left(\begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix} - \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}\right) \overset{a}{\sim} \mathcal{N}(0, \mathbf{JVJ}'), \tag{A.5}$$

where the matrix $\mathbf{J}$ is Jacobian associated with Eq. (A.4):

$$\mathbf{J} = \frac{1}{(1 + a + b)^2} \begin{bmatrix} 1 + b & -a \\ -b & 1 + a \end{bmatrix}. \tag{A.6}$$

Then we have

$$\begin{aligned} &\mathbf{JVJ}' \\ &= \frac{1}{(1+a+b)^3} \\ &\quad \times \begin{bmatrix} -(1+b)(-1+a-ab+b^2) & b(-1+a-ab+b^2) \\ b(-1+a-ab+b^2) & -(-1+b)(-1+a-ab+b^2) \end{bmatrix} \end{aligned} \tag{A.7}$$

and solving for a and b as a function of θ_1 and θ_2 using Eq. (A.4) and substituting these expressions into Eq. (A.7) yields the desired result.

The process X_t is invertible if and only if the roots of characteristic polynomial

$$f(x) = \theta_0 x^2 + \theta_1 x + \theta_2 \tag{A.8}$$

lie inside the unit circle in the complex plane. It is easy to see that this is equivalent to the condition that the roots of

$$f(x) = f\left(\frac{z+1}{z-1}\right) = \frac{z^2 + 2(1 - \theta_1 - 2\theta_2)z + 1 - 2\theta_1}{(z-1)^2} \tag{A.9}$$

lie in the left half-plane (Samuelson, 1941, was perhaps the first to state this result). Applying the Routh-Hurwitz necessary and sufficient conditions to Eq. (A.9) then yields the desired result.

A.2. Proof of Proposition 4

Theorem 1 (Herrndorf, 1984). *If* $\{\varepsilon_t\}$ *satisfies the following assumptions*:

(A1) $E[\varepsilon_t] = \mu$ *for all* t.
(A2) $\sup_t E[|\varepsilon_t - \mu|^\beta] < \infty$ *for some* $\beta > 2$.
(A3) $0 < \sigma_o^2 = \lim_{T \to \infty} E\left[\frac{1}{T}(\sum_{j=1}^n (\varepsilon_j - \mu))^2\right] < \infty$.
(A4) $\{\varepsilon_t\}$ *is strong-mixing with mixing coefficients* α_k *that satisfy*

$$\sum_{j=1}^\infty \alpha_j^{1-2/\beta} < \infty \tag{A.10}$$

then as n *increases without bound*,

$$W_n(s) \equiv \frac{1}{\sigma_o \sqrt{n}} \sum_{j=1}^{[ns]} (\varepsilon_j - \mu) \;\;\Rightarrow\;\; W(s), \quad s \in [0, 1] \tag{A.11}$$

where [ns] *denotes the greater integer less than or equal to ns and* '⇒' *denotes weak convergence.*

With these results in hand, we are ready to prove Proposition 4. Let returns R_t be given by

$$R_t = \varepsilon_t, \tag{A.12}$$

where ε_t satisfies assumptions (A.1) through (A.4) of Theorem 1, and recall:

$$\hat{\mu} = \frac{1}{T} \sum_1^T \varepsilon_t, \tag{A.13}$$

$$\hat{\sigma}_{\text{NW}}^2 = \frac{1}{T} \sum_1^T (\varepsilon_t - \hat{\mu})^2 + \frac{2}{T} \sum_{j=1}^\theta \left(1 - \frac{j}{\theta+1}\right) \sum_{t=j+1}^T (\varepsilon_t - \hat{\mu})(\varepsilon_{t-j} - \hat{\mu}), \tag{A.14}$$

$$= I_1 + I_2, \tag{A.15}$$

where

$$I_1 \equiv \frac{1}{T} \sum_1^T (\varepsilon_t - \hat{\mu})^2 \tag{A.16}$$

and

$$I_2 \equiv \frac{2}{T} \sum_{j=1}^{\theta} \left(1 - \frac{j}{\theta+1}\right) \sum_{t=j+1}^{T} (\varepsilon_t - \hat{\mu})(\varepsilon_{t-j} - \hat{\mu}). \tag{A.17}$$

Observe that

$$I_1 = \frac{1}{T} \sum_1^T (\varepsilon_t - \hat{\mu})^2 = \frac{1}{T} \sum_1^T \varepsilon_t^2 - \hat{\mu}^2 \xrightarrow{p} \sigma_{\varepsilon}^2, \tag{A.18}$$

where

$$\sigma_{\varepsilon}^2 \equiv \lim_{t \to \infty} \frac{1}{T} \sum_1^T \varepsilon_t^2 = \mathrm{E}[\varepsilon_t^2]. \tag{A.19}$$

Now the second term I_2 can be written as

$$I_2 \frac{T(m+1)}{2} = \sum_{j=0}^{m} (m+1-j) \sum_{t=j+1}^{T} (\varepsilon_t - \hat{\mu})(\varepsilon_{t-j} - \hat{\mu}) = J_1 + J_2 + J_3, \tag{A.20}$$

where

$$J_1 \equiv \sum_{j=0}^{m} (m+1-j) \sum_{t=j+1}^{T} \varepsilon_t \varepsilon_{t-j}, \tag{A.21}$$

$$J_2 \equiv \sum_{j=0}^{m} (m+1-j) \sum_{t=j+1}^{T} \hat{\mu}(\varepsilon_t + \varepsilon_{t-j}) \tag{A.22}$$

and

$$J_3 \equiv \sum_{j=0}^{m} (m+1-j) \sum_{t=j+1}^{T} \hat{\mu}^2. \tag{A.23}$$

Consider the first term in Eq. (A.20)

$$
\begin{aligned}
J_1 &= \sum_{j=0}^{m}(m+1-j)\sum_{t=j+1}^{T}\varepsilon_t\varepsilon_{t-j} = m(\varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \cdots + \varepsilon_{T-1}\varepsilon_T) \\
&\quad + (m-1)(\varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_4 + \cdots + \varepsilon_{T-2}\varepsilon_T) \\
&\quad + (\varepsilon_1\varepsilon_{m+1} + \varepsilon_2\varepsilon_{m+2} + \cdots + \varepsilon_{T-m}\varepsilon_T)
\end{aligned}
\tag{A.24}
$$

$$
\begin{aligned}
&= \varepsilon_1((S_2 - S_1) + (S_3 - S_1) + \cdots + (S_{m+1} - S_1)) + \cdots \\
&\quad + \varepsilon_{T-m}((S_{T-m+1} - S_{T-m}) + \cdots + (S_T - S_{T-m})) \\
&\quad + \varepsilon_{T-m+1}((S_{T-m+2} - S_{T-m+1}) + \cdots + 2(S_T - S_{T-m+1})) + \cdots \\
&\quad + \varepsilon_{T-1}(m(S_T - S_{T-1}))
\end{aligned}
\tag{A.25}
$$

$$
\begin{aligned}
&= \varepsilon_1(S_2 + S_3 + \cdots + S_{m+1}) - m\varepsilon_1 S_1 + \cdots \\
&\quad + \varepsilon_{T-m}(S_{T-m+1} + \cdots + S_T) - m\varepsilon_{T-m}S_{T-m} \\
&\quad + \varepsilon_{T-m+1}(S_{T-m+2} + \cdots + 2S_T) - m\varepsilon_{T-m+1}S_{T-m+1} + \cdots \\
&\quad + \varepsilon_{T-1}mS_T - m\varepsilon_{T-1}S_{T-1}
\end{aligned}
\tag{A.26}
$$

$$
\begin{aligned}
&= S_2 S_1 + S_3 S_2 + \cdots + S_{m+1}S_m + S_{m+2}(S_{m+1} - S_1) + \cdots \\
&\quad + S_T(S_{T-1} - S_{T-m-1}) + S_T(\varepsilon_{T-m+1} + \cdots + m\varepsilon_{T-1}) - m\sum_{t=1}^{T-1}\varepsilon_t S_t
\end{aligned}
\tag{A.27}
$$

$$
\begin{aligned}
&= \sum_{t=1}^{T-1}S_t^2 - (m-1)\sum_{t=1}^{T-1}\varepsilon_t S_t - \sum_{t=1}^{T-(m+1)}S_t S_{t+m+1} \\
&\quad + S_T\left(mS_{T-1} - \sum_{t=T-m}^{T-1}S_t\right).
\end{aligned}
\tag{A.28}
$$

