MIT Sloan Finance Problems and Solutions Collection Finance Theory I

Part 2

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1.4 Forward and Futures

1. During the summer you had to spend some time with your uncle, who is a wheat farmer. Your uncle, knowing you are studying for an MBA at Sloan, asked your help. He is afraid that the price of wheat will fall, which will have a severe impact on his profits. Thus he asks you to compute the 1yr forward price of wheat. He tells you that its current price is $3.4 per bushel and interest rates are at 4%. However, he also says that it is relatively expensive to store wheat for one year. Assume that this cost, which must be paid upfront, runs at about $0.1 per bushel. What is the 1yr forward price of wheat?

2. The Wall Street Journal gives the following futures prices for gold on September 6, 2006:

Maturity	Oct	Dec	Jun 07	Dec 07
Futures price ($/oz)	635.60	641.80	660.60	678.70

and the spot price of gold is $633.50/oz. Compute the (effective annualize) interest rate implied by the futures prices for the corresponding maturities.

3. Suppose that in 3 months the cost of a pound of Colombian coffee will be either $1.25 or $2.25. The current price is $1.75 per pound.

(a) What are the risks faced by a hotel chain who is a large purchaser of coffee?

(b) What are the risks faced by a Colombian coffee farmer?

(c) If the delivery price of coffee turns out to be $2.25, should the farmer have forgone entering into a futures contract? Why or why not?

4. Consider a 6-month forward contract (delivers one unit of the security) on a security that is expected to pay a $1 dividend in three months. The annual risk-free rate of interest is 5%. The security price is $20. What forward price should the contract stipulate, so that the current value of entering into the contract is zero?

5. Spot and futures prices for Gold and the S&P in September 2007 are given below.

	07-September	07-December	08-June
COMEX Gold ($/oz)	$693	$706.42	$726.7
CME S&P 500	$1453.55	$1472.4	$1493.7

Table 1: Gold and S&P 500 Prices on September 7, 2007

(a) Use prices for Gold to calculate the effective annualized interest rate for Dec 2007 and June 2008. Assume that the convenience yield for Gold is zero.

(b) Suppose you are the owner of a small gold mine and would like to fix the revenue generated by your future production. Explain how the futures market enables such hedges.

6. Use the same set of information given in the problem above.

(a) Use S&P 500 future prices to calculate the implied dividend yield on S&P 500. For simplicity, assume you can borrow or deposit money at the rates implied by Gold's futures prices.

(b) Now suppose you believe that we are headed for a slow-down in economic activity and that the dividend yield will be lower than the value implied in part (a). What June-2008 contracts you would buy or sell to make money, assuming your view is correct? Again, assume you can borrow or deposit money at the rates implied by Gold's futures prices.

7. The Wall Street Journal gives the following futures prices for crude oil on September 6, 2006:

Maturity	Oct	Dec	Jun 07	Dec 07
Futures price ($/barrel)	67.50	69.60	72.66	73.49

and the spot price of oil is $67.50/barrel. Use the interest rates you found in the previous problem.

(a) Compute the net convenience yield (in effective annual rate) for these maturities. (You can use the market information provided in the above problem.)

(b) Briefly discuss the convenience yield you obtained.

8. The data is the same as in the two problems above. You are running a refinery and need 10 million barrels of oil in three months.

(a) How do you use oil futures to hedge the oil price risk? The contract size is 1,000 barrels for futures.

(b) Suppose that you can also rent a storing facility for 10 million barrels of oil for three months at an annualized cost of 5% (in terms of the value of oil stored). Describe how you can utilize it to lock into a fixed oil price for your future demand.

(c) Which of these two strategies is better? Explain why.

9. The data is the same above. Now suppose instead that you are not in the oil business but can also rent the storage facility at the same cost. Can you take advantage of the current market conditions and the rental opportunity? If yes, please explain how (i.e., describe the actions you need to take). If not, briefly explain why.

10. The current price of silver is \$13.50 per ounce. The storage costs are \$0.10 per ounce per year payable quarterly at the beginning of each quarter and the interest rate is 5% APR compounded quarterly (1.25% per quarter).

(a) Calculate the future price of silver for delivery in nine months. Assume that silver is held for investment only and that the convenience yield of holding silver is zero.

(b) Suppose the actual price of the futures contract traded in the market is below the price you calculated in part (a). How would you construct a risk-free trading strategy to make money? What if the actual price is higher? To get full credit, say precisely what you will buy or sell, and how much money you will borrow or deposit into a bank account and for how long.

11. A pension plan currently has \$50M in S&P 500 index and \$50M in one-year zero-coupon bonds. Assume that the one-year interest rate is 6%. Assume that the current quote on the S&P 500 index is $1,350$, each futures contract is written on 250 units of the index and the dividend yield on the index is approximately 3% per year, i.e., \$$1,000$ invested in the index yields \$30 in dividends at the end of the year.

(a) Suppose you invest \$$1,350 \times 250$ in one-year zero-coupon bonds and at the same time enter into a single futures contract on S&P 500 index with one year to maturity. Assume that in one year the index finishes at $1,200$. What is the total value of your position? How does this compare with buying 250 units of the index and holding them for a year? Assume that in one year the index finishes at $1,400$. Repeat the analysis.

(b) If this plan decides to switch to a 70/30 stock/bond mix for a period of one year, how would you implement this strategy using S&P 500 futures? How many contracts with one year to maturity would you need? Assume that the index finishes the year at 1,400, describe the plan's portfolio in one year and one day from now (right after the futures expire). What is the stock/bond mix?

12. Spot price for soybean meal is $152.70 per ton and the 12-month soybean-meal futures is traded at $148.00. The 1-year interest rate is 3%.

(a) What is the net convenience yield on soybean meal for the 12 month period?

(b) You need 1,000 tons of soybean meal in 12 months. How would you lock into a price today using the futures contracts? (The size for each soybean-meal futures contract is 100 tons.)

13. The spot price for smoked salmon is $5,000 per ton and its six-month futures price is $4,800. The monthly interest rate is .0025 (.25%).

(a) What is the average monthly net convenience yield on smoked salmon for the next six months?

(b) If you are a manager of Bread&Circus and need 10 tons of smoked salmon in six months. How can you avoid the risk in the price of smoked salmon over the next six months using futures?

(c) Suppose that your net convenience yield for smoked salmon is 1.2%. How does this change your hedging strategy?

14. A wine wholesaler needs 100,000 gallons of Cheap Chardonnay for delivery in Boston in June 2007. A producer offers to deliver the wine at that time for $500,000 paid now, in December 2006.

The wholesaler can also buy Cheap Chardonnay futures contracts for June 2007. The current futures price is $51,000 for each 10,000 gallon futures contract.

The wholesaler is determined to lock in the cost of the 100,000 gallons needed in June.

(a) The wholesaler considers the futures contract, but worries that the contract will not lock in her cost, because futures prices may fluctuate widely between now and June. Is her concern justified? Why or why not?

(b) Do you recommend that the wholesaler pay the producer now or take a long position in Chardonnay futures? (Additional assumptions may be needed to answer. Make sure they are reasonable.) Explain briefly.

15. You are a distributor of canola seed and need to make deliveries of 10,000 bushels one month from now. You currently have no canola seed in inventory. The current spot price of canola seed is \$7.45 per bushel and the futures price for delivery in one month is \$7.65. You would like to hedge the uncertainty about the spot price one month from now.

(a) If your storage cost is \$.15 per bushel (paid at the end of month), what would you do?

(b) Suppose that in the short run, your storage cost increases to \$.25 per bushel. What would you do?

16. Assume perfect markets: no transaction costs and no constraints. In addition assume that the one-month risk-free interest rate will remain constant over a three-month period. Two futures contracts with two and three months maturity are traded on a financial asset without any intermediate payout. The price for these contracts are $F_2 = \$100$ and $F_3 = \$101$, respectively.

(a) What is the spot price of the underlying asset today?

(b) Suppose that a one-month futures contract is trading at price $F_1 = \$98$. Does this imply an arbitrage opportunity? How would you take advantage of this opportunity? To get full credit, be precise on what you would buy or sell, and how much money you would deposit into a bank account and/or borrow.

17. Assume perfect markets: no transaction costs and no constraints. The one-month risk-free interest rate will remain constant over a six-month period. Two futures contracts are traded on a financial asset without payouts: a three-month (futures price $F(t, t+3)$) and a six-month (futures price $F(t, t+6)$) contract. You can observe that $F(t, t+3) = \$120$ and $F(t, t+6) = \$122$.

(a) What is the spot price of the underlying asset at time t?

(b) Suppose that a three-month futures contract is trading at price $F(t, t+3) = \$119.5$. Does this imply an arbitrage opportunity? How would you take advantage of this opportunity?

1.5 Options

1. A stock price is currently \$50. It is known that at the end of two months it will be either \$53 or \$48. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of a two-month European call option with a strike price of \$49?

2. A stock price is currently \$80. It is known that at the end of four months it will be either \$75 or \$85. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a four-month European put option with a strike price of \$80?

3. Today's price of three traded call options on BackBay.com, all expiring in one month, are as follows:

Strike Price	Option Price
\$50	$\$7\frac{1}{2}$
60	\$3
70	$\$1\frac{1}{2}$

You are considering buying a "butterfly spread" consisting of the following positions:

- Buy 1 call at strike price of \$50
- Sell (write) 2 calls at strike price of \$60
- Buy 1 call at strike price of \$70.

(a) Plot the payoff of your total position for different values of the stock price on the maturity date.

(b) What is the dollar investment required to establish the spread?

(c) For what stock prices on the maturity date will you be making an overall profit?

4. You are given the following prices:

Security	Maturity (years)	Strike	Price (\$)
JEK stock	-	-	94
Put on JEK stock	1	50	3
Put on JEK stock	1	60	5
Call on JEK	1	50	?
Call on JEK	1	60	?
Tbill (FV=100)	1	-	91

What is the price of the two call options?

5. (a) Here are the payoff diagrams of some popular trading strategies using just put and call options with same maturities. How would you replicate them? Identifyfigthe number and strikes of call or put options that have to be bought or sold in order to generate these payoffs. (All angles are 45 degrees!)

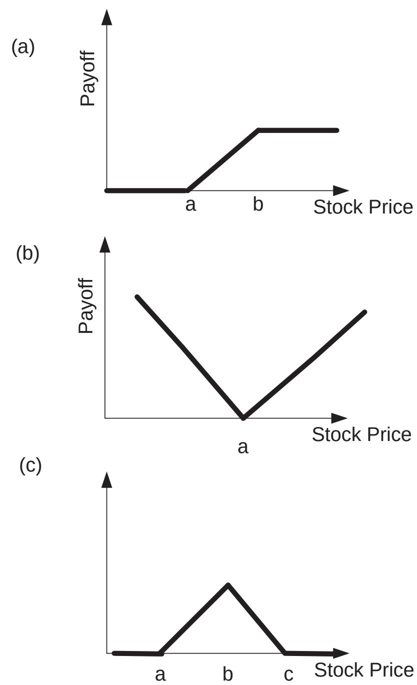


Figure 1: Payoff Diagrams

(b) Now suppose for institutional reasons, you are short on volatility in this market, i.e. you will lose money if the market becomes volatile. For example you can imagine that your are an investment bank working on a few major M&A deals which may fall apart if the market goes down too much or goes up too much. In either case, you will lose money if the market becomes volatile. Explain which if any of the above three payoffs would work well to hedge your exposure. What is the cost of this hedge to you?

6. A *butterfly spread* is a combination of option positions that involve three strike prices. To create a butterfly spread, a trader purchases an option with a low strike price and an option with a high strike price and sells two options with an intermediate strike price. For this problem, assume that the intermediate strike price is halfway between the low and the high strike prices and that the options are European. Denote the intermediate strike price by X, the low strike price by $X - a$, and the high strike price by $X + a$, $a > 0$.

(a) Graph the payoff diagrams at maturity of the butterfly spread in which the underlying options are call options. Holding the intermediate strike price fixed, what happens to the payoffs as the low and high strike price converge to the intermediate price?

(b) Suppose that a trader purchases a butterfly spread (using call options) for which the intermediate strike price equals to today's stock price. Based only on this trade, what is the trader's view of the future direction of the market?

7. IBM shares are now traded at \$80.00. The term structure of interest rates is flat at 5%.

(a) Plot the terminal payoff from a European put option on 1 share of IBM with a exercise price of \$85 and a maturity of 3 months (not including the price of the option).

(b) Suppose that you purchase the put option at \$4.00 from the market. Specify the ranges of IBM share price at the options maturity date for which you will be making a net profit.

8. Suppose that after a recent news about the economy, IBM share price remains the same, but the prices of its options shot up. How is this possible?

9. You are given the following information. Use this information to determine the unknown prices.

Table 2: Stock, Options, and T-bill prices

Secutiry	Maturity (years)	Strike	Price(\$)
401 Stock	-	-	\$100
Put on 401 Stock	1	\$50	\$3
Put on 401 Stock	1	\$60	\$5
Calle on 401 Stock	1	\$50	\$57.50
Calle on 401 Stock	1	\$60	?
Tbill(FV=100)	1	-	?

10. Joseph Jones, a manager at Computer Science, Inc. (CSI), received 1,000 shares of company stock as part of his compensation package. The stock currently sells at \$40 at share. Joseph would like to defer selling the stock until the next tax year. In January, however, he will need to sell all his holdings to provide for a down payment on his new house. Joseph is worried about the price risk involved in keeping his shares. At current prices, he would receive \$40,000 for the stock. If the value of his stock holdings falls below \$35,000, his ability to come up with the necessary down payment would be jeopardized. On the other hand, if the stock value rises to \$45,000, he would be able to maintain a small cash reserve even after making the down payment. Joseph considers three investment strategies:

(a) Strategy A is to write January call options on the CSI shares with strike price \$45. There calls are currently selling for \$3 each.

(b) Strategy B is to buy January put options on CSI with strike price \$35. These options also sell for \$3 each.

(c) Strategy C is to establish a zero-cost collar by writing the January calls and buying the January puts.

Evaluation each of these strategies with respect to Joseph's investment goals. What are the advantages and disadvantages of each? Which would you recommend?

11. You write a call option with strike \$50 and buy a call with strike \$60. The options are on the same stock and have the same maturity date. One of the calls sells for \$3; the other sells for \$9. (Assume zero interest rate.)

(a) Draw the payoff graph for this strategy at the option maturity date.

(b) Draw the profit graph for this strategy.

(c) What is the break-even point for this stratgy? Are you bullish or bearish on the stock?

12. Consider an increasingly popular deposit contract with payoffs linked to the performance on the S&P 500 Index on the U.S. stock market. For every dollar invested in the contract, the rate of return in one year is equal to 60% of the realized rate of return of the S&P 500 Index during this year if this rate of return is positive; otherwise, you get your money back. In essence, you are protected from the downside risk of the S&P 500 Index, while you are still able to participate in the upside potential of the stock market. The one year riskless interest rate is 10%. For simplicity, assume that the stocks in the index do not pay dividends.

(a) Draw a graph for the payoff one year from now for a one dollar investment in the contract with the horizontal axis being the realized rate of return on the S&P 500 Index. Also write down the payoff symbolically.

(b) Show that the payoff one year from now for a one dollar investment in this contract is the payoff to a portfolio of a default-free bond and a European call option on the S&P 500 Index.

(c) Suppose that the rates of return on the S&P 500 Index can take two possible values one year from now, 20% and -20% with probabilities 60% and 40%, respectively. Do you make money or lose money investing in this contract? If so, how much?

13. What is a lower bound for the price of 3-month call option on a non-dividend-paying stock when the stock price is $50, the strike price is $45, and the 3-month risk-free interest rate is 8%? Explain briefly.

14. Draw position (payoff) diagrams for each of the following trades. Each put or call option is written on 100 shares of the same stock and has the same 6-month maturity. The current stock price is $50 per share.

(a) Buy 100 shares, buy a put with an exercise price of $40, sell a call with an exercise price of $60.

(b) Same as (a), except that you borrow $4902. The semi-annual interest rate is 2%, so you will have to repay $\$4902 \times 1.02 = \$5000$ after six months.

(c) Buy a put and a call with exercise price of $50, sell a put with exercise price of $40, sell a call with an exercise price of $60.

15. Explain how you could generate the same payoffs as in part a of last question without purchasing any shares.

16. Ineffable Corporations stock price is currently $100. At the end of 3 months it will be either $110 or $90.91. The risk-free interest rate is 2% per annum. What is the value of a 3-month European call option with a strike price of $100? Calculate your answer to this problem using

(a) replication.

(b) the risk-neutral method.

17. State whether the following statements are true or false. In each case, provide a brief explanation.

(a) In a risk averse world, the binomial model states that, other things being equal, the greater the probability of an up movement in the stock price, the lower the value of a European put option.

(b) By observing the prices of call and put options on a stock, one can recover an estimate of the expected stock return.

(c) An investor would like to purchase a European call option on an underlying stock index with a strike price of 210 and a time to maturity of 3 months, but this option is not actively traded. However, two otherwise identical call options are traded with strike prices of 200 and 220 respectively, hence the investor can replicate a call with a strike price of 210 by holding a static position in the two traded calls.

(d) In a binomial world, if a stock is more likely to go up in price than to go down, an increase in volatility would increase the price of a call option and reduce the price of a put option. Note that a *static position* is a position that is chosen initially and not rebalanced through time.

Draw a diagram showing an investor's profit and loss with the terminal stock price for a portfolio consisting of:

18. (a) One share of stock and a short position in one call option

(b) Two shares of stock and a short position in one call option

(c) One share of stock and a short position in two call options

(d) One share of stock and a short position in four call options

You should take into account the cost from purchasing the stock and revenue from selling the calls. For simplicity ignore discounting when combining these costs and revenues with the terminal payoff of the portfolio. For simplicity also assume that the current stock price is equal to the strike price, K, of the call. Denote the current call price by c, and the terminal stock price by S_T.

19. Stock XYZ is worth $S = \$80$ today. Every 6 months the stock price goes either up by $u = 1.3$ or down by $d = 0.8$. The riskless rate is 6% APR with semiannual compounding. The stock pays no dividends.

(a) Compute the price of a European call with a maturity of 1 year and a strike price of $X = \$95$.

(b) Compute the price of an American call with a maturity of 1 year and a strike price of $X = \$95$.

(c) Compute the price of a European put with a maturity of 1 year and a strike price of $X = \$95$.

20. In August 1998 the Bank of Thailand was reported as offering to foreign investors in troubled banks the opportunity to resell their shares back to the central bank within a period of five years for the original purchase price. "This is to guarantee that at least they will not lose any of the money they plan to invest," said the Deputy Governor. (*The Wall Street Journal Europe*, August 6, 1998, p.20.) Suppose that(a) the standard deviation of Thai bank shares was about 50 percent a year, (b) the interest rate on teh Thai baht was 15%, and (c) the banks were not expected to pay a dividend in this five-year period. How much was this option worth? Assume an investment of 100 million baht.

21. Shares of ePet.com are traded at \$60. In six months, share price could either be \$66 or \$54 with probability 0.6 and 0.4, respectively. The current 6-month risk-free rate is 6%. What is the price of a European put on 100 ePet shares with a strike price of \$64 per share? Would your answer be different if the option is American?

22. Consider again ePet. You want to use ePet shares and the risk-free bond to replicate a payoff in six months that equals the square of ePet's share price. That is, when ePet price goes up to \$66, you have a payoff of $66^2 = \$4,356$ and when the price goes down to \$64, you have a payoff of $54^2 = \$2,916$. Describe the strategy that gives these payoffs. What is the present value of these payoffs?

23. The price of the stock of NewWorld Chemicals Company is \$80. The standard deviation of NewWorld's stock returns is 50%. The 1-year interest rate is 6%.

(a) What should be the price of a call on one share of NewWorld with a maturity of 1 year and strike price of \$85? Use the Black-Scholes formula.

(b) What should be the price of a put on one share of NewWorld with the same maturity and strike price?

24. You are asked to price some options on ABC stock. ABC's stock price can go up by 15 percent every year, or down by 5 percent. Both outcomes are equally likely. The risk free interest rate is 5 percent per year for the next two years, and the current stock price of ABC is \$100.

(a) Find the risk neutral probabilities

(b) What is the price of a European Call option on ABC, with strike 100 and maturity 2 years?

(c) Describe the strategy to replicate the payoff of the call using the stock and the risk-free bond.

(d) What is the price of an American option with the same characteristics?

25. You are asked to price some options on KYC stock. KYC's stock price can go up by 15 percent every year, or down by 10 percent. Both outcomes are equally likely. The risk free rate is 5 percent, and the current stock price of KYC is 100.

(a) Price a European Put option on KYC with maturity of 2 years and a strike price of 100.

(b) Price an American Put option on KYC with the same characteristics. Is the price different? Why or why not?

26. IBM is currently trading at $90.29 per share. You believe that IBM will have an expected return of 7% with volatility of 26.1% per year, while annual interest rates are at 0.95%. What is the price of an European put on IBM with a strike price of $90 and maturity of 1 year?

27. Shares of Ontel will sell for either $150 or $80 three months later, with probabilities 0.60 and 0.40, respectively. A European call with an exercise price of $100 sells for $25 today, and an identical put sells for $8. Both options mature in three months. What is a price of a three-month zero-coupon bond with a face of $100?

28. 401.com's stock is trading at $100 per share. The stock price will either go up or go down by 25% in each of the next two years. The annual interest rate is 5%.

(a) Determine the price of a two-year European call option with the strike price $X = \$110$.

(b) Determine the price of a two-year European put option with the strike price $X = \$110$.

(c) Verify that the put-call parity holds.

(d) Determine the price of a two-year American put option with the strike price $X = \$110$.

(e) What is the replicating portfolio (at every node of the tree) for the American put option with the strike price $X = \$110$?

29. For this problem assume that the risk-free rate of interest for one year loans is 5%. Google stock is selling today for $500 a share. Assume that in one year Google will either be worth $600 a share or $475 a share and that Google will pay no dividends for at least two years. A call option with an exercise price of \$550 and one year to go until expiration is available for Google stock. What is the value of this call option?

30. A particular stock follows the price movement below.

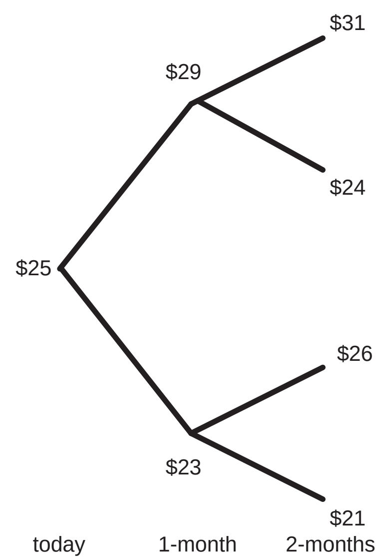


Figure 2: Stock Price Movement

(a) For this part, suppose the interest rate is fixed at 1% per month. What is the price of a put option with maturity two months, and strike of \$26 ?

(b) Again, suppose the interest rate is fixed at 1% per month. What is the price of an exotic derivative that in 2-months has a pay off that is a function of the maximum price of the stock during the two month period given by

$$\max(\hat{S} - \$25, 0),$$

where

$$\hat{S} = \max_{0 \le t \le 2} S_t.$$

and t is measured in months.

31. Intel stock is trading at \$120 per share, and the company will not pay any dividends over the next year. Consider an Intel European call option and a European put option, both having an exercise price of

\$124 and both maturing in exactly one year. The simple (annualized) interest rate for borrowing and lending between now and one year from now is 3% for each 6 month period (6.09% per year).

Assume that there are no arbitrage opportunities. Is there enough information to determine which option has the higher market value? If so, which option, the call or the put, has higher market value?

32. Calculate the price of a three-month European put option a non-dividend paying stock with a strike price of \$50 when the current stock price is \$50, the risk free rate is 10% per annum, and the volatility is 30% per annum. What difference does it make to your calculations if a dividend of \$1.50 is expected in two months? Assume that the assumptions made to derive the Black-Scholes formula are valid.

33. It is possible to buy three-month call options and three-month put options on stock X. Both have an exercise price of \$60 and both are worth \$10. Is a six-month call with an exercise price of \$60 more or less valuable than a similar six-month put?

1.6 Risk & Portfolio Choice

1. True or false or "it depends"?

(a) Briefly explain or qualify your answer: diversification can reduce risk only when asset returns are negatively correlated.

(b) If the returns on all risky assets in the world were uncorrelated with each other, the expected return of each risky asset should be the same.

2. True or false or "it depends"? Optimal portfolios should exclude individual assets whose expected return and risk (measured by its standard deviation) are dominated by other available assets.

3. Is the following statement true or false? Explain. As more securities are added to a portfolio, total risk would typically be expected to fall at a decreasing rate.

4. You need to invest \$10M in two assets: a risk-free asset with an expected return of 5% and a risky asset with an expected return of 12% and a standard deviation of 40%. You face a cap of 30% on the portfolio's standard deviation (the "risk budget"). What is the maximum expected return you can achieve on your portfolio?

5. Are the following statements true, false or uncertain? Briefly explain your answer.

(a) Diversification over a large number of assets completely eliminates risk.

(b) Diversification works only when asset returns are uncorrelated.

(c) A stock with high standard deviation may contribute less to portfolio risk than a stock with lower standard deviation.

(d) Diversification reduces the expected return on the portfolio as its risk decreases.

6. Are the following statements true or false? Give brief but precise explanations for your answers.

(a) Stock A has expected return 10% and standard deviation 15%, and stock B has expected return 12% and standard deviation 13%. Then, no investor will buy stock A.

(b) Diversification means that the equally weighted portfolio is optimal.

7. Which statement about portfolio diversification is correct?

(a) Proper diversification can reduce or eliminate systematic risk.

(b) Diversification reduces the portfolio's expected return because it reduces the portfolio's total risk.

(c) As more securities are added to a portfolio, total risk would typically be expected to fall at a decreasing rate.

(d) The risk-reducing benefits of diversification do not occur meaningfully until at least 30 individual securities are included in the portfolio.

8. Which of the following portfolios can not be on the Markowitz efficient frontier? Explain briefly.

Portfolio	Expected Return	Standard Deviation
Q	10%	15%
R	10.5%	16.5%
S	11.5%	18.5%
T	12.5%	20%

9. You have decided to invest all your wealth in two mutual funds: A and B. Their returns and risks are as follows:

- the mean returns are $\tilde{r}_A = 15\%$ and $\tilde{r}_B = 11\%$
- the covariance matrix is

	r_A	r_B
r_A	.04	.025
r_B	.025	.032

You want your total portfolio to yield a return of 12%. What proportions of your wealth should you invest in A and B? What is the standard deviation of the return on your portfolio?

10. There are only two securities (A and B, no risk free asset) in the market. Expected returns and standard deviations are as follows:

Security	Expected return	standard Deviation
Stock A	25%	20%
Stock B	15%	25%

(a) The correlation between stocks A and B is 0.8. Compute the expected return and standard deviation of a portfolio that has 0% of A, 10% of A, 20% of A, etc, until 100% of A. Plot the portfolio frontier formed by these portfolios.

(b) Repeat the previous question, assuming that the correlation is −0.8.

(c) Explain intuitively why the portfolio frontier is different in the two cases.

11. Stock A and B have the following characteristics:

	E(r)	σ
A	8%	20%
B	8%	40%

Their correlation is 0. The risk-free interest rate is 2%.

(a) Consider a portfolio, P, with 90% in stock A and 10% in the risk-free asset. What is the mean and standard deviation of portfolio P's return?

(b) Consider another portfolio, Q, which consists of 80% of stock A and 20% of stock B. What is the mean and standard deviation of portfolio Q's return?

(c) You need to choose a portfolio to invest all your wealth in. Between portfolio P and Q, which one is better? Explain why.

(d) Given that stock A dominates stock B (A has the same mean but lower risk), explain why you ever include stock B in your portfolio.

12. You can form a portfolio of two assets, A and B, whose returns have the following characteristics:

Stock	E[R]	Standard Deviation	Correlation
A	0.10	0.20	0.5
B	0.15	0.40	

If you demand an expected return of 12%, what are the portfolio weights? What is the portfolio's standard deviation?

13. Your have decided to invest all your wealth in two mutual funds: A and B. Their returns are characterized as follows:

- the mean returns are $\bar{r}_A = 20\%$ and $\bar{r}_B = 15\%$
- the covariance matrix is

	r_A	r_B
r_A	0.3600	0.0840
r_B	0.0840	0.1225

You want your total portfolio to yield a return of 18%. What proportion of your wealth should you invest in fund A and B? What is the standard deviation of the return on your portfolio?

14. In addition to the fund A and B in the previous question, now you decide to include fund C to your portfolio. Its expected return is $\bar{r}_C = 10\%$. The covariance matrix of the three funds is

	r_A	r_B	r_C
r_A	0.3600	0.0840	0.1050
r_B	0.0840	0.1225	0.0700
r_C	0.1050	0.0700	0.0625

Your portfolio now consists of fund A, B and C. You would like to have an expected return of 16% on your portfolio and a minimum risk (measured by standard deviation of the return). What portfolio should you hold? What is the return standard deviation of your portfolio? (Hint: You would need to use Excel Solver or some other optimization software to solve the optimal portfolio.)