When the Functional Central Limit Theorem is applied to Eq. (A.28), we have

$$
\begin{aligned}
J_1/T^2 &\Rightarrow \sigma^2\left(\int_0^1 W(r)(W(r) - W(\min(r+\lambda,1)))\,dr\right) \\
&\quad - \frac{(m-1)}{2T}(\sigma^2 W^2(1) + \sigma_\varepsilon^2) + \frac{m}{T}\sigma^2 W^2(1).
\end{aligned}
\tag{A.29}
$$

Now the second term in Eq. (A.20) can be rewritten as

$$
J_2 = -\sum_{j=1}^{m}(m+1-j)\sum_{t=j+1}^{T}\hat{\mu}(\varepsilon_t + \varepsilon_{t-j})
\tag{A.30}
$$

$$
= -\hat{\mu}\sum_{j=1}^{m}(m+1-j)(S_T - S_j + S_{T-j})
\tag{A.31}
$$

$$
= -\frac{m(m+1)}{2}\hat{\mu}S_T - \hat{\mu}\sum_{j=1}^{m}(m+1-j)(S_{T-j} - S_j)
\tag{A.32}
$$

and applying the Functional Central Limit Theorem to Eq. (A.32) yields

$$J_2/T^2 \Rightarrow -\frac{m(m+1)}{2T^2}\sigma^2 W^2(1)$$

$$- \sigma^2 W(1) \int_0^\lambda (\lambda - r)(W(1-r) - W(r))\, dr. \tag{A.33}$$

The third term of Eq. (A.20) can be rewritten in a similar manner

$$J_3 = \sum_{j=1}^m (m+1-j) \sum_{t=j+1}^T \hat{\mu}^2 = \hat{\mu}^2 \sum_{j=1}^m (m+1-j)(T-j-1) \tag{A.34}$$

$$= \hat{\mu}^2 \sum_{j=1}^m (j^2 - j(T+m) + (m+1)(T-1)) \tag{A.35}$$

$$= \hat{\mu}^2 \frac{m(m+1)(3T-m-5)}{6} \tag{A.36}$$

and applying the Functional Central Limit Theorem to Eq. (A.36) yields

$$J_3/T^2 \Rightarrow \sigma^2 W^2(1)\frac{(1+m)m(3T-m-5)}{6T^3}. \tag{A.37}$$

Combining Eqs. (A.29), (A.33) and (A.37) shows that

$$I_2 \Rightarrow \frac{2}{\lambda}\sigma^2 \left(\int_0^1 W(r)[W(r) - W(\min(r+\lambda,1))]\, dr \right.$$

$$- W(1) \int_0^\lambda (\lambda - r)(W(1-r) - W(r))\, dr \Big)$$

$$+ \sigma^2 W^2(1)\left(1 - \frac{\lambda^2}{3}\right) - \sigma_\varepsilon^2 \tag{A.38}$$

and summing Eq. (A.18) and Eq. (A.38) yields the desired result.

A.3. Proof of Proposition 5

Given the weak convergence of $\hat{\bar{\sigma}}^2_{\mathrm{NW}}$ to the functional $f(W)$ in Eq. (87), Proposition 5 is an almost trivial consequence of the following well-known result:

Theorem 2 (Extended Continuous Mapping Theorem). [23] *Let h_n and h be measurable mappings from $D[0,1]$* (*the space of all functions on $[0,1]$ that are right*

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[23] See Billingsley (1968) for a proof.

continuous with left-hand limits) to itself and denote by E the set of $x \in D[0, 1]$ such that $h_n(x_n) \rightarrow h(x)$ fails to hold for some sequence x_n converging to x. If $W_n(s) \Rightarrow W(s)$ and E is of Wiener-measure zero, i.e. $P(W \in E) = 0$, then $h_n(W_n) \Rightarrow h(W)$.

A.4. TASS fund category definitions

The following is a list of category descriptions, taken directly from TASS documentation, that define the criteria used by TASS in assigning funds in their database to one of 17 possible categories:

Equity Hedge: This directional strategy involves equity-oriented investing on both the long and short sides of the market. The objective is not to be market neutral. Managers have the ability to shift from value to growth, from small to medium to large capitalization stocks, and from a net long position to a net short position. Managers can use futures and options to hedge. The focus can be regional, such as long/short US or European equity, or sector specific, such as long and short technology or healthcare stocks. Long/short equity funds tend to build and hold portfolios that are substantially more concentrated than those of traditional stock funds. US equity hedge, European equity hedge, Asian equity hedge and Global equity hedge is the regional focus.

Dedicated Shortseller: Short biased managers take short positions in mostly equities and derivatives. The short bias of a manager's portfolio must be constantly greater than zero to be classified in this category.

Fixed-Income Directional: This directional strategy involves investing in Fixed income markets only on a directional basis.

Convertible Arbitrage: This strategy is identified by hedge investing in the convertible securities of a company. A typical investment is to be long the convertible bond and short the common stock of the same company. Positions are designed to generate profits from the fixed income security as well as the short sale of stock, while protecting principal from market moves.[24]

Event-Driven: This strategy is defined as special situations investing designed to capture price movement generated by a significant pending corporate event such as a merger, corporate restructuring, liquidation, bankruptcy, or reorganization. There are three popular subcategories in event-driven strategies: risk (merger) arbitrage, distressed/high yield securities, and Regulation *D*.

Nondirectional/Relative Value: This investment strategy is designed to exploit equity and/or fixed-income market inefficiencies and usually involves being simultaneously long and short matched-market portfolios of the same size within a country. Market-neutral portfolios are designed to be either beta or currency neutral, or both.

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[24] Note that the category closest to this definition in the TASS database is Convertible fund (long only), which is related to but different from convertible arbitrage—see footnote 16 for details. TASS recently changed their style classifications, and now defines Convertible arbitrage as a distinct category.

Global Macro: Global macro managers carry long and short positions in any of the world's major capital or derivative markets. These positions reflect their views on overall market direction as influenced by major economic trends and or events. The portfolios of these funds can include stocks, bonds, currencies, and commodities in the form of cash or derivatives instruments. Most funds invest globally in both developed and emerging markets.

Global Opportunity: Global macro managers carry long and short positions in any of the world's major capital or derivative markets on an opportunistic basis. These positions reflect their views on overall market direction as influenced by major economic trends and/or events. The portfolios of these funds can include stocks, bonds, currencies, and commodities in the form of cash or derivatives instruments. Most funds invest globally in both developed and emerging markets.

Natural Resources: This trading strategy has a focus on natural resources around the world.

Leveraged Currency: This strategy invests in currency markets around the world.