15. You can only invest in two securities: ABC and XYZ. The correlation between the returns of ABC and XYZ is 0.2. Expected returns and standard deviations are as follows:

Security	E[R]	σ(R)
ABC	20%	20%
XYZ	15%	25%

a) It seems that ABC dominates XYZ in that it has a higher expected return and lower standard deviation. Would anyone ever invest in XYZ? Why?

b) What is the expected return and standard deviation of a portfolio that invests 60% in ABC and 40% in XYZ?

c) Suppose instead that you want your portfolio to have an expected return of 19.5%. What portfolio weights do you select now? What is the standard deviation of this portfolio?

16. You have the same data as the previous question. In addition, you have a risk-free security with a guaranteed return of 5%. The tangency portfolio has an expected return of ?? and standard deviation of ??.

(a) What weights are placed on ABC and XYZ in the tangency portfolio?

(b) What portfolio weights will you choose for the risk-free asset and the tangency portfolio to get an expected return of 19.5%.

(c) Compare this portfolio with the one you obtained in (c) of the previous question. Comment.

17. Calculate the expected return and standard deviation of a portfolio of stocks A, B and C. Assume an equal investment in each stock.

	Expected Return	Standard Deviation	Correlations		
			A	B	C
A	11%	30%	1.0	.3	.15
B	14.5%	45%	.3	1.0	.45
C	9%	30%	.15	.45	1.0

18. Your employer offers two funds for your pension plan, a money market fund and an S&P 500 index fund. The money market fund holds 3-month Treasury bills, which currently offer a 3% safe return per year. The S&P 500 index fund offers an expected return of 10% per year with a standard deviation of 20%.

(a) You want to achieve an expected return of 8% per year for your portfolio. What should be the composition of your portfolio? What is the standard deviation of its returns?

(b) Now your employer adds an emerging-market fund to the two existing funds. The emerging-market fund offers an expected return of 10% per year, the same as the S&P 500 index fund, but with a standard deviation of 30%, higher than the S&P 500 index fund. Would you consider including the emerging-market fund as part of your portfolio? Explain.

(c) The correlation between the S&P 500 index fund and the emerging-market fund is zero. Consider portfolio A, which consists of 80% in the S&P index fund and 20% in the emerging-market fund. Calculate portfolio A's expected return and standard deviation.

(d) If you mix portfolio A with the money-market fund to achieve an expected return of 8%, is it better than the portfolio in part (a) of this question? Explain.

19. You are given data on the following stocks:

Stock	E[R]	←	V[R]	→	Price	Mkt. Cap.
A	0.10	0.0625	0.0437	0.0525	$50	$105M
B	0.15	-	0.1225	0.0420	$30	$40M
C	0.20	-	-	0.0900	$27	$75M

a) Compute the expected return and variance of an equally weighted portfolio (i.e. weights of 1/3 on each of the stocks).

Consider alternative portfolios formed using assets A, B, and C. For instance, a value-weighted portfolio places weights on each assets in proportion to their market capitalizations. The S&P500 index is an example of a value-weighted portfolio. A price-weighted portfolio places weights in proportion to prices. The Dow Jones is an example of a price-weighted portfolio.

b) What are the expected returns and variance of a value-weighted portfolio.

c) What are the expected returns and variance of a price-weighted portfolio.

20. Your current portfolio consists of three assets, the common stock of IBM and GM combined with an investment in the riskless asset. You know the following about the stocks ($\rho_{i,j}$ denotes the correlation between asset i and asset j, and M denotes the market portfolio):

$$\rho_{IBM,M} = 0.60 \quad \rho_{GM,M} = 0.80$$
$$\sigma^2_{IBM} = 0.0900 \quad \sigma^2_{GM} = 0.0625$$

You also have the following data about the market portfolio M and the riskless asset F:

$$\bar{r}_M = 0.13 \quad r_F = 0.04$$
$$\sigma^2_M = 0.04$$

Suppose that individuals can borrow and lend at r_F and that the Capital Asset Pricing Model (CAPM) describes expected returns on assets. You have $200,000 invested in IBM, $200,000 invested in GM, and $100,000 invested in the riskless asset.

(a) What are the expected rates of return on IBM stock and GM stock?

(b) Assume that the correlation between IBM and GM, $\rho_{IBM,GM}$, is 0.40. What is the variance of your portfolio? What is its beta, $\beta_{p,M}$?

(c) Suppose that you can also invest in the market portfolio. Find an efficient portfolio that has the *same standard deviation* as your portfolio, but has the highest expected rate of return possible. What is the expected rate of return on this portfolio?

21. There are three assets, A, B and C, where A is the market portfolio and C is the risk-free asset. The return on the market has a mean of 12% and a standard deviation of 20%. The risk-free asset yields a return of 4%. Asset B is a risky asset whose return has a standard deviation of 40% and a market beta of 1. Assume that the CAPM holds.

(a) Compute the expected return of asset B and its covariances with asset A (the market portfolio) and asset C (the risk-free asset), respectively.

(b) Consider a portfolio of the two risky assets, A and B, with weight w in asset A (the market portfolio) and $1-w$ in asset B. Compute the expected return and return standard deviation of the portfolio with w being 0, 1/2, and 1, respectively, and enter them into the following table:

Portfolio weight w	0	1/2	1
Expected return			
Standard deviation			

(c) Can you rank the three portfolios in the question above? Explain.

(d) Consider a portfolio with equal weights in asset B and C (the risk-free asset). Denote this portfolio as asset D. Compute the return standard deviation and expected return of asset D.

(e) Consider a portfolio of asset A (the market portfolio) and C. Find the portfolio weight such that its return standard deviation is the same as that of asset D in Question (d). What is the expected return of this portfolio?

(f) What can you say about the mean-variance efficiency of asset A, B and C (i.e., are they efficient portfolios)? Explain.

(g) Construct an efficient portfolio from the assets A, B and C with an expected return of 10%.

22. Assume that an investor must put all of her money in the following four stocks.

	Expected Return	Standard Deviation	Correlations			
			A	B	C	D
A	11%	25%	1.0	.3	.15	.4
B	14.5%	35%	.3	1.0	.45	.2
C	9%	30%	.15	.45	1.0	.25
D	11.5%	32%	.4	.2	.25	1.0

(a) What is the expected return and standard duration of an equally weighted portfolio of the four stocks?

(b) Calculate the mean-variance efficient portfolios that can be constructed from the four stocks. (Hint: use Excel Solver)

(c) Assume the investor can borrow or lend at a 5% risk-free rate. What is the best portfolio of the four stocks?

23. Assets X, Y, and Z have the following characteristics:

Asset	Exp. Ret. ($\bar{r}$)	Std. Dev. (σ)
X	15%	20%
Y	10%	15%
Z	10%	15%

Correlation	X	Y	Z
X	1	0.7	0.3
Y		1	0.2
Z			1

Consider the portfolio frontier of the three assets. *Without solving for the portfolio weights*, answer the following questions:

(a) The frontier portfolio with mean 12% has higher weight on

i. Y

ii. Z

(b) The frontier portfolio with mean 9% has higher weight on

i. Y

ii. Z

(c) The frontier portfolio with mean 20% has higher weight on

i. Y

ii. Z

Explain the intuition for your results.

24. Consider a sample of 1000 randomly selected stocks, and assume for simplicity that each stock has a standard deviation of 35%. The correlation coefficient between each pair of stocks is .3. What is the standard deviation of an equal-weighted portfolio of 10 stocks? 100 stocks? 1000 stocks?

25. Assume that you can borrow and lend at a riskless rate of 5% and that the tangency portfolio of risky assets has an expected return of 13% and a standard deviation of return of 16%.

(a) What is the highest level of expected return that can be obtained if you are willing to take on a standard deviation of returns that is at most equal to 24%? Answer and explain below.

(b) What is the fraction of your wealth (in percent) invested in the riskless asset in the portfolio you found in part (a) (the mean-variance efficient portfolio with a standard deviation of 24%)? What is the fraction invested in the tangency portfolio of risky assets?

26. For this problem assume that it is possible to borrow and lend risklessly at a rate of 4%. Also assume that the expected return on the tangency (i.e., the optimal) portfolio composed only of risky assets is 13% with a standard deviation of 18%. Below we list 6 pairs of expected return and standard deviation combinations. For each pair determine whether or not the pair is feasible. If it is feasible, then there is at least one investment that can be made using risky assets and riskless borrowing or lending that produces this level of expected return and standard deviation. Then, if the pair is feasible, determine whether it is efficient or not. It is efficient if the expected return is the highest level that can be obtained for the associated level of standard deviation.

Pair	Standard Deviation	Expected Return
a	20.00%	24.75%
b	12.00%	18.00%
c	30.00%	19.00%
d	60.00%	50.00%
e	10.00%	4.00%
f	45.00%	56.50%

27. Parmacheenee Belle's entire common stock portfolio ($500,000) is allotted to an index fund tracking the Standard & Poors 500 index. The expected rate of return on the index is 9.5% and the standard deviation is 18% per year. The one-year risk-free rate is 2.0%.

Now Ms. Belle receives a strongly favorable security analyst's report on Mycronics Corp. The analyst projects a return of 25%. Myroncis has a high volatility (40% annual standard deviation) but its correlation coefficient with the S&P 500 is only .3.

Assume the return in the analyst's report is an unbiased forecast. Should Ms. Belle sell part of her index fund holdings and invest in Myronics? If so, how much? Note: Ms. Belle can also lend or borrow at the 2.0% risk-free rate.

28. Samantha Darling's entire common stock portfolio ($100,000) is allotted to an index fund tracking the Standard & Poors 500 index. The expected rate of return on the index is 12% and the standard deviation is 16% per year. The one-year risk-free rate is 5.5%.

Now Samantha receives a strongly favorable security analyst's report on e.Coli Corp. The analyst projects a return for e.Coli of 25%. e.Coli has a high volatility (50% annual standard deviation) but its correlation coefficient with the S&P 500 is only .4.

Assume the analyst's report is accurate. Should Samantha sell part of her index fund holdings and invest in e.Coli? If so, now much? Note: Samantha can also lend or borrow at the 5.5% risk-free rate.

29. You are a salesman/investment advisor working for a major investment bank. Whenever clients contact you with money to invest, your job is to help them find an appropriate mutual fund to invest in given their financial position. The available investments are:

Fund	E[R]	σ(R)
A	10%	15%
B	20%	45%
C	20%	55%

Assume throughout this problem that you only recommend *one* of the three funds to your clients (possibly a different recommendation for different clients though).

a) Would you recommend investment C to someone who comes to you with all his investment funds? Explain.

b) Which investment would you recommend to Keith Richards, the really, really old rock star from the Rolling Stones? Assume he invests all his wealth in your particular recommendation.

c) Might your answer to b) change if Keith invests only half his wealth in your particular recommendation? If so, under what circumstances?

30. Sarah runs an investment consulting business offering advice to clients on portfolio choices, using what she has learned in 15.401. Her analysis shows that the efficient frontier of risky assets can be obtained by mixing two portfolios, a portfolio of "large cap" stocks (L) and a portfolio of "small cap" stocks (S). In addition, she can also invest in riskless T-Bills (F).

For a very risk-averse retiree, Sarah has recommended the following portfolio: 70% in F, 20% in L and 10% in S. For a young, less risk-averse executive, however, Sarah recommends only 10% in F and the rest in the two risky portfolios.

Assume that Sarah has chosen the optimal portfolios for both the old retiree and the young executive. What are the weights for the young executive on the "large cap" and "small cap" portfolios, respectively? (Hint: The tangent portfolio should a combination of portfolios L and S.)

31. Which of the following common stock portfolios is best for a conservative, risk-averse investor? Explain briefly.

	Expected Return	Expected Risk Premium	Standard Deviation of Return
Portfolio A	19%	13%	20%
Portfolio B	16%	10%	16%
Portfolio C	13%	7%	12.5%

Note: the risk premium is calculated by subtracting a 6% Treasury bill rate from the expected rate of return. The investor can also buy Treasury bills.

32. Suppose the overall stock market is divided in four asset classes: large-cap growth stocks (LGR, 40% of the market), large-cap income stocks (LINC, 35% of the market), small-cap growth stocks (SMGR, 15% of the market) and small-cap income stocks (SMINC, 10% of the market). Forecasted returns, standard deviations (σ) and correlation coefficients for these asset classes are given on the table below. You can borrow or lend at the risk-free interest rate of 5%. You have $1 million to invest in some combination of the four asset classes. (You can buy index funds or exchange traded portfolios tracking the asset classes.)

	LGR	LINC	SMGR	SMINC
% of market	0.40	0.35	0.15	0.10
$\bar{r}$	0.1438	0.1092	0.1329	0.0931
σ	0.28	0.20	0.30	0.22

Correlations:

	LGR	LINC	SMGR	SMINC
LGR	1	0.65	0.70	0.30
LINC	0.65	1	0.40	0.55
SMGR	0.70	0.40	1	0.45
SMINC	0.30	0.55	0.45	1

(a) What are the expected rate of return and standard deviation of the market portfolio? What is the market's Sharpe ratio (the ratio of expected risk premium to standard deviation)?

(b) Can you improve the portfolio's Sharpe ratio by investing more in any of the asset classes? (Hint: Analyze a two-asset portfolio, with the market as one asset and a particular asset class as the other. If you sell some of the market portfolio and put the proceeds in that asset class, you end up over-weighting the asset class.)

1.7 CAPM

1. What is the beta of a portfolio with $E(r_p) = 18\%$, if $r_f = 6\%$ and $E(r_M) = 14\%$?

2. You are a consultant to a large manufacturing corporation that is considering a project with the following net after-tax cash flows (in millions of dollars):

Years from Now	After-Tax Cash Flow
0	-40
$1-10$	15

The project's beta is 1.8. Assuming that $r_f = 8\%$ and $E(r_M) = 16\%$, what is the net present value of the project? What is the highest possible beta estimate for the project before its NPV becomes negative?

3. Are the following true or false?

(a) Stocks with a beta of zero offer an expected rate of return of zero.

(b) The CAPM implies that investors require a higher return to hold highly volatile securities.

(c) You can construct a portfolio with a beta of 0.75 by investing 0.75 of the investment budget in bills and the remainder in the market portfolio.

4. Assume that the risk-free rate of interest is 6% and the expected rate of return on the market is 16%. A share of stock sells for \$50 today. It will pay a dividend of \$6 per share at the end of the year. Its beta is 1.2. What do investors expect the stock to sell for at the end of the year?

5. Assume that the risk-free rate of interest is 6% and the expected rate of return on the market is 16%. A stock has an expected rate of return of 4%. What is its beta? Why would anyone consider buying this risky asset which provides an expected return less than the risk-free rate?

6. In 1997 the rate of return on short-term government securities (perceived to be risk-free) was about 5%. Suppose the expected rate of return required by the market for a portfolio with a beta measure of 1 is 12%. According to the capital asset pricing model (security market line):

(a) What is the expected rate of return on the market portfolio?

(b) What would be the expected rate of return on a stock with $\beta = 0$?