Managed Futures: This strategy invests in listed financial and commodity futures markets and currency markets around the world. The managers are usually referred to as Commodity Trading Advisors, or CTAs. Trading disciplines are generally systematic or discretionary. Systematic traders tend to use price and market specific information (often technical) to make trading decisions, while discretionary managers use a judgmental approach.

Emerging Markets: This strategy involves equity or fixed-income investing in emerging markets around the world.

Property: The main focus of the investments is property.

Fund of Funds: A multi-manager fund will employ the services of two or more trading advisors or hedge funds who will be allocated cash by the trading manager to trade on behalf of the fund.

A.5. Supplementary empirical results

In Tables A.1–A.7, we provide corresponding empirical results to Tables 7–14 but with Live and Graveyard funds separated in order to show the effects of survivorship bias. Of course, since we still apply our five-year minimum returns history filter to both groups, there is still some remaining survivorship bias. Tables A.1 and A.2 contain summary statistics for the two groups of funds; Tables A.3 and A.4 report summary statistics for the maximum likelihood estimates of the smoothing model Eqs. (21) through (23); Tables A.5 and A.6 report similar statistics for the regression model estimates Eq. (62) of the smoothing model, and Table A.7 contains smoothing-adjusted Sharpe ratios for the two groups of funds. Finally, Table A.8 corresponds to the regressions of Table 11 but with dependent variables $\hat{\theta}_0$ and $\hat{\xi}$ estimated by maximum likelihood under the alternate constraint Eq. (75).

Table A.1

Means and standard deviations of basic summary statistics for funds in the TASS Hedge Fund Live database

The means and standard deviations of basic summary statistics for 651 hedge funds in the TASS Hedge Fund Live database with at least five years of returns history during the period from November 1977 to January 2001. The columns 'p-Value(Q)' contain means and standard deviations of p-values for the Box-Pierce Q-statistic for each fund using the first six autocorrelations of returns.

Category	N	Annual mean		Annual SD		Skewness		Kurtosis		$\hat{\rho}_1(\%)$		$\hat{\rho}_2(\%)$		$\hat{\rho}_3(\%)$		p-Value(Q)	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	44	12.08	6.04	13.08	10.44	0.05	0.73	2.37	2.54	8.8	15.9	2.3	14.5	1.4	9.8	31.8	32.0
US Equity Hedge	139	23.53	10.92	21.86	11.95	0.20	0.98	3.79	4.89	8.0	13.7	0.7	11.4	−4.1	11.2	33.0	29.0
European Equity Hedge	19	18.75	7.33	15.36	6.53	0.51	0.73	3.10	2.86	13.0	11.1	13.5	9.0	−2.9	8.9	16.4	20.7
Asian Equity Hedge	5	9.16	7.96	21.13	6.64	0.50	0.85	2.32	0.73	11.7	13.0	5.0	5.6	−5.4	13.4	40.2	21.1
Global Equity Hedge	24	15.04	7.67	18.28	6.31	−0.16	1.06	4.17	5.83	10.5	9.9	−0.6	8.8	−4.1	5.4	41.8	26.9
Dedicated Shortseller	6	−0.17	11.15	23.01	14.51	0.44	0.17	2.24	2.63	2.6	7.4	−0.9	9.0	−5.6	7.0	28.3	28.7
Fixed-Income Directional	12	10.17	3.39	9.80	9.08	−1.00	1.33	5.78	7.49	22.8	15.8	14.9	13.4	1.7	14.1	17.6	21.8
Convertible Fund (Long Only)	12	14.05	5.54	11.67	7.39	−0.26	1.88	7.16	10.93	22.9	11.9	5.3	14.9	−1.0	11.9	15.2	22.1
Event-Driven	97	15.61	6.42	8.96	6.66	−0.58	1.74	8.10	11.71	20.0	16.6	5.8	13.2	−0.9	12.1	25.2	28.6
Nondirectional/Relative Value	63	13.23	5.91	7.89	4.84	−0.61	1.74	6.08	9.52	18.4	26.0	15.3	14.0	5.4	15.8	14.6	24.0
Global Macro	15	14.92	7.82	17.34	10.46	0.79	1.07	4.67	4.79	5.3	11.8	−1.3	12.2	−0.9	8.4	41.9	24.2
Global Opportunity	1	−17.39	—	31.03	—	−0.48	—	2.80	—	23.1	—	14.1	—	0.0	—	20.8	—
Natural Resources	1	14.49	—	18.42	—	0.30	—	−0.14	—	−10.1	—	−1.6	—	7.8	—	40.6	—
Pure Leveraged Currency	15	10.36	5.23	15.27	6.25	0.81	0.75	3.22	4.04	7.4	8.7	−3.3	9.9	−3.1	7.1	31.2	18.7
Pure Managed Futures	28	10.54	6.19	16.46	7.62	0.42	1.55	5.92	12.20	0.9	13.2	−4.1	11.4	−2.5	8.6	32.4	29.9
Pure Emerging Market	54	9.08	10.21	26.12	11.66	−0.56	1.21	4.97	5.86	18.3	12.0	4.1	11.9	−1.7	9.2	30.7	29.4
Pure Property	1	3.96	—	9.41	—	−1.33	—	5.43	—	−23.1	—	2.3	—	7.3	—	18.9	—
Fund of Funds	115	11.45	4.95	10.16	5.75	−0.29	1.27	4.83	6.17	17.3	14.0	6.2	11.7	−1.2	9.1	26.5	27.9
All live	651	14.88	9.33	15.05	10.65	−0.15	1.36	5.00	7.63	13.7	16.5	4.6	13.1	−1.2	11.1	28.0	28.3

Table A.2

Means and standard deviations of summary statistics for funds in the TASS Hedge Fund Graveyard database

Means and standard deviations of basic summary statistics for 258 hedge funds in the TASS Hedge Fund Graveyard database with at least five years of returns history during the period from November 1977 to January 2001. The columns 'p-Value(Q)' contain means and standard deviations of p-values for the Box-Pierce Q-statistic for each fund using the first six autocorrelations of returns.