(c) Suppose you consider buying a share of stock at \$40. The stock is expected to pay \$3 dividends next year and you expect it to sell then for \$41. The stock risk has been evaluated by $\beta = -.5$. Is the stock overpriced or underpriced?

7. True or False?

(a) CAPM says that all risky assets must have positive risk premium.

(b) The expected return on an investment with a beta of 2.0 is twice as high as the expected return on the market.

(c) If a stock lies below the security market line, it is under valued.

8. If we regress the stocks' average risk premium (return minus the risk-free rate) on their betas, what should be the slope and the intercept according to the CAPM?

9. If we regress a stock's risk premium on the risk premium of the market portfolio, what should be the slope and the intercept according to the CAPM?

10. The risk-free rate is 5%, the expected return on the market portfolio is 14%, and the standard deviation of the return on the market portfolio is 25%. Consider a portfolio with expected return of 16% and assume that it is on the efficient frontier.

(a) What is the beta of this portfolio?

(b) What is the standard deviation of its return?

(c) What is its correlation with the market return?

11. Your future father-in-law is 60 years old. He shows you his portfolio:

Assets	Holdings
Cash	\$ 50,000
S&P 500 Index Fund	100,000
Analog Devices Inc.	200,000

He asks you to forecast how much the portfolio will be worth in 5 years when he retires. The risk-free rate is 6% per year, the average return on the market portfolio is 12%, the beta of the S&P index is 1.0, and the beta of Analog Devices is 1.5.

(a) What is the expected rate of return on the portfolio, assuming the CAPM holds?

(b) What is the forecasted portfolio value after 5 years?

(c) Your future father-in-law is not impressed with this CAPM "theory" since his portfolio has done much better than your forecasted return over the past five years. What would you say about that?

12. Integral Industries, Inc. (III) has three subsidiaries, A, B, and C. You are negotiating to buy subsidiary C. Subsidiary A and B each contribute to 40% of III's market value and have betas of 0.8 and 1.4, respectively. The company as a whole has a beta of 1.0. What is the beta of subsidiary C? If you end up buying it, what would be C's opportunity cost of capital? The current risk-free rate is 6% and the market risk premium is 6%.

13. Stock 1 and 2 have the same beta of 0.8. But stock 1's return has a standard deviation of 40% and stock 2 has a standard deviation of 60%. How would you compare the risk of these two stocks? Which one do you think should have the higher expected returns? Explain briefly.

14. Stock A has a beta of 0.6 and stock B has a beta of 1.2. They both have a return standard deviation of 40% and the market portfolio has a return standard deviation of 25%. What fractions of the total variances of the two stocks' returns are market risks?

15. Five years of monthly risk premiums give the following statistics for Ampersand Electric common stock (risk premium = rate of return - risk-free rate):

- α is 0.4% per month, with a standard error of 1.2%
- β is 1.2, with a standard error of 0.27
- R^2 is 0.30
- σ is 7.2% per month.

(a) What does α measure? What role does it play in the CAPM? Does a positive α indicate a higher-than-normal expected return?

(b) What does R^2 measure? Would a higher R^2 increase your confidence in the estimated β?

Briefly explain your answers.

16. Consider three stocks: Q, R and S.

	Beta	STD (annual)	Forecast for Nov 2008	
			Dividend	Stock Price
Q	0.45	35%	$0.50	$45
R	1.45	40%	0	$75
S	-0.20	40%	$1.00	$20

Use a risk-free rate of 2.0% and an expected market return of 9.5%. The market's standard deviation is 18%. Assume that the next dividend will be paid after one year, at t = 1.

(a) According to the CAPM, what is the expected rate of return of each stock?

(b) What should today's price be for each stock, assuming the CAPM is correct?

17. Assume the Fama-French 3-factor APT holds with the factor risk premiums given in BM Table 8.5, p. 209. What are the expected rates of return for stocks Q, R and S in the previous question? The factor sensitivities are:

	b_{market}	b_{size}	$b_{book-to-market}$
Q	0.45	0.05	0.14
R	1.45	-0.33	-0.22
S	-0.20	1.21	0.64

18. It is November, 2007. The following variance-covariance matrix, for the market (S&P 500) and stocks T and U, is based on monthly data from November 2002 to October 2007. Assume T and U are included in the S&P 500. The betas for T and U are T = 0.727 and U = 0.75.

	$S\&P500$	T	U
$S\&P500$	0.0256	0.0186	0.0192
T	0.0186	0.1225	0.0262
U	0.0192	0.0262	0.0900

Average monthly risk premiums from 2002 to 2007 were:

$S\&P500 : 1.0\%$
$T : 0.6\%$
$U : 1.1\%$

Assume the CAPM is correct, and that the expected future market risk premium is 0.6% per month. The risk-free interest rate is 0.3% per month.

(a) What were the alpha's for stocks T and U over the last 60 months?

(b) What are the expected future rates of return for T and U?

(c) What are the optimal portfolio weights for the S&P 500, T and U? Explain.

19. CML and SML: Using the properties of the capital market line (CML) and the security market line (SML), determine which of the following scenarios are consistent or inconsistent with the CAPM. Explain your answers. Let A and B denote arbitrary securities while F and M represent the riskless asset and the market portfolio respectively.

(a) *Scenario I:*

Security	E[R]	β
A	25%	0.8
B	15%	1.2

(b) *Scenario II:*

Security	E[R]	$\sigma(R)$
A	25%	30%
M	15%	30%

(c) *Scenario III:*

Security	E[R]	$\sigma(R)$
A	25%	55%
F	5%	0%
M	15%	30%

(d) *Scenario IV:*

Security	E[R]	β
A	20%	1.5
F	5%	0
M	15%	1.0

(e) *Scenario V:*

Security	E[R]	β
A	35%	2.0
M	15%	1.0

20. True/False/Depends Questions: Please include brief explanations in your responses:

(a) The average return on stocks in the US over the past 30 years has been 12% annually. You find two portfolios, one with an expected return of 14% and another with an expected return of 19%. This contradicts the CAPM.

(b) All portfolios on the SML have no idiosyncratic risk.

21. Calculating Beta (Part Two): Consider two securities, A and B, along with the market portfolio M. Their variance-covariance matrix is:

Security	Ã	V[R]	!
A	0.0900	0.0420	0.0525
B	-	0.1225	0.0437
M	-	-	0.0625

(a) Calculate the beta of each stock.

(b) Which stock has the highest expected return?

22. Are the following statements true or false? Give brief but precise explanations for your answers.

(a) Stock A has expected return 10% and standard deviation 15%, and stock B has expected return 12% and standard deviation 13%. Then, no investor will buy stock A.

(b) Diversi¯cation means that the equally weighted portfolio is optimal.

(c) The CAPM predicts that the expected return on the market portfolio is always greater than the return on the riskless asset.

(d) The CAPM predicts that a security with a beta of zero o®ers zero expected return.

(e) The CAPM predicts that all investors hold the same portfolio of risky assets.

(f) The CAPM predicts that investors demand higher expected rates of return from stocks that have a high (positive) sensitivity to °uctuations in the stock market.

(g) An investor who puts $10000 in T-bills and $20000 in the market portfolio will have a beta of 2.0.

23. Security B has a price of $50 and a beta of 0.8. The risk-free rate is 3% and the market risk premium is 4%.

(a) According to the CAPM, what return do investors expect on the security?

30. True or False. Briefly explain.

(a) The capital asset pricing model assumes that all investors have the same information and are willing to hold the market portfolio.

(b) Over the long run, average returns on low-beta stocks have been less than predicted by the capital asset pricing model.

31. Suppose that the actual rate of return on the S&P 500 index from December 17, 2001 (today) to December 17, 2002 (12 months hence) is 9.0%, including dividends paid by companies in the index. You are given the following information about the performance of mutual funds X, Y and Z. Each mutual fund invests only in common stocks.

Fund (manager)	Rate of return	Alpha (SE)	Beta (SE)	R^2
X (Gladys Friday)	9.8%	+.48% (1.0%)	1.05 (.05)	.92
Y (Gene Pool)	9.0%	−.65% (3.0%)	1.10 (.07)	.88
Z (Hugh Betcha)	13.4%	+.50% (3.1%)	1.60 (.09)	.65

Alphas and betas are estimated from 52 weekly returns from December 2001 to December 2002. Returns and alphas are given above as annual percentage returns. SE means standard error. The start-of-year risk-free rate is 2.5%.

Based on these statistics, what can you say about the investment strategy and performance of each of the three managers? Explain. Consider risk as well as return before answering.

32. Your company offers three funds to its employees for their pensions: a money-market fund, an S&P 500 index fund and a new-economy equity fund. You need to form a portfolio from these funds for your own pension investments.

The money-market fund is invested in 3-month Treasury bills, now with a risk-free return of 1.5% per annum. The index fund gives a premium of 8% and a standard deviation of 20% per annum. The new-economy fund's return can be described by the following equation:

$$r_t - r_F = \alpha + \beta(r_{Mt} - r_F) + e_t$$

where r_t and r_{Mt} are the fund and market returns, r_F is the risk-free return, α is a constant, and e_t is the part of the fund's returns not explained by the market. The performance of the fund over the last 60 months gives

- $\alpha = 0.0$
- $\beta = 1.2$
- $R^2 = 0.75$ (proportion of the variance of the fund's return explained by the market return).

(a) Compute the expected return of the new-economy fund using CAPM. Use reasonable estimates for the market return and the risk-free return.

(b) If CAPM holds, what is the optimal portfolio to achieve an expected return of 8% per annum.

(c) If instead, the estimate of α is 0.0050 (0.5% per month) with a standard error of 0.0015. Without doing any calculations, discuss how this may affect your portfolio.

33. Two mutual fund managers are being evaluated for their performance in the last ten years. One of them, Mr. Hare, has achieved an eye-popping 34% annual average return; the other, Ms. Tortoise, has obtained a modest 12% annual average return. On closer examination of their portfolios, it is found that Mr. Hare always bet on risky Argentinian stocks (whose beta is 4), whereas Ms. Tortoise always invested in conservative technology firms like IBM (whose beta is

(a) If the risk-free return was 3% every year and the expected market return was 11% every year, who should get the higher bonus? Why? (Credit only if reasoning is correct.)

(b) If the risk-free return was 7% every year and the expected market return was 13% every year, who should get the higher bonus? Why? (Credit only if reasoning is correct)

34. True or false. Explain briefly

By the CAPM, stocks with the same beta have the same variance.

35. True or false. Explain briefly

If CAPM holds, α should be zero for all assets.

36. Does the CAPM provide a good explanation of past average rates of return? How would you (briefly) summarize the evidence?

37. True or false. Explain briefly.

(a) The Sharpe ratio equals average return divided by standard deviation of return.

(b) The average beta of all the assets in the market is 1.

38. Assume that you can borrow and lend at a riskless rate of 5% and that the tangency portfolio of risky assets has an expected return of 13% and a standard deviation of return of 16%.

(a) What is the highest level of expected return that can be obtained if you are willing to take on a standard deviation of returns that is at most equal to 24%? Answer and explain below.

(b) What is the fraction of your wealth (in percent) invested in the riskless asset in the portfolio you found in part (a) (the mean-variance efficient portfolio with a standard deviation of 24%)? What is the fraction invested in the tangency portfolio of risky assets?

1.8 Capital Budgeting

1. Table A gives investments, NPVs, IRRs and the first three years' cash flow for several capital investment projects. Each project's cash flows continue for several more years, longer for some projects than others. The cost of capital is 12% for all projects.

Table A (figures in millions).

Project	Invest in 2000	C_1	C_2	C_3	NPV	IRR
A	100	20	20	20	57	17.8
B	200	0	20	40	64	14.5
C	50	20	20	20	41	37.8
D	75	-10	10	30	0	12
E	30	-10	5	7	-3	11
F	10	3	4	5	5.5	30.2

Projects A and B are mutually exclusive - your firm can take only one. The projects are discrete - you cannot make partial investments in any project.

(a) Suppose the firm rejects all projects with payback periods greater than 3 years. What is the NPV from following this policy?

(b) A manager defends the decisions in part a as a way to avoid taking on risky projects. Does this defense make sense?

(c) Which project would you choose, A or B?

(d) Suppose that the firm now identifies a new project AA with exactly the same cash flows, NPV and IRR as project A. Does the opportunity to invest in AA change your answer to part c?

(e) Suppose the firm has only $200 million to invest - a fixed capital constraint. Which projects would you undertake? (Ignore project AA.)

(f) Now the firm negotiates a line of credit that allows it to borrow up to $100 million at 8%. Would access to additional debt capital at a cost of 8% change your answers to questions c, d or e?

Explain each answer briefly.

2. You own three oil wells in Vidalia, Texas. They are expected to produce 7,000 barrels next year in total, but production is declining by 6 percent every year after that. Fortunately, you have a contract fixing the selling price at $15 per barrel for the next 12 years. What is the present value of the revenues from the well during the remaining life of the contract? Assume a discount rate of 8 percent.

3. Your company's CFO has budgeted \$18 million for capital expenditures during 2000 by your division. Unfortunately the division has good opportunities to invest much more than \$18 million. The cost of capital is 12%.

Project	Investment in 2000	NPV	IRR
Q	10.5	5.5	15
R	2.0	0.5	18
S	6.0	2.5	25
T	7.5	2.0	30
U	1.5	1.0	26
V	3.0	1.0	20

Assume the \$18 million budget can not be increased. Which projects should be undertaken?

4. The DEF Corporation is trying to decide whether to undertake an expansion of its production facilities. The expansion will cost \$8.5 million, to be paid immediately. After tax cash flows generated by the expansion are projected to be \$1 million next year, and will be growing indefinitely with inflation at 2.5% per year. Assume that the CAPM holds, the beta of DEF assets is 1.2, the riskless rate is 5% per year (and the yield curve is flat at this rate) and that the expected return on the market portfolio is 12%. Should DEF undertake the expansion?

5. You have developed the technology to use gold to produce high capacity fiber optic switches. The technology has cost \$ 5 million to develop. You need \$50 million of initial capital investment to start production. Sales of the switch sales will be \$20 million per year for the next 5 years and then drop to zero. The main cost of production is gold. Each year, you need 20,000 ounces of gold. Gold is currently selling for \$250 per ounce. Your supplier thinks that the gold price will appreciated at 5% per year for the next 5 years. The cost of capital is 10% for the fiber-optics business. The tax rate is 35%. The capital investment can be depreciated linearly over the next 5 years.

(a) Calculate the after-tax cash flows of the project.

(b) Should you take the project?

6. Your company considers a new investment project, which lasts for three years. The project requires a purchase of a new machine, which costs \$600,000. This initial investment can be depreciated to zero over the next three years according to a straight line depreciation rule. The machine has no salvage value at the end. Operating revenue is projected to be \$400,000 per year. Operating costs for raw materials are \$100,000 per year. The above data is summarized in the following table (in thousands of dollars):

Capital expenditure	600			
Depreciation		200	200	200
Operating revenue		400	400	400
Operating cost		100	100	100

The corporate tax rate is 30% and the risk-adjusted discount rate is 10%.

(a) Compute after-tax cash flows every year.

(b) Should we take the project and why?

(c) Suppose that you need to purchase an inventory of raw materials now (year 0) instead of year by year in the next three years. This will require an initial expense of $300,000 in inventory, which will then be depleted, by equal amount every year in the next three years. How would your answers to the above questions change?

7. Halliburton is considering for a two-year project. The project requires an initial capital expenditure of $100 million (in year 0), which can be depreciated linearly to zero in the next two years. It generates a revenue of $80 million and a cost of $25 million per year for the next two years (year 1 and 2). The tax rate for Halliburton is 30%.

(a) Compute the net after-tax cash flows of the project.

(b) The cash flows of this project are risk-free. The market gives the following interest rates:

Maturity (years)	1	2	3	⋯
Spot interest rate (%)	4.00	6.00	6.50	⋯

Should Halliburton take this project? Explain.

2 Solutions

2.4 Forward and Futures

1. Use simple arbitrage argument. To deliver one bushel of wheat in a year, you can buy it today for \$3.4 per bushel and store it for one year. The cost of storage is \$0.1 per bushel. So the total cost if \$3.5 per bushel in today's dollar. Project this forward one year using the interest rate of 4% give the price of $\$3.5 \times (1 + 4\%) = \$3.64$ for 1-year forward contract.

2. We assume that the forward price is equal to (or very close) to the futures price. The forward price is given by:

$$H_T = S_0(1 + r_T)^T \cdot$$

Re-arreng this basic formulat to get:

$$r_T = \left(\frac{H_T}{S_0}\right)^{1/T} - 1 \cdot$$

Applying this formula, we get the effetice annualized interest rates:

- October: $r_{Oct} = (635.60/633.50)^{12} - 1 = 0.0405$
- December: $r_{Dec} = (641.80/633.50)^{4} - 1 = 0.0534$
- June, 2007: $r_{Jun,07} = (660.60/633.50)^{12/9} - 1 = 0.0574$
- December, 2007: $r_{Dec,07} = (678.70/633.50)^{12/15} - 1 = 0.0567$

3. (a) On one hand, the hotel chain faces the risk that the price at which it purchases coffee will rise. On the other hand, the hotel chain can pass along these costs to the consumers of the coffee, so it is not clear to what extent the hotel chain will be affected by an increase in coffee prices.

The hotel chain's ability to pass along rising costs to its customers is a type of a *natural hedge*. A company's failure to recognize its natural hedges can lead to "hedges" that actually increase risk (e.g., a hotel chain that locks in its costs by buying forward coffee might be at a competitive disadvantage if coffee prices fall).

(b) The coffee farmer has no natural hedge and is at risk if the price of coffee falls. The coffee farmer may wish to sell forward coffee.

(c) When entering the contract, the coffee farmer presumably understood the risk that a hedge may lock in a lower selling price if coffee prices rise above the futures price. The fact that the farmer lost the upside while protecting the downside does not indicate that there were "losses" or that the farmer was wrong.

4. (a) Assume the Yield Curve is flat at 5%
Price of forward $= \$20 * (1+5\%)^{.5} - \$1 * (1+5\%)^{.25} = \$19.482$

The second terms takes into account that you don't receive the \$1 dividend if you enter into the futures today.

5. (a) The price of futures contract is related to the spot price by the following equation:

$$F = S_0(1+r)^T$$

For 7-Dec contract we have:

$$\$706.42 = \$693\,(1+r_{3-months})^{1/4} \rightarrow r_{3-months} = 7.97\%$$

Similarly, for the 8-Jun contract we have:

$$\$726.7 = \$693\,(1+r_{9-months})^{3/4} \rightarrow r_{9-months} = 6.54\%$$

(b) Since you are the owner of a gold mine, you are naturally long gold in the future. In order to eliminate the uncertainty regarding the price you will be able to sell your product at, you can enter into a futures contract to be able to sell your production at a price that can be fixed today. For example, based on the information given here, you can enter into a contract to sell your December 2007 production at the price of \$706.42 /oz and your June 2008 production at \$726.7 /oz.

6. (a) For a financial asset with a dividend the spot price and the futures price are related as:

$$F = S_0(1+r-y)^T \tag{1}$$

Rearranging this equation you get

$$y = 1 + r - \left(\frac{F}{S_0}\right)^{1/T}$$

You have to be careful the use the relevant interest rates, $r_{3-months}$ or $r_{9-months}$, calculated in the pervious problem for this question. For 7-Dec contract we have:

$$y_{3-months} = 1 + 0.0797 - \left(\frac{1472.4}{1453.55}\right)^4 = 2.68\%$$

For 8-Jun contract we have:

$$y_{9-months} = 1 + 0.0654 - \left(\frac{1493.4}{1453.55}\right)^{4/3} = 2.87\%$$

(b) You can see that in equation 1, the divided yield appears with negative sign, i.e. a decrease in dividend yield will causes the price of a futures contract to increase. Hence, if you believe that the dividend yield in the next 9 months will be lower than 2.84%, it implied that you believe the future is priced too "cheaply", i.e. you think the replicating portfolio for the futures contract would cost more than the price this contract is traded for in the market. To take advantage of that, you must enter into a contract to be able to buy the S&P index in June 2008. As a general rule, you always try to buy the asset that appears to be under priced according to your model or you view in this case. You also need to take appropriate positions in the S&P index in the spot market to hedge your exposure to S&P index. Since you are long the future, the appropriate position you need to take is to sell S&P index (compare this with question one, the logic is the same). But since you don't own the S&P index you need to borrow this to sell it, i.e. you will take a short position in S&P. Let's assume that you borrow the S&P index from a mutual fund to sell in the market for $1453.55 today. You will deposite this money into a bank for 9 months which will grow to $1524.23. As time progresses, you need to pay the equivalent amount of divided paid by the stocks in the S&P index to the mutual fund that lent you the index. If your view is current, the total dividend you have to pay out is less than $1524.23-$1453.55=$30.53 and you will end up being positive - remember you need to buy back the S&P index in 9 months to return it to the mutual fund that lent you the index initially but you can do this at the price of $1453.55 which you agreed upon initially. In short, you have a positive exposure, i.e. you make money, if your view is correct about slow down in the rate of dividend yield.

7. Recall futures pricing formula:

$$H = S_0(1 + r_t - \hat{y}_T)^T$$

Rearrange this to get:

$$\hat{y}_T = 1 + r_T - \left(\frac{H}{S_0}\right)^{1/T}$$

where r_T is the annualized interest rate and $\hat{y}_T$ is the annualized convenience yield over the time horizon indicated by T. Apply this to the market data:

- October: $\hat{y}_{Oct} = 1 + 0.0405 - (67.50/67.50)^{12} = 0.0405$

- December: $\hat{y}_{Dec} = 1 + 0.0534 - (69.60/67.50)^4 = -0.0769$
- June, 2007: $\hat{y}_{Jun,07} = 1 + 0.0575 - (72.66/67.50)^{12/9} = -0.0458$
- December, 2007: $\hat{y}_{Dec,07} = 1 + 0.0566 - (73.49/67.50)^{12/15} = -0.0137$

8. (a) You can go long 10,000,000/1,000=10,000 future contract with December maturity. You then get a lock in price of \$69.60 per barrel.

(b) You should buy oil in the spot market and store it until December. The total cost will be $67.50 \times (1 + 5\% + 5.34\%)^{3/12} = \$69.18$. here we assumed the rent of 5% is compounded monthly and you should also include the time value of money. For simplicity, we have assumed the rent payment are paid upfront.

(c) You should follow option b because the cost as calculated above is slightly less than the current market price of \$69.60 for these contracts. In fact, you should do even more than hedging and try to eliminate this arbitrage. You can do this because you have access to cheaper storage facility than what is priced into the future prices through the convenience yield (5% vs. 7.69%) but the difference is small.

9. We already eluded to this answer above. Here is what you should to precisely.

- Sell future contract as the price of \$69.60.
- Borrow \$68.70 at the rate of 5.34% for 3-month. This will grow to exactly \$69.60 in 3-months.
- Use \$67.50 of the above money to buy oil in the spot market
- Pay $\$67.50 \times (1 + 5\%)^{1/4} - \$67.50 = \$0.83$ of the storage cost upfront (again depending on how the rent is quouted, compounded, and paid this calculation may change)
- Pocket the difference today, i.e. take the arbitrage profit of \$68.70-\$67.50-\$0.83=\$0.37.
- In December, take deliver the oil, get \$69.60 and pay back your debt.

10. (a)

$$\begin{aligned} F &= \$13.50\left(1+\frac{0.05}{4}\right)^3 + \frac{0.1}{4}\left(1+\frac{0.05}{4}\right)^3 + \frac{0.1}{4}\left(1+\frac{0.05}{4}\right)^2 + \frac{0.1}{4}\left(1+\frac{0.05}{4}\right)^1 \\ &= \$14.09 \end{aligned}$$

Here is the arbitrage argument of why this has to be the market price. This is basically the same argument that you have seen in class. Suppose you enter into a contract to sell silver in 9-months at price F. The fair price F must be equal to the cost of replicating portfolio, or otherwise there is an arbitrage opportunity. We will discuss the case that there is a mispricing and a strategy to take advantage of that in part b. Here is how the replicating portfolio will be constructed.

To replicate the future contract you would borrow money today (\$13.50 to buy an ounce of silver), buy silver and store that for 9 months. Note that your initial net cash flow is zero. You will have to also pay storage fees once every quarter. By the end of 9 months, you will owe $13.50\left(1+\frac{0.05}{4}\right)^3 = 14.013$ due to your initial borrowing and, in addition, you will owe $\frac{0.1}{4}\left(1+\frac{0.05}{4}\right)^3 + \frac{0.1}{4}\left(1+\frac{0.05}{4}\right)^2 + \frac{0.1}{4}\left(1+\frac{0.05}{4}\right)^1 = 0.077$ due to the storage fees that you have had to pay. On the other hand, you own an ounce of silver which you can sell at agreed price of F. In order to have the net cash flow zero at maturity, F must be equal to the total money you owe. Putting all these together you get that F must be equal to the value given above in order to have no arbitrage.

(b) If the actual futures price is below \$14.09 you would like to take advantage of this mispricing by buying the futures contract, i.e. entering into the contract to buy Silver in 9 months. This is also sometimes referred to as "taking a long position in the futures contract". You can then use the reverse of the above replication argument to synthesize a short position in the future to offset your long position. Alternatively, if the price is above, you would enter in a short position in the futures contract, i.e. agree to sell Silver in 9-months at a level above \$14.09, and use the above replication strategy to replicate a long position in the future. The main idea here is that the cost of the replicating strategy is \$14.09. Hence if you can buy the future at lower price or sell at higher, you should do that and use the replication strategy to offset your position. This way you will eliminate all your risk and make risk-free or arbitrage money. The argument works exactly the same way, with signs reversed, in both case. We consider both cases below.

For concreteness, let's assume you can sell the contract at \$14.30, which is higher that the fair price you calculated in part a. To take advantage of this, you would do the following:

i. Enter into a contract to sell Silver in 9 months at \$14.30

ii. Borrow \$13.50 now and buy an ounce of silver. You will have to borrow this money for 9 months.

iii. Store the silver, which will cost you \$0.10 per quarter - meaning you would have to take additional loans from the bank every quarter to pay for this cost.
iv. By the end of 9-months, you will owe the total of \$14.09 - see part a to understand where this number is coming from.
v. Sell your ounce of Silver at the price \$14.30, i.e the price that was agreed upon at time zero
vi. Use \$14.09 to pay your debt and pocket the \$0.21 on money

The argument if the price was below \$14.09 is exactly reversed. Again, For concreteness let's assume the price is \$13.90. You would enter into a contact to buy silver in 9 months at this price. You would short sell silver today, i.e. borrow silver to sell it for \$13.50 today, and deposite the money into a bank for 9 months. A potentially confusing concept here in this case is understanding what happens to the storage cost. Since you have borrowed the silver to short sell, the person who you borrowed the silver from would owe you the storage cost - think that he would have had to pay the cost to someone else to store the Silver for him which he must now pay you in this case. You would deposite the storage cost you receive every quarter into your bank account as well. By the end of 9 months, you will have \$14.09 in the bank. Use \$13.90 to buy silver and return that to the person who lent you silver. The difference, \$0.19 in this case, is the risk free arbitrage money you have made.

11. (a) First, we calculate the total investment in TBills for one year $\$1350 \times 250 = \$337,500$. Second, lets list everything given in the problem:

$$S(t) = \$1350,\ S(T) = \$1,200 \text{ or } \$1,400,\ r_F = 0.06,\ D = 0.03,$$

and, finally, each future contract has 250 units of the index. Using formula (23) from Lecture 2 we can calculate the index futures price

$$F(t,T) = S(t)\times(1+r)-D = \$1350\times(1+6\%)-\$1350\times3\% = \$1390.5$$

Where D denotes the value of future dividends at the end of 1 year.
Tables below present results for the required position: buy TBills and one futures contract on S&P.

Transaction	Payoff at $T = 12$ months, $S(T) = \$1,200$
Buy TBills	$\$337,500 \times (1+6\%) = \$357,750$
Long Futures	$\$250 \times (1200 - 1390.5) = -\$47625$
Net	$\$310,125$
Index	$250 \times (\$1200 + \$1350 \times 0.03) = \$310,125$

Transaction	Payoff at $T = 12$ months, $S(T) = \$1,400$
Buy TBills	$\$337,500 \times (1 + 6\%) = \$357,750$
Long Futures	$250 \times (\$1400 - \$1390.5) = \$2375.0$
Net	$\$360,125$
Index	$250 \times (\$1400 + \$1350 \times 0.03) = \$360,125$

Therefore the payoffs are the same in both cases! Hence we did a synthetic replication of the underlying by taking positions in futures and bonds.

(b) For a 70/30 mix we will have to transfer \$20 Million from bonds to stocks. We can replicate the long position in S&P as long futures. The price of the futures contract is \$1390.5. The number of contracts required is equal to $\frac{\$20M}{250 \times \$1350} = 59$. The payoff at the termination of the contract:

The payoff from Futures $= 250 \times 59 \times (\$1400 - \$1390.5) = \$140,741$.

Therefore the final position is : S&P $= \$50M \times \frac{1400}{1350} = \$53.852M$. The final position in bonds $= \$50M \times (1 + 6\%) = \$53M$. The final position in cash: $\$50 \times (1 + 6\%) + \$140,741 + \$50M \times *.03 = \$53.290M$ The stock/bond ratio is 50.684/49.316. (Percentage in stock = value of stock/ (value of stock + total cash)

12. (a) Recall futures pricing formula:

$$H = S_0(1 + r - \hat{y})^T$$

Rearrange this to get:

$$\hat{y} = 1 + r - \left(\frac{H}{S_0}\right)^{1/T}$$

Apply this to the data to get

$$\hat{y} = 1 + 3\% - \left(\frac{\$148}{\$152.70}\right)^1 = 6.08\%$$

(b) Simply enter into a future contract to be able to buy soybean at the price of \$148 in one year.