Category	N	Annual mean		Annual SD		Skewness		Kurtosis		$\hat{\rho}_1(\%)$		$\hat{\rho}_2(\%)$		$\hat{\rho}_3(\%)$		p-Value(Q)	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	67	9.92	7.48	17.26	11.52	0.17	1.23	4.53	5.88	3.4	17.5	−0.1	14.9	−0.1	12.4	39.0	29.2
US Equity Hedge	23	16.47	7.82	20.68	9.67	0.03	0.83	3.97	5.90	7.2	12.5	0.4	9.7	−5.3	10.1	38.9	27.6
European Equity Hedge	3	13.33	3.85	10.58	4.17	−0.38	1.06	2.15	1.89	10.5	11.7	−3.7	14.8	3.8	7.6	51.4	9.1
Asian Equity Hedge	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Global Equity Hedge	3	3.06	7.16	10.41	5.29	−0.88	0.97	2.29	3.48	27.5	12.9	3.2	16.9	7.8	12.4	42.2	36.6
Dedicated Shortseller	1	3.32	—	12.78	—	0.78	—	2.46	—	15.5	—	−13.5	—	−12.0	—	52.4	—
Fixed-Income Directional	1	5.58	—	6.49	—	−0.15	—	0.50	—	7.7	—	4.2	—	4.1	—	24.0	—
Convertible Fund (Long Only)	3	18.55	2.34	8.48	2.77	0.09	0.31	2.05	2.71	20.8	16.5	9.7	4.5	−6.5	16.7	25.2	40.7
Event-Driven	12	13.22	8.67	13.98	22.43	−0.66	2.45	9.55	11.85	26.6	17.8	11.4	14.8	5.9	13.1	29.7	37.5
Nondirectional/Relative Value	22	10.58	8.26	9.02	5.67	−1.18	1.61	7.47	9.56	17.8	14.5	3.5	16.3	3.9	10.4	24.4	26.8
Global Macro	10	19.49	7.70	21.29	7.83	0.14	0.86	4.64	9.40	10.1	19.2	1.6	8.7	−0.8	8.0	20.6	13.3
Global Opportunity	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Natural Resources	2	9.84	5.78	19.15	1.38	0.56	0.59	1.95	1.24	12.5	9.0	13.6	11.9	−1.0	6.7	51.1	61.2
Pure Leveraged Currency	11	7.83	9.18	19.78	11.48	−0.07	1.28	3.90	6.13	2.2	8.2	−10.1	7.8	−6.1	9.3	41.4	34.8
Pure Managed Futures	65	9.19	10.35	24.02	19.41	0.13	1.25	4.51	5.98	−0.6	12.9	−3.7	9.4	−3.9	11.5	36.7	28.9
Pure Emerging Market	18	11.06	13.11	28.05	21.07	−1.08	2.12	9.79	12.04	20.2	11.6	5.1	10.9	−0.7	8.4	38.1	34.0
Pure Property	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Fund of Funds	17	8.24	5.82	9.50	4.87	−0.04	1.53	4.39	6.25	19.2	14.6	2.2	9.6	2.9	10.2	24.6	24.9
All Graveyard	258	10.80	9.17	18.61	15.21	−0.14	1.45	5.17	7.40	8.2	16.8	0.2	12.7	−1.0	11.4	35.2	29.3

Table A.3

Means and standard deviations of maximum likelihood estimates of an MA(2) smoothing process for funds in the TASS Hedge Fund Live database

Means and standard deviations of maximum likelihood estimates of an MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$, for 651 funds in the TASS Hedge Fund Live database with at least five years of returns history during the period from November 1977 to January 2001.

Category	N	MA(2) with constrained sum								MA(2) with constrained σ_η							
		$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$		$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	44	0.973	0.273	0.049	0.149	−0.023	0.174	1.075	0.704	0.942	0.269	0.046	0.145	−0.015	0.160	1.006	0.584
US Equity Hedge	139	0.951	0.213	0.065	0.159	−0.016	0.156	1.004	0.562	0.974	0.194	0.071	0.147	−0.008	0.151	1.035	0.416
European Equity Hedge	19	0.790	0.115	0.077	0.088	0.133	0.073	0.673	0.196	0.902	0.170	0.096	0.101	0.156	0.095	0.893	0.317
Asian Equity Hedge	5	0.942	0.296	0.053	0.188	0.005	0.109	0.998	0.712	0.904	0.198	0.064	0.167	0.013	0.096	0.883	0.404
Global Equity Hedge	24	0.926	0.143	0.095	0.091	−0.020	0.099	0.903	0.271	0.994	0.173	0.100	0.098	−0.018	0.100	1.045	0.346
Dedicated Shortseller	6	1.031	0.220	0.008	0.109	−0.039	0.135	1.131	0.528	1.149	0.152	0.024	0.115	−0.031	0.142	1.368	0.363
Fixed-Income Directional	12	0.752	0.177	0.160	0.105	0.088	0.126	0.652	0.280	0.733	0.203	0.157	0.104	0.093	0.128	0.633	0.303
Convertible Fund (Long Only)	12	0.868	0.392	0.189	0.069	−0.057	0.387	1.074	1.516	0.896	0.300	0.202	0.086	−0.027	0.263	0.997	0.650
Event-Driven	97	0.816	0.170	0.148	0.128	0.036	0.115	0.747	0.285	0.856	0.240	0.154	0.126	0.038	0.113	0.842	0.439
Nondirectional/Relative Value	63	0.809	0.253	0.077	0.279	0.114	0.133	0.831	0.697	0.764	0.296	0.070	0.275	0.110	0.127	0.777	0.793
Global Macro	15	1.006	0.186	0.046	0.115	−0.053	0.168	1.088	0.422	1.054	0.214	0.052	0.120	−0.049	0.188	1.205	0.525
Global Opportunity	1	0.737	—	0.162	—	0.100	—	0.580	—	0.632	—	0.139	—	0.086	—	0.426	—
Natural Resources	1	1.111	—	−0.113	—	0.003	—	1.246	—	0.861	—	−0.088	—	0.002	—	0.748	—
Pure Leveraged Currency	15	0.970	0.132	0.074	0.091	−0.044	0.121	0.986	0.271	0.926	0.099	0.076	0.084	−0.035	0.115	0.892	0.186
Pure Managed Futures	28	1.138	0.305	−0.034	0.171	−0.104	0.219	1.470	0.904	1.129	0.253	−0.030	0.165	−0.090	0.190	1.406	0.621
Pure Emerging Market	54	0.836	0.165	0.148	0.087	0.016	0.139	0.774	0.328	0.797	0.162	0.141	0.082	0.021	0.136	0.706	0.296
Pure Property	1	1.232	—	−0.306	—	0.074	—	1.618	—	0.951	—	−0.236	—	0.057	—	0.963	—
Fund of Funds	115	0.848	0.220	0.119	0.132	0.034	0.136	0.817	0.522	0.859	0.232	0.122	0.126	0.035	0.133	0.841	0.470
All	651	0.891	0.231	0.093	0.160	0.016	0.158	0.906	0.578	0.899	0.242	0.096	0.155	0.021	0.148	0.923	0.512

Table A.4

Means and standard deviations of maximum likelihood estimates of an MA(2) smoothing process for funds in the TASS Hedge Fund Graveyard database

Means and standard deviations of maximum likelihood estimates of an MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$, for 257 funds in the TASS Hedge Fund Graveyard database with at least five years of returns history during the period from November 1977 to January 2001.

Category	N	MA(2) with constrained sum								MA(2) with constrained σ_η							
		$\hat\theta_0$		$\hat\theta_1$		$\hat\theta_2$		$\hat\xi$		$\hat\theta_0$		$\hat\theta_1$		$\hat\theta_2$		$\hat\xi$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	67	1.079	0.353	−0.009	0.194	−0.070	0.267	1.399	1.226	0.993	0.268	−0.007	0.165	−0.049	0.214	1.130	0.613
US Equity Hedge	23	0.945	0.179	0.053	0.113	0.002	0.115	0.951	0.363	0.993	0.212	0.065	0.126	0.000	0.123	1.063	0.471
European Equity Hedge	3	0.984	0.251	0.073	0.107	−0.057	0.147	1.041	0.544	0.859	0.201	0.065	0.094	−0.049	0.127	0.788	0.378
Asian Equity Hedge	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Global Equity Hedge	3	0.790	0.099	0.220	0.043	−0.011	0.057	0.683	0.131	0.670	0.196	0.179	0.003	−0.014	0.044	0.508	0.242
Dedicated Shortseller	1	0.999	—	0.166	—	−0.165	—	1.052	—	1.035	—	0.172	—	−0.171	—	1.131	—
Fixed-Income Directional	1	0.902	—	0.064	—	0.033	—	0.820	—	0.977	—	0.069	—	0.036	—	0.961	—
Convertible Fund (Long Only)	3	0.753	0.121	0.138	0.113	0.109	0.013	0.617	0.166	0.885	0.062	0.182	0.155	0.131	0.033	0.854	0.158
Event-Driven	12	0.761	0.174	0.184	0.135	0.055	0.101	0.670	0.286	0.740	0.270	0.166	0.115	0.058	0.099	0.666	0.438
Nondirectional/Relative Value	22	0.844	0.196	0.142	0.099	0.014	0.137	0.796	0.317	0.768	0.203	0.135	0.101	0.014	0.127	0.673	0.332
Global Macro	9	0.869	0.155	0.135	0.061	−0.005	0.126	0.813	0.302	0.818	0.188	0.123	0.050	−0.002	0.124	0.732	0.351
Global Opportunity	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Natural Resources	2	0.811	0.115	0.079	0.030	0.110	0.085	0.687	0.164	0.845	0.050	0.084	0.039	0.119	0.099	0.741	0.055
Pure Leveraged Currency	11	1.272	0.446	−0.024	0.147	−0.248	0.331	1.980	2.016	1.107	0.163	0.000	0.107	−0.183	0.169	1.319	0.424
Pure Managed Futures	65	1.129	0.325	−0.053	0.233	−0.076	0.159	1.467	1.295	1.082	0.244	−0.037	0.193	−0.066	0.144	1.292	0.642
Pure Emerging Market	18	0.817	0.148	0.154	0.085	0.029	0.109	0.730	0.254	0.789	0.181	0.145	0.078	0.028	0.107	0.692	0.313
Pure Property	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Fund of Funds	17	0.845	0.165	0.168	0.103	−0.013	0.107	0.789	0.250	0.823	0.208	0.158	0.090	−0.012	0.106	0.761	0.324
All	257	1.001	0.314	0.042	0.190	−0.042	0.196	1.177	1.064	0.950	0.259	0.046	0.164	−0.031	0.162	1.025	0.569