13. (a) Recall futures pricing formula:

$$H = S_0(1 + r - \hat{y})^T$$

Rearrange this to get:

$$\hat{y} = 1 + r - \left(\frac{H}{S_0}\right)^{1/T}$$

Apply this to the data to get

$$\hat{y} = 1 + 0.25\% - \left(\frac{\$4,800}{\$5,000}\right)^2 = 8.1\%$$

(b) Simply enter into a future contract to be able to buy Salmon at the price of \$4,800/ton in six months.

(c) The implied future price based on your convenience yiled is (i.e. the price if you were to enter into the arbitrage type of argument, buy the salmon today and store if for six months):

$$S_0(1 + 0.25\% - 1.2\%)^{1/2} = \$4,976$$

So, you should use the capital market to hedge your risk. It is not beneficial to you to try to replicate the payoff of the futures contract by buying in the spot market and storing.

14. (a) No, she is needs wine in 6-months and by following either approach she will be laying off this risk to the market. The interim price fluctuations are irrelevant.

(b) In order to answer this, you need to compare the cost of two option. In one option, the wholesaler will pay $\$500,000$ now and receive the desired amount of wine in June. In the second option, the wholesaler will have to pay $\$510,000$ in June. The annual interest rate that equates this options is $1-(510,000/500,000)^2 = 4.04\%$. So if the interest rate the wholesaler can raise money at is less than 4.04%, she should pay now; otherwise she should enter into the futures contract. To do this analysis, we have ignored margin requirement and any money that may exchange hand during the 6-month life of the contract as the price of wine fluctuates.

15. Assume payment of storage cost would occur at $t = 0$.

(a) You have to compare the cost of two options. If you buy in the spot market and store, it will cost you $\$7.45 \times (1 + r/12) + \$.15$. Alternatively, buyin in the futures market you will fix your cost (paid in 1 month) at \$7.65. The breakeven interst rate is 8.05%. So if you can borrow at a lower rate, you should buy in the spot and store. Otherwise, you should use the futures markets. This is assuming you get zero convinience yield from this. We wil discuss this issue in the next part.

(b) If you apply the same methodology here, you will get the breakeven interest rate of -8.05%. This basically means that the market is attributing such a large convininece yield to this that basically makes the storage option not preferable even if you have a zero interest rate. Of course, you may derive evne a higher covinience yield from holding bushels and that would change the situation but here we have, again,. assumed your convinience yield is zero. (Recall that $H = S_0(1 + r_T - \hat{y}_T)^T$)

16. (a) Let the 1-month risk free rate (not annualized) be r_1. You can write the following two equations

$$F_2 : \$100 = S_0(1 + r_1)^2$$
$$F_3 : \$101 = S_0(1 + r_1)^3$$

Solving these equations you get: $r_1 = 1\%$ and $S_0 = 98.03$

(b) The 1-month futures price according spot and the interest rate calculated above should be

$$F_1 = \$98.03(1 + 0.01) = \$99.01$$

Since the market price of the future is less than this amount, you would like to buy the future contract in the market, i.e. enter into a contract to buy the asset at price of \$98 in a month, and take an offsetting position. In this case, what you have to do is to sell the underlying asset, i.e. take a short position in the underlying asset. This transaction would give you \$98.03 today. You will deposite this into a bank account for 1-month. In a month time, your deposite will be worth \$98.03(1+0.01)=\$99.01. You can use \$98 of this money to buy the asset, remember you entered into a contract that gives you the right to do this, and close your short position. You will be able to pocket the difference of \$1.01. This means you will receive \$1.01 in 1 month with out taking any risk, Alternatively, you can only deposite \$97.03 of the money you receive today due to the short sell in the bank and take \$1 for yourself today. The money you deposite, \$97.03, will grow to \$97.03(1+0.01)=\$98 which would be exactly enough to buy the asset and close your short position. In either case, you make money without taking any risk, i.e. there is an arbitrage.

17. (a) Let's calculate the one-month risk-free rate r(1), using the formula

$$F(t, T) = S(t)(1 + r(T - t)) = S(t) * (1 + r(1))^{T-t}.$$

In our case $T_1 = t+3$ and $T_2 = t+6$ since $T-t$ is measured in monthly terms. Substituting these maturities in the above formula gives

$$F(t,t+3) = \$120 = S(t) * (1+r(3)) = S(t) * (1+r(1))^3, \quad (2)$$

Notice $(1+r(3)) = (1+r(1))^3$ as the one month rate will stay constant

$$F(t,t+6) = \$122 = S(t) * (1+r(6)) = S(t) * (1+r(1))^6. \quad (3)$$

Dividing equation (2) by equation (1) leads to the following expression

$$\frac{\$122}{\$120} = \frac{S(t) * (1+r(1))^6}{S(t) * (1+r(1))^3} = (1+r(1))^3,$$

which can be solved to yield

$$r(1) = (\frac{\$122}{\$120})^{1/3} - 1 = 0.5525\%.$$

We can now use either equation (1) or (2) to find $S(t)$

$$\$120 = S(t) * (1+r(1))^3 \Rightarrow S(t) = \$118.033$$

(b) We can check for the arbitrage opportunity by calculating the spot price implied by the risk-free rate from part (a) and futures price $F(t,t+1)$

$$S(t) = F(t,t+1)/(1+r(1))^3 = \frac{\$119.5}{(1+0.5525\%)^3} = \$117.541.$$

This value is smaller than \$118.033 found in part (a), thus implying the arbitrage opportunity. The following strategy exploits this arbitrage opportunity:

Transaction	Payoff at t	Payoff at $t+1$
1. Long futures	0	$\$S(t+1) - 119.5$
2. Long asset	\$118.033	-$S(t+1)$
3. Invest in Risk-free	-\$117.541	\$119.5
Net:	0.492	0

2.5 Options

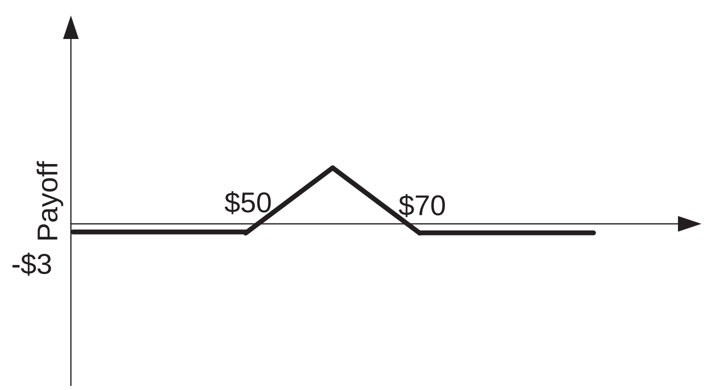
1. At the end of two months the value of the option will be either $C_u = \$4$ (if the stock price is \$53) or $C_d = \$0$ (if the stock price \$48). Moreover, $u = \frac{53}{50} = 1.06$, $d = \frac{48}{50} = .96$ and $R = e^{10\% \times \frac{2}{12}} = 1.017$ so that the risk-neutral probability of "up" is $q = \frac{R-d}{u-d} = 0.57$. The value of the option is therefore

$$C = \frac{1}{R} \times [qC_u + (1-q)C_d] = \$2.235.$$

2. At the end of four months the value of the option will be either $P_d = \$5$ (if the stock price is \$75) or \$0 (if the stock price $P_u = \$85$). Moreover, $u = \frac{85}{80} = 1.063$, $d = \frac{75}{80} = .94$ and $R = e^{5\% \times \frac{4}{12}} = 1.017$ so that the risk-neutral probability of "up" is $q = \frac{R-d}{u-d} = 0.634$. The value of the option is therefore

$$P = \frac{1}{R} \times [qP_u + (1-q)P_d] = \$1.797$$

3. (a) The payoff is as follows:



(b)

$$\begin{aligned} \text{Cost} &= \text{(Long 50 Call)} - 2 \times \text{(Short 60 Call)} + \text{(Long 70 Call)} \\ \$3.00 &= \$7.50 - 2 \times \$3.00 + \$1.50 \end{aligned}$$

This position requires an initial investment of \$3.00

(c) There will be a positive profit for stock prices at option maturity between \$53.00 and \$67.00

4. Use the Put-Call parity twice. Recall that by Put-Call parity, we have:

$$\text{Call} - \text{Put} = \text{Stock} - PV(\text{Strike})$$

So,

$$\text{Call}_{50} = \$94 - PV(\$50) + \text{Put}_{50} = \$51.50$$

and

$$\text{Call}_{60} = \$94 - PV(\$60) + \text{Put}_{60} = \$44.40$$

Note From the T-bill data point, you know $PV(\$100) = 91 \rightarrow PV(\$50) = 91/2$, etc.

5. (a) i. Buy one call option with strike price a and sell a call option with strike price b

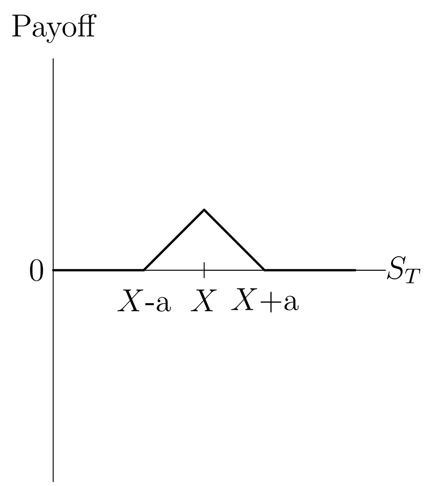
ii. Buy one call option with strike price a and buy one put option with strike price a

iii. Buy one call option with strike price a, sell two call options with strike price b, and buy another call option with strike price c

(b) The second diagram is a payoff that would hedge your risk to market volatility, i.e. if market drops or increases significantly this structure would have a positive cash flow and, hence, it can be used to hedge your risk to volatility. The cost of this hedge is that amount you have to pay to buy one call and one put options.

6. (a) Payoff diagram:

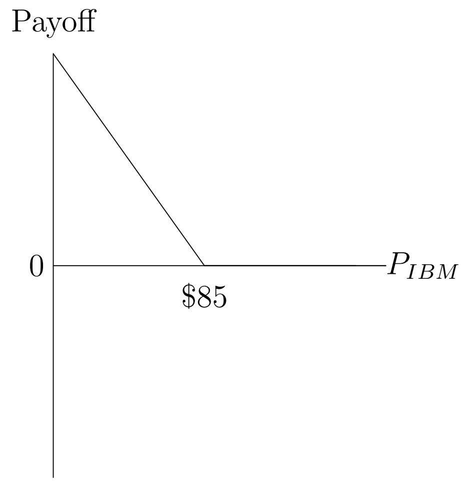
Butterfly Spread



As the low and high strike price converge to the intermediate price, the payoffs converge to 0.

(b) Based only on this trade, we may conclude that the trader believes that the stock price will not move far from its current value.

7. (a) Payoff diagram:



(b) You will make money as long as $P_{IBM} \leq \$85 - FV(\$4) = \$85 - \$4.05 = \$80.95$

8. The price of an option depends on many factors including the volatility of the underlying asset, in this case one IBM share. So if the news happens to increase that volatility, for example increase the uncertainty or range of possible future growth potential of IBM, it will increase the option price even though the news many not change the price of a share. Of course, alternatively the news may change the interest rate for the relevant maturity time, and hence change the price as well.

9. You need to apply the Put-Call parity twice. Remember that according to Put-Call parity, the following relation exits between the price of put and call option written on a stock with the same maturity and strike price:

$$Call - Put = Stock - PV(Strike)$$

Applying this to prices for option with strike \$50, we get:

$$57.50 - 3 = 100 - PV(50) \rightarrow PV(50) = 45.5$$

This implies that the present value of T-bill with future price of \$100 must be $2 \times 45.5 = 91$. Now, apply the same logic to price of options with strike price fo \$60.

$$Call - 5 = 100 - PV(60) \rightarrow Call = 100 - 60\frac{91}{100} + 5 \rightarrow Call = 50.4$$

So the price of Call option with strike price $60 should be $50.4.

10. Let us examine the payoffs of each of the strategies:

(a) By writing call options (these are called covered calls given the long position in the stock), Jones takes in premium income of $3,000. If the price of the stock in January is less than or equal to $45, he will have his stock plus the premium income. But the most he can have is $45,000 + $3,000 because the stock will be called away from him if its price exceeds $45. (We are ignoring interest earned on the premium income from writing the options in this very short period of time.) The payoff as a function of the stock price in January, S_T, is

Stock price	Portfolio value
$S_T \leq \$45$	$(1,000)S_T + 3,000$
$S_T > \$45$	$45,000 + 3,000 = \$48,000$

This strategy offers some extra premium income (by selling the upside) but leaves substantial downside risk. At an extreme, if the stock price fell to zero, Jones would be left with only $3,000. The strategy also puts a cap on the final value at $48,000, but this is more than sufficient to purchase the house.

(b) By buying put options with a $35 strike price, Jones will be paying $3,000 in premiums to insure a minimum level for the final value of his portfolio. That minimium value is $(35)(1,000) - 3,000 = \$32,000$. This strategy allows for upside gain, but exposes Jones to the possibility of a moderate loss equal to the cost of the puts. The payoff structure is

Stock price	Portfolio value
$S_T \leq \$35$	$35,000 - 3,000 = \$32,000$
$S_T > \$35$	$(1,000)S_T - 3,000$

(c) The cost of the collar is zero. The value of the portfolio in January will be as follows:

Stock price	Portfolio value
$S_T \leq \$35$	$\$35,000$
$\$35 < S_T < \$45$	$(1,000)S_T$
$S_T \geq \$45$	$45,000

If the stock price is less than or equal to \$35, the collar preserves the \$35,000 in principal. If the stock price exceeds \$45, the value of the portfolio can rise to a cap of \$45,000. In between, the proceeds equal 1,000 times the stock price

Given the objective of Jones, the best strategy would be (c) since it satisfies the two requirements of preserving the \$35,000 in principal while offering a chance of getting \$45,000. Strategy (a) should be ruled out since it leaves Jones exposed to the risk of substantial loss of principal.

11. This strategy is a bearish spread. The initial proceeds are $9 - 3 = \$6$. The payoff is either negative or zero:

	$S_T \leq 50$	$50 < S_T < 60$	$S_T \geq 60$
Buy call with $K = \$60$	0	0	$S_T - 60$
Write call with $K = \$50$	0	$-(S_T - 50)$	$-(S_T - 50)$
Final Payoff	0	$-(S_T - 50)$	-10
Initial proceeds	\$6	\$6	\$6
Total	\$6	\$56-S_T	$-\$4$

The profit graph is given by the payoff graph plus a constant of \$6 (ignoring the interest).

The strategy breaks even when the payoffs offsets the initial proceeds of \$6, which occurs at a stock price of $S_T = \$56$.

12. (a) The payoff from investing \$1 in "the contract" can be expressed as

$$\text{payoff} = \max\left[.6\left(\frac{S^\star - S_0}{S_0}\right) + 1,\ 1\right]$$

where $S^\star$ is the S&P index value one year from now, and S_0 is its current value.

(b) Since the contract pays off at least \$1 for each dollar invested *plus* 60% of the return on the market (if that return is positive), the contract can be replicated by a portfolio consisting of

- a risk-free discount bond with face value \$1
- .6 call options that pay $\max\left(\frac{S^\star - S_0}{S_0},\ 0\right)$.

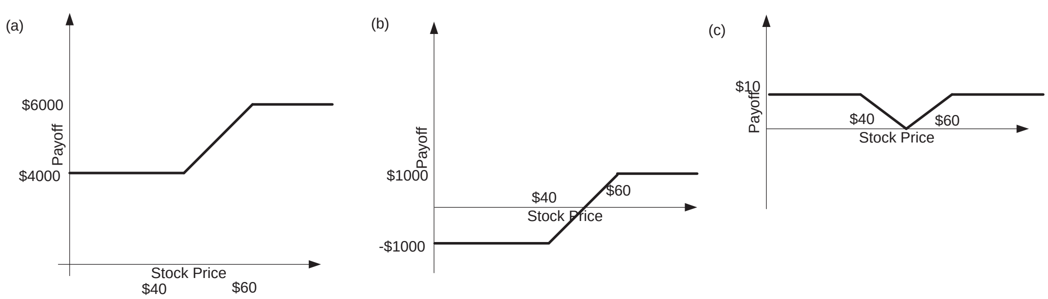
(c) The tree for the S&P 500 is:
implying that:

$$\begin{aligned} R &= 1.1 \\ u &= 1.2;\ \ d = 0.8 \\ q_u &= \frac{1.1 - .8}{1.2 - .8} = .75;\ \ q_d = \frac{1.2 - 1.1}{1.2 - .8} = 0.25. \end{aligned}$$

Using risk-neutral pricing, the value of this contract is $\frac{1}{1.1}[.75(1.12) + .25(1)] = 0.9909$. Given that the contract costs one dollar, you lose money by investing in this seemingly attractive contract!

13. The lower bound will be $50 - PV(45) = 50 - 45/1.08 = 8.33$

14. The payoff are given as follows:



15. Buy a call with an exercise price of \$40. Sell a call with an exercise price of \$60 Assuming the semi annual interest rate is 2%, invest \$3921.57, so that the payoff of your investment in 6 months will be \$4,000. Alternatively, you could also use the put call parity to replicate the stock.

16. (a) First, calcualte the option's hedge ratio:

$$\Delta = \frac{10 - 0}{110 - 90.01} = 0.5238$$

To replicate the call, you need to buy Δ shares and finance this by borrowing at r_f. Assuming the given rate is the APR, the amount you need to borrow is the solution of the following equation:

$$X(1 + 0.02/4) = 10 \rightarrow X = \$47.38$$

Hence the price of the call option is

$$0.5238 \times 100 - \$47.38 = \$5.00$$

Note: We have used a slight shortcut in this question to calculate the hedge ratio. You could have calculated the hedge ratio and the required amount to borrow by setting up the replication equations from the first principles as has been done in several other questions dealing with this.

(b) Solve for p_u and $p_d = 1 - p_u$.

$p_u \times \$110 + (1 - p_u) \times \$90.91 = 100 \times (1 + 0.02/4) \rightarrow p_u = 0.503$ and $p_d = 0.497$

Use the payoff of call and the probabilities to calculat the call price.

$$\text{Call Price} = \frac{0.503 \times 10 + 0.497 \times 0}{1 + 0.02/4} = \$5.00$$

17. (a) *False.* You can replicate the pay-off of the put option using a replication portfolio that is independent of the probabilities of the up or down movement. Hence, by the arbitrage argument, the probability of the up or down movement is irrelevant for the option pricing as long as the price of the stock is given and fixed.

(b) *False.* By observing the put and call prices, one can recover the stock price using the put-call parity. However that does not give you an estimate of the stock return.

(c) *False.* Suppose the statement were true. Denote C_{200}, C_{210} and C_{220} to be prices of calls with strikes 200, 210, and 220, respectively. Need to replicate the payoff $\max[S_T - 210, 0]$ by choosing a static trading strategy consisting of x_1 units of C_{200} and x_2 units of C_{220}. That is, x_1 and x_2 need to be such that at maturity of the calls, $x_1 C_{200} + x_2 C_{220} = C_{210}$. Suppose the time-$T$ value of the stock turns out to be $S_T = 205$. In this case, C_{210} and C_{220} are worth zero, and C_{200} is worth $205 - 200 = 5$. This pins down the weight x_1 in the replicating portfolio:

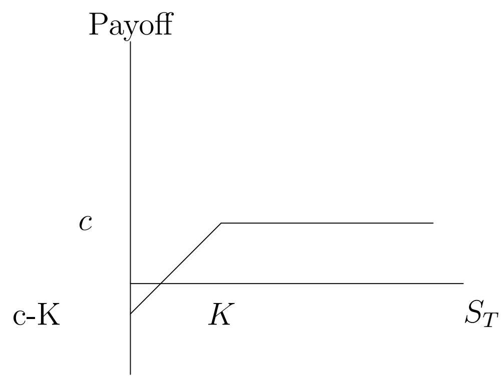
$$x_1 \times 5 + x_2 \times 0 = 0$$

$$\Rightarrow x_1 = 0.$$

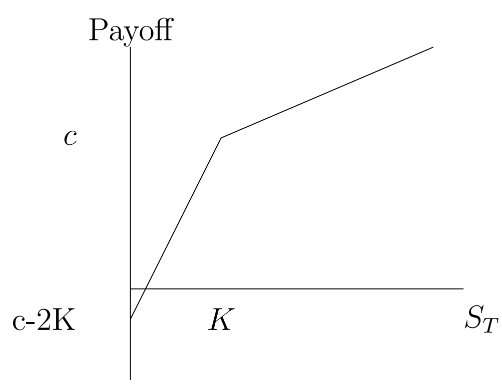
Now suppose the time-T value of the stock is $S_T = 215$. C_{210} is then worth $215 - 210$. The replicating portfolio has, however, zero value since $x_1 = 0$ and C_{220} is worth zero. Therefore, it is not possible to find a static portfolio of x_1 units of the call with the strike of 200 and x_2 units of the call with the strike of 220 that replicates the call with the strike of 210.

(d) *False.* Due to put call parity, if stock price and risk-free rate stays constant, an increase in call price will necessarily lead to increase in put price.

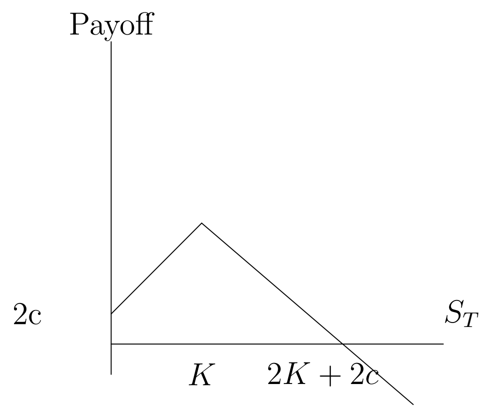
18. (a) Payoff diagram:



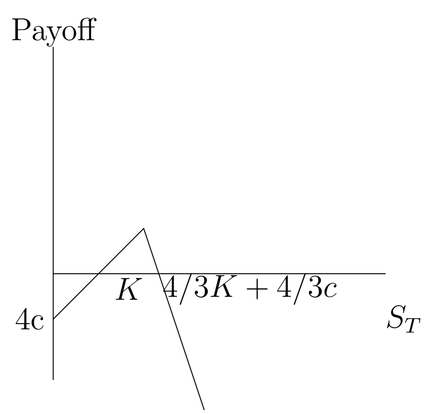
(b) Payoff diagram:



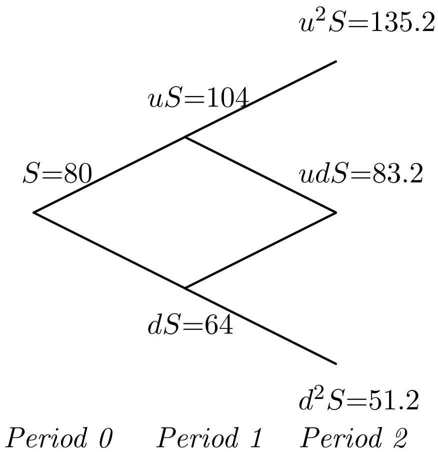
(c) Payoff diagram:



(d) Payoff diagram:



19. We have the following stock price movement diagram (Parameters: $u = 1.3$, $d = 0.8$)



Let's solve this using the risk-neutral probability approch. Let π_u and π_d be risk-neutral probability of the two possible outcomes. The following relationships mush hold

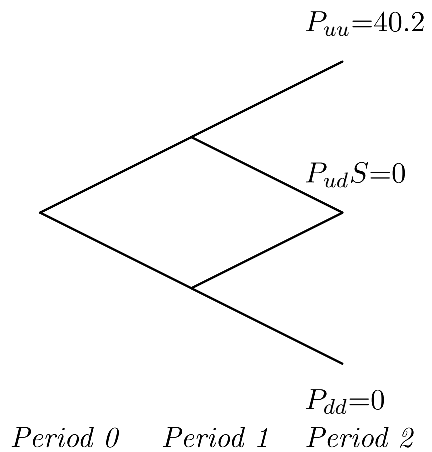
$$\begin{aligned} \frac{\pi_u + \pi_d}{1.03} &= \frac{1}{1.03} \\ \frac{\pi_u \times 1.3 \times \$80 + \pi_d \times 0.8 \times \$80}{1.03} &= \$80 \end{aligned}$$

Recall that the first equation comes from pricing of the risk-free asset and the second one from pricing the stock. Solving this system will give the following solution for the risk-neutral probabilities:

$$\begin{aligned} \pi_u &= 0.46 \\ \pi_d &= 0.54 \end{aligned} \tag{4}$$

Note also that these equations are the same for stages (the intial step as well as the step after 6 months) and there is no need to solve a new set of equations.

(a) The payoff of the European call options is:

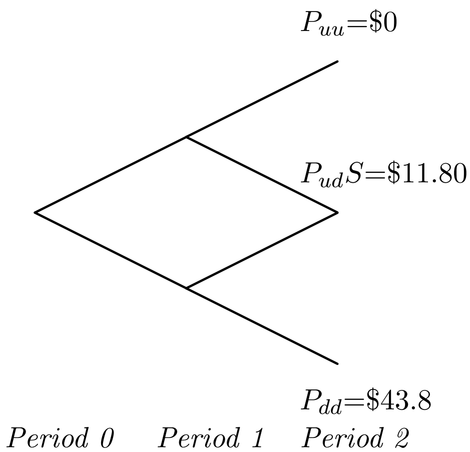


Using the risk neural probabilities finding the price is very simple: Price $= \frac{P_{uu}\times\pi_u^2}{(1.03)^2} = \frac{40.2\times0.46^2}{(1.03)^2} = \$8.02$

(b) This is now a bit more complicated because we need to compare the excercise value of the call with its price if we were to hold the option until year two at each point. The maximum would be the price at that node.

If price moves up after the first step, the immediate excercise value is \$104-\$95=\$9. The value if not excercised is $\frac{P_{uu}\times\pi_u}{(1.03)} = \frac{40.2\times0.46}{1.03} =$ \$17.95. Clearly it is not benefitial to excercise early. If the price moves down, the option will be under-water (so no immediate excercise value) and also there is no value if held becuse if both possible final outcomes the payoff is zero. So the price of the option in this case is \$0. The intial price can now be calculated as: Price $= \frac{\$17.95\times\pi_u+\$0\pi_d}{(1.03)} = \frac{40.2\times0.46}{1.03} = \$8.02$. It is the same as European call because it was not optimal to exercise early here so this extra level of flexibility was worth zero. As it turns out, it is never optimal to excercise an American call on a non-divided paying stock early. So the price of a European call and American call in such cases are always equal.

(c) The payoff of the European put options is:



Using the risk neural probabilities finding the price is very simple: Price $= \frac{P_{uu}\times\pi_u^2+2\times P_{ud}\times\pi_u\times\pi_d+P_{dd}\times\pi_d^2}{(1.03)^2} = \frac{2\times 11.80\times 0.46\times 0.54+43.8\times 0.54^2}{(1.03)^2} =$ \$17.56

20. The date for the underlying asset, the interest rate and the option is as follows:

$$\begin{aligned}\sigma &= 0.5\\ r &= 0.15\\ t &= 5\\ S/K &= 1.\end{aligned}$$

Ratio of PV of strike price to stock price $(K(1+r)^{-5}/S)$ is 49.72%. $\sigma\sqrt{T} = (0.5)\sqrt{5} = 1.1180$. By the Black-Scholes formula, the call price as percentage of the stock price is 62.05%. (Note that the interest rate you put in the Black-Scholes in the continuously compounding interest rate. So you should plug $\ln(1+15\%)$ in the formula to take this into account). By put-call parity, the put price with the same strike is 11.77% of the stock price.

21. First find the risk-neutral probabilities by solving for q_u and $q_d = 1-q_u$

$$60\times(1.06) = 66\times q_u + 54\times(1-q_u) \rightarrow q_u = 0.8 \text{ and } q_d = 0.2$$

alternately, remember the explicit formulas for risk-neutral probabilities:

$$q_u = \frac{(1+r_f)-d}{u-d} \text{ and } q_d = \frac{u-(1+r_f)}{u-d}$$

In the upstate, put payoff is max(64-66,0)=0. In the downstate, put payoff is max(64-54,0)=10. We can discount risk-neutral probability weighted payoffs by the risk-free rate to obtain PVs. Thus the price of the put is

$$\frac{q_u \times 0 + q_d \times 10}{1.06} = \$1.89$$

22. We use the risk-neutral probabilities to replicate and price this exotic security. Use a system of 2 equations and 2 unknowns to solve for the amount of stock and bond necessary to replicate the payoffs.

upstate equation: $4356 = 66 \times S + 1.06 \times B$

downstate equation: $2916 = 54 \times S + 1.06 \times B$

$\rightarrow S = 120, B = -3362.26$

So, to replicate this payoff you need to borrow \$3362.26 of the risk-free bond and buy 120 shares of ePet. The PV of this position is $q_u \times 4356 + q_d \times 2916 = \$3837.74$

23. (a) From Black-Scholes, we have:

$$C(S, K, T) = SN(d_1) - KR^{-T}N(d_2)$$

where

$$d_1 = \frac{\ln(S/KR^{-T})}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

In this case, $S = \$80$, $K = \$85$, $\sigma = 0.5$, $R = 1.06$, $T = 1$. Plugging these values in the above equation, you will find that the value of the call option is approximately \$15.71.