Table A.5
Means and standard deviations of linear regression estimates of an MA(2) smoothing process under the assumption of a linear single-factor model for hedge funds in the TASS Hedge Fund Live database

Means and standard deviations of linear regression estimates of an MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$ under the assumption of a linear single-factor model for R_t where the factor is the total return of the S&P 500 Index, for 651 hedge funds in the TASS Hedge Fund Live database with at least five years of returns history during the period from November 1977 to January 2001.

Category	N	$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$		D.W.		$R^2(\%)$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	44	1.598	4.141	−0.613	3.212	0.015	2.294	34.919	108.677	1.81	0.32	15.0	13.9
US Equity Hedge	139	0.713	1.557	0.080	0.433	0.207	1.268	4.747	41.449	1.80	0.28	24.8	17.4
European Equity Hedge	19	0.581	0.113	0.344	0.735	0.075	0.756	1.527	4.614	1.76	0.19	18.0	9.3
Asian Equity Hedge	5	0.405	0.252	0.323	0.179	0.272	0.088	0.425	0.086	1.82	0.24	7.4	4.5
Global Equity Hedge	24	0.913	0.323	0.028	0.331	0.059	0.354	1.163	0.956	1.69	0.20	24.3	19.9
Dedicated Shortseller	6	0.886	0.189	0.104	0.106	0.010	0.202	0.869	0.376	1.96	0.17	41.1	15.0
Fixed-Income Directional	12	−1.495	6.678	4.689	15.156	−2.193	8.493	346.600	1198.277	1.61	0.30	14.1	9.7
Convertible Fund (Long Only)	12	0.513	0.239	0.354	0.160	0.133	0.162	0.506	0.152	1.51	0.22	23.8	15.2
Event-Driven	97	0.550	0.279	0.330	0.288	0.120	0.232	0.638	0.661	1.64	0.36	18.0	11.4
Nondirectional/Relative Value	63	0.607	2.010	0.762	1.808	−0.369	3.342	19.268	80.833	1.63	0.51	12.7	9.5
Global Macro	15	0.541	0.428	0.182	0.424	0.277	0.161	0.765	0.491	1.84	0.29	12.0	7.0
Global Opportunity	1	0.619	—	0.191	—	0.191	—	0.455	—	1.55	—	30.9	—
Natural Resources	1	0.640	—	0.175	—	0.186	—	0.474	—	2.20	—	2.3	—
Pure Leveraged Currency	15	0.049	1.238	0.596	0.971	0.354	1.241	4.234	10.045	1.85	0.15	4.7	7.4
Pure Managed Futures	28	−0.290	1.930	0.776	2.494	0.514	2.027	14.502	41.277	1.96	0.26	3.8	3.4
Pure Emerging Market	54	0.928	0.916	0.125	0.527	−0.053	0.506	2.226	7.470	1.52	0.28	22.0	10.8
Pure Property	1	1.319	—	−0.024	—	−0.295	—	1.828	—	2.31	—	15.4	—
Fund of Funds	115	0.616	1.287	−0.070	2.694	0.454	3.231	19.776	183.336	1.63	0.30	22.0	14.8
All	651	0.636	1.888	0.256	2.649	0.109	2.284	16.252	184.926	1.71	0.34	19.1	14.7

Table A.6
Means and standard deviations of linear regression estimates of an MA(2) smoothing process under the assumption of a linear single-factor model for hedge funds in the TASS Hedge Fund Graveyard database

Means and standard deviations of linear regression estimates of an MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$ under the assumption of a linear single-factor model for R_t where the factor is the total return of the S&P 500 Index, for 257 hedge funds in the TASS Hedge Fund Graveyard database with at least five years of returns history during the period from November 1977 to January 2001.

Category	N	$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$		D.W.		$R^2(\%)$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	67	0.042	10.672	1.163	11.752	−0.206	2.251	254.635	1841.566	1.89	0.33	12.6	18.5
US Equity Hedge	23	0.586	1.129	0.182	0.266	0.232	0.900	2.492	8.220	1.81	0.26	33.8	16.6
European Equity Hedge	3	0.434	0.329	0.312	0.173	0.254	0.275	0.493	0.173	1.78	0.17	18.3	21.0
Asian Equity Hedge	—	—	—	—	—	—	—	—	—	—	—	—	—
Global Equity Hedge	3	0.308	0.626	0.294	0.271	0.397	0.361	0.737	0.293	1.40	0.26	4.1	1.3
Dedicated Shortseller	1	1.040	—	0.176	—	−0.216	—	1.159	—	1.84	—	54.7	—
Fixed-Income Directional	1	−0.738	—	0.748	—	0.990	—	2.084	—	1.83	—	1.6	—
Convertible Fund (Long Only)	3	0.636	0.383	0.295	0.127	0.068	0.272	0.655	0.428	1.63	0.32	30.1	16.3
Event-Driven	12	0.837	1.223	−0.008	1.244	0.171	0.159	3.543	10.629	1.50	0.38	18.1	11.0
Nondirectional/Relative Value	22	0.748	7.341	1.275	3.420	−1.023	7.574	120.592	461.012	1.59	0.28	10.5	8.2
Global Macro	9	0.587	0.173	0.214	0.139	0.198	0.234	0.522	0.138	1.63	0.22	16.0	9.3
Global Opportunity	—	—	—	—	—	—	—	—	—	—	—	—	—
Natural Resources	2	0.717	0.184	0.072	0.089	0.211	0.095	0.589	0.211	1.71	0.19	2.7	1.2
Pure Leveraged Currency	11	−0.255	1.590	0.037	1.150	1.218	2.690	11.631	36.324	1.95	0.16	4.0	3.5
Pure Managed Futures	65	0.079	1.418	0.662	1.132	0.260	1.406	5.700	15.336	2.00	0.26	6.8	6.5
Pure Emerging Market	18	0.811	0.231	0.256	0.151	−0.067	0.237	0.852	0.460	1.46	0.30	24.9	9.8
Pure Property	—	—	—	—	—	—	—	—	—	—	—	—	—
Fund of Funds	17	1.064	2.613	0.093	1.469	−0.158	1.366	11.379	40.134	1.59	0.30	19.1	17.0
All	257	0.346	5.928	0.643	6.112	0.010	2.680	79.904	950.698	1.80	0.34	14.4	15.6

Table A.7

Means and standard deviations of Sharpe ratios of funds in the TASS Hedge Funds Live and Graveyard databases

Means and standard deviations of Sharpe ratios of 651 funds in the TASS Hedge Funds Live database and 257 funds in the TASS Hedge Fund Graveyard database, both with at least five years of returns history during the period from November 1977 to January 2001. SR is the standard Sharpe ratio, SR* is the smoothing-adjusted Sharpe ratio of Lo (2002), and SR** is the smoothing-adjusted Sharpe ratio using $\hat{\sigma}_{NW}$. All Sharpe ratios are computed with respect to a zero benchmark.

Category	Sharpe ratios for live funds							Sharpe ratios for graveyard funds						
	N	SR		SR*		SR**		N	SR		SR*		SR**	
		Mean	SD	Mean	SD	Mean	SD		Mean	SD	Mean	SD	Mean	SD
Not Categorized	44	1.55	1.34	1.41	1.00	1.33	0.94	67	0.84	0.79	0.83	0.70	0.77	0.65
US Equity Hedge	139	1.31	0.77	1.36	0.76	1.27	0.70	23	0.99	0.54	1.06	0.60	1.00	0.57
European Equity Hedge	19	1.44	0.78	1.46	0.85	1.36	0.78	3	1.36	0.57	1.26	0.49	1.18	0.46
Asian Equity Hedge	5	0.50	0.39	0.52	0.39	0.49	0.37	0	—	—	—	—	—	—
Global Equity Hedge	24	0.95	0.60	1.00	0.66	0.94	0.61	3	0.55	0.75	0.51	0.59	0.48	0.55
Dedicated Shortseller	6	0.28	0.65	0.33	0.70	0.31	0.67	1	0.26	—	0.30	—	0.28	—
Fixed-Income Directional	12	2.11	2.42	1.87	2.31	1.75	2.14	1	0.86	—	0.87	—	0.84	—
Convertible Fund (Long Only)	12	1.67	1.18	1.48	0.72	1.38	0.69	3	2.45	1.26	2.39	1.07	2.23	1.00
Event-Driven	97	2.45	1.46	2.31	1.58	2.17	1.47	12	1.61	1.16	1.47	1.39	1.37	1.29
Nondirectional/Relative Value	63	2.36	1.82	2.21	2.52	2.06	2.35	22	1.73	1.93	1.53	1.91	1.42	1.76
Global Macro	15	1.12	0.77	1.30	0.86	1.22	0.82	9	1.02	0.51	0.87	0.34	0.82	0.32
Global Opportunity	1	−0.56	—	−0.39	—	−0.37	—	0	—	—	—	—	—	—
Natural Resources	1	0.79	—	0.75	—	0.69	—	2	0.50	0.27	0.46	0.23	0.43	0.22
Pure Leveraged Currency	15	0.83	0.50	0.83	0.50	0.78	0.47	11	0.35	0.32	0.41	0.39	0.38	0.37
Pure Managed Futures	28	0.81	0.71	0.92	0.66	0.87	0.63	65	0.43	0.43	0.51	0.52	0.48	0.49
Pure Emerging Market	54	0.39	0.48	0.37	0.48	0.34	0.44	18	0.39	0.36	0.34	0.30	0.32	0.27
Pure Property	1	0.42	—	0.45	—	0.41	—	0	—	—	—	—	—	—
Fund of Funds	115	1.48	1.02	1.35	0.90	1.27	0.83	17	1.15	0.98	0.95	0.70	0.89	0.65
All	651	1.50	1.29	1.45	1.35	1.36	1.26	257	0.85	0.95	0.83	0.90	0.78	0.84

Table A.8
Regressions of the maximum likelihood estimated smoothing coefficient and smoothing index on indicator variables for hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database
Regressions of the maximum likelihood estimated smoothing coefficient $\hat{\theta}_0$ and smoothing index $\hat{\xi}$ on indicator variables for 908 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database with at least five years of returns history during the period from November 1977 to January 2001, where the maximum likelihood estimator $\hat{\sigma}_\eta$ is constrained to equal a nonparametric estimator $\tilde{\sigma}_\eta$ of the innovation standard deviation. Absolute values of t-statistics are given in parentheses. The indicator variables are OPEN (one if the fund is open, zero otherwise); the fund categories (one if the fund belongs to the category, zero otherwise); USBASED (one if the fund is based in the US, zero otherwise); and geographical focus categories (one if the geographical focus of the fund is in a given region, zero otherwise, where the regions are USA, Asia Pacific, Western Europe, Eastern Europe, and Africa, respectively).

Regressor	$\hat{\theta}_0$	$\hat{\xi}$	$\hat{\theta}_0$	$\hat{\xi}$
Constant	1.007	1.152	1.029	1.212
	(42.02)	(21.91)	(35.56)	(19.04)
US Equity Hedge	0.020	−0.009	0.026	−0.004
	(0.69)	(0.15)	(0.85)	(0.07)
European Equity Hedge	−0.063	−0.175	−0.120	−0.281
	(1.19)	(1.50)	(1.97)	(2.10)
Asian Equity Hedge	−0.054	−0.167	−0.030	−0.131
	(0.52)	(0.73)	(0.28)	(0.54)
Global Equity Hedge	0.005	−0.053	0.032	0.006
	(0.11)	(0.49)	(0.64)	(0.05)
Dedicated Shortseller	0.178	0.291	0.184	0.293
	(2.01)	(1.50)	(2.06)	(1.49)
Fixed-Income Directional	−0.203	−0.386	−0.196	−0.375
	(3.04)	(2.63)	(2.92)	(2.54)
Convertible Fund (Long Only)	−0.060	−0.073	−0.064	−0.086
	(0.95)	(0.53)	(1.01)	(0.62)
Event-Driven	−0.116	−0.228	−0.124	−0.247
	(3.74)	(3.36)	(3.85)	(3.49)
Nondirectional/Relative Value	−0.194	−0.303	−0.180	−0.281
	(5.88)	(4.18)	(5.26)	(3.73)
Global Macro	0.004	−0.029	0.034	0.026
	(0.09)	(0.26)	(0.63)	(0.22)
Global Opportunity	−0.314	−0.598	−0.309	−0.59
	(1.37)	(1.19)	(1.35)	(1.18)
Natural Resources	−0.096	−0.281	−0.095	−0.284
	(0.72)	(0.96)	(0.72)	(0.97)
Pure Leveraged Currency	0.042	0.019	0.045	0.022
	(0.85)	(0.17)	(0.89)	(0.20)
Pure Managed Futures	0.133	0.267	0.142	0.284
	(4.16)	(3.78)	(4.25)	(3.87)
Pure Emerging Market	−0.161	−0.344	−0.146	−0.311
	(4.63)	(4.50)	(3.53)	(3.42)
Pure Property	−0.056	−0.190	−0.028	−0.146
	(0.25)	(0.38)	(0.12)	(0.29)
Fund of Funds	−0.099	−0.211	−0.092	−0.199
	(3.32)	(3.21)	(3.03)	(2.96)
OPEN	−0.061	−0.128	−0.055	−0.118
	(3.27)	(3.13)	(2.78)	(2.73)

Table A.8 (*continued*)

Regressor	$\hat{\theta}_0$	$\hat{\xi}$	$\hat{\theta}_0$	$\hat{\xi}$
USBASED			0.012	0.022
			(0.68)	(0.58)
GF-GLB			−0.034	−0.078
			(1.69)	(1.78)
GF-USA			−0.043	−0.087
			(2.09)	(1.92)
GF-APC			−0.051	−0.104
			(1.73)	(1.61)
GF-WEU			0.048	0.079
			(1.70)	(1.28)
GF-EEU			−0.058	−0.133
			(1.46)	(1.51)
GF-AFR			0.071	0.137
			(1.27)	(1.12)
Sample size	908	908	891	891
Adjusted R^2(%)	15.7	11.4	16.9	12.6

References

Ackermann, C., McEnally, R., Ravenscraft, D., 1999. The performance of hedge funds: risk, return, and incentives. Journal of Finance 54, 833–874.

Agarwal, V., Naik, N., 2000a. Performance evaluation of hedge funds with buy-and-hold and option-based strategies. Hedge Fund Centre Working Paper No. HF-003, London Business School.

Agarwal, V., Naik, N., 2000b. On taking the "alternative" route: the risks, rewards, and performance persistence of hedge funds. Journal of Alternative Investments 2, 6–23.

Agarwal, V., Naik, N., 2000c. Multi-period performance persistence analysis of hedge funds source. Journal of Financial and Quantitative Analysis 35, 327–342.

Agarwal, V., Naik, N., 2002. Risks and portfolio decisions involving hedge funds. Institute of Finance and Accounting Working Paper No. 364, London Business School.

Asness, C., Krail, R., Liew, J., 2001. Do hedge funds hedge? The Journal of Portfolio Management 28, 6–19.

Atchison, M., Butler, K., Simonds, R., 1987. Nonsynchronous security trading and market index autocorrelation. Journal of Finance 42, 111–118.

Baquero, G., Horst, J., Verbeek, M., 2002. Survival, look-ahead bias and the performance of hedge funds. Unpublished working paper. Erasmus University, Rotterdam, Netherlands.

Beneish, M., 2001. Earnings management: a perspective. Managerial Finance 27, 3–17.

Bhargava, R., Bose, A., Dubofsky, D., 1998. Exploiting international stock market correlations with open-end international mutual funds. Journal of Business, Finance, and Accounting 25, 765–773.

Bhattacharya, S., Pfleiderer, P., 1985. Delegated portfolio management. Journal of Economic Theory 36, 1–25.

Billingsley, P., 1968. Convergence of Probability Measures. Wiley, New York.

Black, F., 1986. Noise. Journal of Finance 41, 529–544.

Boudoukh, J., Richardson, M., Subrahmanyam, M., Whitelaw, R., 2002. Stale prices and strategies for trading mutual funds. Financial Analysts Journal 58, 53–71.

Brockwell, P., Davis, R., 1991. Time Series: Theory and Methods, 2nd Edition. Springer, New York.

Brown, S., Goetzmann, W., 2001. Hedge funds with style. NBER Working Paper No. 8173.

Brown, S., Goetzmann, W., Ibbotson, R., Ross, S., 1992. Survivorship bias in performance studies. Review of Financial Studies 5, 553–580.

Brown, S., Goetzmann, W., Ibbotson, R., 1999. Offshore hedge funds: survival and performance 1989–1995. Journal of Business 72, 91–118.

Brown, S., Goetzmann, W., Liang, B., 2002. Fees on fees in funds of funds. Yale International Center for Finance (ICF) Working Paper No. 02-33.

Brown, S., Goetzmann, W., Park, J., 1997. Conditions for survival: changing risk and the performance of hedge fund managers and CTAs. Yale School of Management Working Paper No. F-59.

Brown, S., Goetzmann, W., Park, J., 2000. Hedge funds and the Asian currency crisis. Journal of Portfolio Management 26, 95–101.

Brown, S., Goetzmann, W., Park, J., 2001. Careers and survival: competition and risks in the hedge fund and CTA industry. Journal of Finance 56, 1869–1886.

Campbell, J., Lo, A., MacKinlay, C., 1997. The Econometrics of Financial Markets. Princeton University Press, Princeton, NJ.

Carpenter, J., 2000. Does option compensation increase managerial risk appetite? Journal of Finance 55, 2311–2331.

Carpenter, J., Dybvig, P., Farnsworth, H., 2001. Portfolio performance and agency. Unpublished working paper. Stern School of Business, New York University.

Carpenter, J., Lynch, A., 1999. Survivorship bias and attrition effects in measures of performance persistence. Journal of Financial Economics 54, 337–374.

Cecchetti, S., Mark, N., 1990. Evaluating empirical tests of asset pricing models. American Economic Review 80, 48–51.

Chalmers, J., Edelen, R., Kadlec, G., 2001. On the perils of security pricing by financial intermediaries: the wildcard option in transacting mutual-fund shares. Journal of Finance 56, 2209–2236.

Chandar, N., Bricker, R., 2002. Incentives, discretion, and asset valuation in closed-end mutual funds. Journal of Accounting Research 40, 1037–1070.

Chordia, T., Roll, R., Subrahmanyam, A., 2000. Commonality in liquidity. Journal of Financial Economics 56, 3–28.

Chordia, T., Roll, R., Subrahmanyam, A., 2001. Market liquidity and trading activity. Journal of Finance 56, 501–530.

Chordia, T., Roll, R., Subrahmanyam, A., 2002. Order imbalance, liquidity, and market returns. Journal of Financial Economics 65, 111–130.

Cohen, K., Hawawini, G., Maier, S., Schwartz, R., Whitcomb, D., 1983a. Estimating and adjusting for the intervalling-effect bias in beta. Management Science 29, 135–148.

Cohen, K., Hawawini, G., Maier, S., Schwartz, R., Whitcomb, D., 1983b. Friction in the trading process and the estimation of systematic risk. Journal of Financial Economics 12, 263–278.

Cohen, K., Maier, S., Schwartz, R., Whitcomb, D., 1978. The returns generation process, returns variance, and the effect of thinness in securities markets. Journal of Finance 33, 149–167.

Cohen, K., Maier, S., Schwartz, R., Whitcomb, D., 1979. On the existence of serial correlation in an efficient securities market. TIMS Studies in the Management Sciences 11, 151–168.

Cohen, K., Maier, S., Schwartz, R., Whitcomb, D., 1986. The Microstructure of Securities Markets. Prentice-Hall, Englewood Cliffs, NJ.

Dimson, E., 1979. Risk measurement when shares are subject to infrequent trading. Journal of Financial Economics 7, 197–226.

Edwards, F., Caglayan, M., 2001. Hedge fund and commodity fund investments in bull and bear markets. The Journal of Portfolio Management 27, 97–108.

Elton, E., Gruber, M., 2003. Incentive fees and mutual funds. Journal of Finance, forthcoming.

Fama, E., 1970. Efficient capital markets: a review of theory and empirical work. Journal of Finance 25, 383–417.

Farmer, D., 1998. Market force, ecology, and evolution. Santa Fe Institute Working Paper No. 98-12-118.

Farmer, D., Lo, A., 1999. Frontiers of finance: evolution and efficient markets. Proceedings of the National Academy of Sciences 96, 9991–9992.

Fisher, J., Gatzlaff, D., Geltner, D., Haurin, D., 2003. Controlling for the impact of variable liquidity in commercial real estate price indices. Real Estate Economics 31, 269–303.

Fisher, J., Geltner, D., Webb, R., 1994. Value indices of commercial real estate: a comparison of index construction methods. Journal of Real Estate Finance and Economics 9, 137–164.

Fisher, L., 1966. Some new stock market indexes. Journal of Business 39, 191–225.

Fung, W., Hsieh, D., 1997a. Empirical characteristics of dynamic trading strategies: the case of hedge funds. Review of Financial Studies 10, 275–302.

Fung, W., Hsieh, D., 1997b. Investment style and survivorship bias in the returns of CTAs: the information content of track records. Journal of Portfolio Management 24, 30–41.

Fung, W., Hsieh, D., 1999. A primer on hedge funds. Journal of Empirical Finance 6, 309–331.

Fung, W., Hsieh, D., 2000. Performance characteristics of hedge funds and commodity funds: natural versus spurious biases. Journal of Financial and Quantitative Analysis 35, 291–307.

Fung, W., Hsieh, D., 2001. The risk in hedge fund strategies: theory and evidence from trend followers. Review of Financial Studies 14, 313–341.

Getmansky, M., Lo, A., 2004. A system dynamics model of the hedge fund industry. Unpublished working paper. MIT Laboratory for Financial Engineering.

Goetzmann, W., Ingersoll, J., Ross, S., 1997. High water marks and hedge fund management contracts. Journal of Finance, forthcoming.

Goetzmann, W., Ingersoll, J., Spiegel, M., Welch, I., 2002. Sharpening Sharpe ratios. National Bureau of Economic Research Working Paper No. W9116.

Goetzmann, W., Ivkovic, Z., Rouwenhorst, G., 2001. Day trading international mutual funds: evidence and policy solutions. Journal of Financial and Quantitative Analysis 36, 287–309.

Goodwin, T., 1993. Business-cycle analysis with a Markov-switching model. Journal of Business and Economic Statistics 11, 331–339.

Granger, C., 1980. Long memory relations and the aggregation of dynamic models. Journal of Econometrics 14, 227–238.

Granger, C., 1988. Aggregation of time series variables—a survey. Federal Reserve Bank of Minneapolis Institute for Empirical Macroeconomics, Discussion Paper 1.

Greene, J., Hodges, C., 2002. The dilution impact of daily fund flows on open-end mutual funds. Journal of Financial Economics 65, 131–158.

Grossman, S., 1976. On the efficiency of competitive stock markets where trades have diverse information. Journal of Finance 31, 573–585.

Grossman, S., Stiglitz, J., 1980. On the impossibility of informationally efficient markets. American Economic Review 70, 393–408.

Gyourko, J., Keim, D., 1992. What does the stock market tell us about real estate returns. AREUEA Journal 20, 457–486.

Hamilton, J., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57, 357–384.

Hamilton, J., 1990. Analysis of time series subject to changes in regime. Journal of Econometrics 45, 39–70.

Hamilton, J., 1996. Specification testing in Markov-switching time series models. Journal of Econometrics 70, 127–157.

Hansen, L., 1982. Large sample properties of generalized method of moments estimators. Econometrica 50, 1029–1054.

Healy, P., Wahlen, J., 1999. A review of the earnings management literature and its implications for standard setting. Accounting Horizons 14, 365–383.

Hendricks, D., Patel, J., Zeckhauser, R., 1997. The J-shape of performance persistence given survivorship bias. Review of Economics and Statistics 79, 161–170.

Herrndorf, N., 1984. A functional central limit theorem for weakly dependent sequences of random variables. Annals of Probability 12, 141–153.

Horst, J., Nijman, T., Verbeek, M., 2001. Eliminating look-ahead bias in evaluating persistence in mutual fund performance. Journal of Empirical Finance 8, 345–373.

Jobson, J., Korkie, R., 1981. Performance hypothesis testing with the Sharpe and Treynor measures. Journal of Finance 36, 889–908.

Kadlec, G., Patterson, D., 1999. A transactions data analysis of nonsynchronous trading. Review of Financial Studies 12, 609–630.

Kandel, S., Stambaugh, R., 1991. Asset returns and intertemporal preferences. Journal of Monetary Economics 27, 39–71.

Kao, D., 2002. Battle for alphas: hedge funds versus long-only portfolios. Financial Analysts Journal 58, 16–36.

Leroy, S., 1973. Risk aversion and the Martingale property of stock returns. International Economic Review 14, 436–446.

Liang, B., 1999. On the performance of hedge funds. Financial Analysts Journal 55, 72–85.

Liang, B., 2000. Hedge funds: the living and the dead. Journal of Financial and Quantitative Analysis 35, 309–326.

Liang, B., 2001. Hedge fund performance: 1990–1999. Financial Analysts Journal 57, 11–18.

Lo, A., 2001. Risk management for hedge funds: introduction and overview. Financial Analysts Journal 57, 16–33.

Lo, A., 2002. The statistics of Sharpe ratios. Financial Analysts Journal 58, 36–50.

Lo, A., MacKinlay, C., 1988. Stock market prices do not follow random walks: evidence from a simple specification test. Review of Financial Studies 1, 41–66.

Lo, A., MacKinlay, C., 1990. An econometric analysis of nonsynchronous trading. Journal of Econometrics 45, 181–212.

Lo, A., MacKinlay, C., 1999. A Non-Random Walk Down Wall Street. Princeton University Press, Princeton, NJ.

Lochoff, R., 2002. Hedge funds and hope. The Journal of Portfolio Management 28, 92–99.

Lucas, R., 1978. Asset prices in an exchange economy. Econometrica 46, 1429–1446.

Newey, W., West, K., 1987. A simple positive definite heteroscedasticity and autocorrelation consistent covariance matrix. Econometrica 55, 703–705.

Niederhoffer, V., 1998. The Education of a Speculator. Wiley, New York.

Pastor, L., Stambaugh, R., 2003. Liquidity risk and expected stock returns. Journal of Political Economy 111, 642–685.

Richardson, M., Stock, J., 1989. Drawing inferences from statistics based on multiyear asset returns. Journal of Financial Economics 25, 323–348.

Ross, S., Zisler, R., 1991. Risk and return in real estate. Journal of Real Estate Finance and Economics 4, 175–190.

Rubinstein, M., 1976. The valuation of uncertain income streams and the pricing of options. Bell Journal of Economics 7, 407–425.

Samuelson, P., 1941. Conditions that the roots of a polynomial be less than unity in absolute value. Annals of Mathematical Statistics 45, 689–693.

Samuelson, P., 1965. Proof that properly anticipated prices fluctuate randomly. Industrial Management Review 6, 41–49.

Schneeweis, T., Spurgin, R., 1996. Survivor bias in commodity trading advisor performance. Journal of Futures Markets 16, 757–772.

Scholes, M., Williams, J., 1977. Estimating betas from nonsynchronous data. Journal of Financial Economics 5, 309–328.

Schwert, G., 1977. Stock exchange seats as capital assets. Journal of Financial Economics 4, 51–78.

Shanken, J., 1987. Nonsynchronous data and the covariance-factor structure of returns. Journal of Finance 42, 221–232.

Sharpe, W., 1994. The Sharpe ratio. Journal of Portfolio Management 21, 49–58.

Spurgin, R., 2001. How to game your Sharpe ratio. The Journal of Alternative Investments 4, 38–46.

Turner, C., Startz, R., Nelson, C., 1989. A Markov model of heteroskedasticity, risk, and learning in the stock market. Journal of Financial Economics 25, 3–22.

Weisman, A., 2002. Informationless investing and hedge fund performance measurement bias. The Journal of Portfolio Management 28, 80–91.

Zitzewitz, E., 2003. Who cares about shareholders? Arbitrage-proofing mutual funds. Journal of Law, Economics, and Organization 19, 245–280.