(b) Using put-call parity, you have

$$\text{Value}_{\text{Put}} = \text{Value}_{\text{Call}} - S + \frac{K}{(1+r)^T}$$

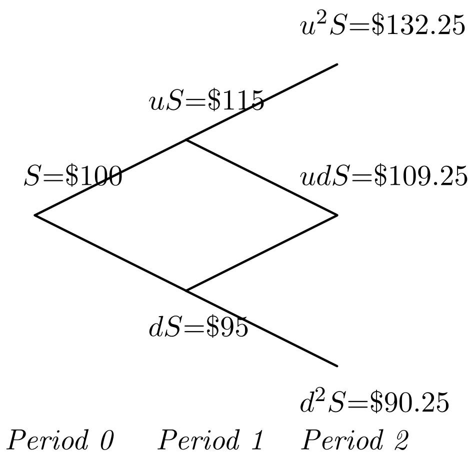
So the value of a put is $\text{Value}_{\text{Put}} = \$15.71 - \$80 + \$85/1.06 \approx \$15.91$

24. (a) Let the risk neural probabilties be π_u and π_d. The following set of equations should hold:

$$\begin{aligned} \pi_u + \pi_d &= 1 \\ 1.15 \times \pi_u + 0.95 \times \pi_d &= 1.05 \end{aligned}$$

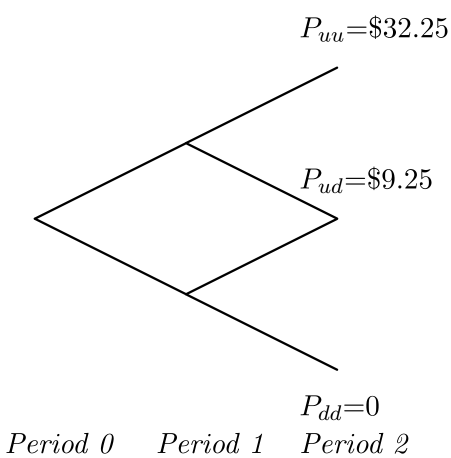
Solving this you will find that $\pi_u = 0.5$ and $\pi_d = 0.5$.

(b) The stock has the following price path: Parameters: $u = 1.15$, $d = 0.95$. Stock tree:



The call option has the following payoffs:

The stock has the following price path: Parameters: $u = 1.15$, $d = 0.95$. Stock tree:



Finding the price using the risk-neutral probabilities is very simple.

$$\text{Price(Call)} = \frac{\$32.25 \times \pi_u^2 + \$9.25 \times \pi_u \pi_d \times 2}{1.05^2} = \$11.51$$

(c) Start from the last node. First condier the case that price increases in the first step. Let Δ_u and B_u be the number of shares and the amount of investment in the risk-free (bond) asset in the "up" note. These quantities can be determined by solving these equations:

$$\begin{aligned} B_u \times 1.05 + \Delta_u \times 1.15 &= \$32.25 \\ B_u \times 1.05 + \Delta_u \times 0.95 &= \$9.25 \end{aligned}$$

The solution is $B_u = -95.23$ (you need to borrow) and $\Delta_u = \$115$. The total value of the holding is \$19.76 (note that this is the price of the call option in this node, you can verify this using the risk neutral approach as well). Do the exact same for the case that the stock price moves down in the first step. You need to solve:

$$\begin{aligned} B_d \times 1.05 + \Delta_d \times 1.15 &= \$9.25 \\ B_d \times 1.05 + \Delta_d \times 0.95 &= \$0 \end{aligned}$$

The solution is $B_d = -41.84$ (you need to borrow) and $\Delta_d = \$46.25$. The total value of the holding is \$4.40. Now solve the a similar set of equations for the first step. The value of the portfolio you construct in each of the two possible outcomes (i.e. if the stock price moves up or down) should be equal to the total value of the portfolio you calculated above. You simply need to solve the following equations:

$$\begin{aligned} B \times 1.05 + \Delta \times 1.15 &= \$19.76 \\ B \times 1.05 + \Delta \times 0.95 &= \$4.40 \end{aligned}$$

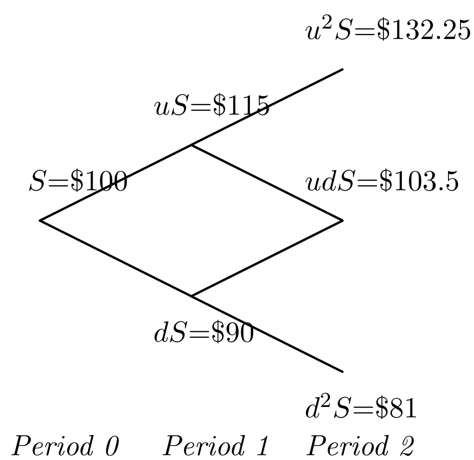
The solution is $B = -65.29$ and $\Delta = 76.80$. The value of this portfolio is \$11.51. So you can sell a call option for this price (it is the same value we calculated in part a) and invest the proceeds into stock and bond according such that \$76.80 is invested in the stock and you finance this by borrowing \$65.29. In the next step, if the price moves up, you need to adjust your holding of the stock up such that you have \$115 invested in stock and borrowed \$95.23. If the price moves down, you need to reduce you stock holding to \$46.25 and borrowed \$-41.84. If you follow this strategy, you are certain to have enough money at time 2 to cover the payoff of the stock.

(d) You can go through the analysis and workout the price of the American option by considering payoff due to early exercise at each node and compare that to the value of the option. As we calculated above, the value of the option in the "up" node at period 1 is \$19.76. The value of exercise is \$15, so early exercise is not optimal. In the "down" node, the value of the early exercise is 0 (option in out of the money) and hence, again, the early exercise is not optimal. Putting these together, early exercise in not optimal in this case so the value of the American call is the same as the value of the European call that we calculated above. In general, early exercise of a call option on a non-dividend paying stock is never optimal, so the value of American and European call options in those cases is the same.

25. (a) Let the risk neural probabilties be π_u and π_d. The following set of equations should hold:

$$\begin{aligned} \pi_u + \pi_d &= 1 \\ 1.15 \times \pi_u + 0.90 \times \pi_d &= 1.05 \end{aligned}$$

Solving this you will find that $\pi_u = 0.6$ and $\pi_d = 0.4$. The stock has the following price path (parameters: $u = 1.15$, $d = 0.9$):



The payoff of the put are $P_{uu} = 0$,$P_{ud} = P_{du} = 0$ and $P_{dd} = \$19$. Use the risk neutral pricing approach to get:

$$\text{P(Put)} = \frac{\$19 \times \pi_d^2}{1.05^2} = \$2.76$$

(b) This is a bit more complicated. If the price moves up in the first step, the value of the put will be zero since in both possible outcomes in step 2 the final payoff will be zero (i.e. $P_{uu} = 0$ and $P_{ud} = 0$). If the price moves down, however, the value of the immediate exercise is \$10 (=\$100-\$90). The value if you keep the option (again using the risk neutral approach) is:

$$\frac{\$19 \times \pi_d + \$3.5 \times \pi_u}{1.05} = \$9.24$$

In this case, it paysoff to excercise early. So the value of the put at the first step is

$$\text{P(Put)} = \frac{\$10 \times \pi_d + \$0 \times \pi_u}{1.05} = \$3.80$$

So the value is different and higher by almost $1. The reason is that it is benefitial to exercise the option early in case the price drops at the first step.

26. From Black-Scholes, we have:

$$C(S, K, T) = SN(d_1) - KR^{-T}N(d_2)$$

where

$$d_1 = \frac{\ln(S/KR^{-T})}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

From Put-Call parity, we have:

$$P(S, K, T) = C(S, K, T) - S + KR^{-T}$$

This simplifies to:

$$\begin{aligned} P(S, K, T) &= S(N(d_1) - 1) - KR^{-T}(N(d_2) - 1) \\ &= S(-N(-d_1)) - KR^{-T}(-N(-d_2)) \\ &= KR^{-T}N(-d_2) - SN(-d_1) \end{aligned}$$

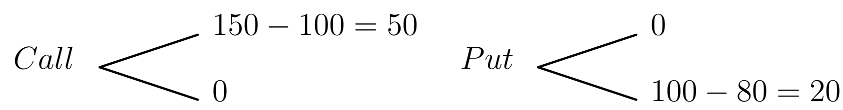
In this case, we have:

$$\begin{aligned} d_1 &= \frac{\ln(90.29/(90 \times (1 + 0.95\%)^{-1}))}{26.1\% \times \sqrt{1}} + \frac{1}{2}26.1\% \times \sqrt{1} \\ &= 0.1066 \\ d_2 &= d_1 - 26.1\% \times \sqrt{1} \\ &= -0.1544 \end{aligned}$$

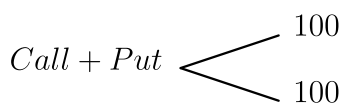
Putting this into the put price formula given above you get:

$$\begin{aligned} P(50, 50, 1) &= 90 \times (1 + 0.095\%)^{-1}N(0.1544) - 90.29 \times N(-0.1066) \\ &= 90 \times 1.0095^{-1} \times 0.5614 - 90.29 \times 0.4576 \\ &= 8.74 \end{aligned}$$

27. Call and put payoffs:



Notice that the payoff of 2 call + 5 put is



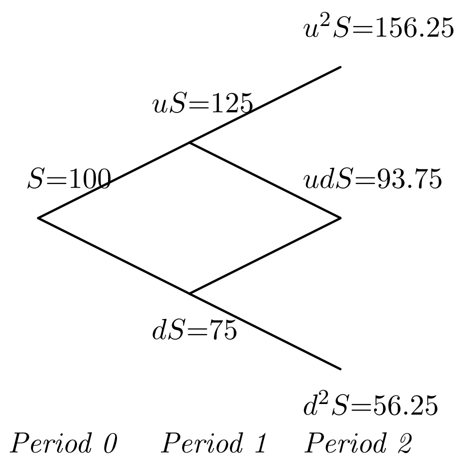
Riskless payoff!

By no-arbitrage, then, the price of 1 zeros with a face of $100 maturing in three months is equal to

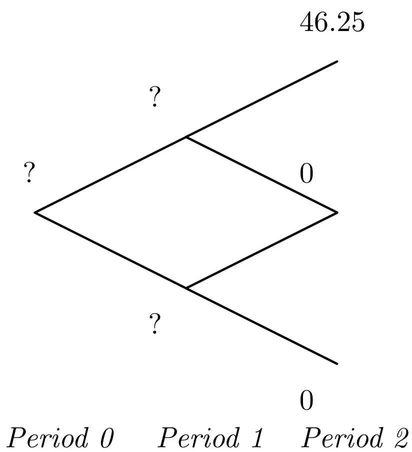
$$\text{Price of 2 call} + \text{Price of 5 put} = 2 * 25 + 5 * 8 = 90.$$

Hence, the price of the zero is 90.

28. Parameters: $u = 1.25$, $d = 0.75$. Stock tree:

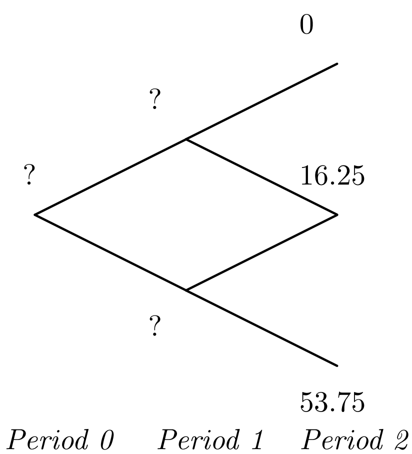


(a) Call payoffs:



First compute RNP: $q = \frac{1+0.05-0.75}{1.25-0.75} = 0.6$. Then use the risk-neutral valuation formula to compute $C_u = 26.43$, $C_d = 0$ and $C = 15.10$.

(b) Put payoffs:



Again, using the risk-neutral valuation, we find that P_u=6.19, $P_d = 29.76$, $P = 14.88$.

(c) Put-call parity: $C = S + P - \frac{K}{(1+r)^T}$.

$$\text{Period 0:} \qquad 15.10 = 100 + 14.88 - 110\frac{1}{(1+5\%)^2} \quad \checkmark$$

$$\text{Period 1, } u\text{-node:} \qquad 26.43 = 125 + 6.19 - 110\frac{1}{(1+5\%)} \quad \checkmark$$

$$\text{Period 1, } d\text{-node:} \qquad 0 = 75 + 29.76 - 110\frac{1}{(1+5\%)} \quad \checkmark$$

(d) and (e)

u-node: There is no early exercise because the option is out-of-money

$$\begin{aligned} 156.25\Delta + 100B &= 0 \\ 93.75\Delta + 100B &= 16.25 \end{aligned}$$

Solving this system, we have $\Delta = -0.26$, $B = 0.4063$. So the value of the replicating portfolio in the u-node is $-0.26 \times 125 + 0.4063 \times 100/1.05 = 6.19$ at this node. We will use this value shortly.

d-node: Payoff from exercising = 110-75 = 35 > Value of holding on ($P_d = 29.76$), therefore the put is exercised, hence the portfolio here will consist of \$35 cash and no stock.

initial-node:

$$\begin{aligned} 125\Delta + 100B &= 6.19 \\ 75\Delta + 100B &= 35 \end{aligned}$$

Solving this system, we have $\Delta = -0.5762$, $B = 0.7822$. So the value of the initial replicating portfolio is $-0.5762 \times 100 + 0.7822 \times 100/1.05 = \$16.87$.

29. Let's solve this using the digital security approach. Assume you have two securities that each pay \$1 in one of the two possible states depending contingent on where the price of the Google stock ends up at in 1-year. Let p_u and p_d be the today's price of these securities. The following relationships mush hold

$$\begin{aligned} p_u + p_d &= 1/1.05 \\ p_u \times \$600 + p_d \times \$475 &= \$500 \end{aligned}$$

Solve this sytems of equations. The solution is

$$\begin{aligned} p_u &= \$0.3810 \qquad (5) \\ p_d &= \$0.5714 \end{aligned}$$

Now, the call option pays \$50 if the price of one share of Google ends at \$600 in one year, and zero otherwise. Putting this together, the price of the call option should be $50 \times \$0.3810 = \$19.05$

Note: You can solve this problem using alternative approaches such as the risk-neutral approach or the replication argument.

30. (a) We can use the concept of replicating portfolios. To do this, we need to look at the problem backward and consider two cases depending on what happens to the stock in the first month. First, consider the case that the price moves to \$29 in 1 month. You need to construct a portfolio of a shares and b dollars invested in bonds such that the portfolio has the exact same payoff as the put option. This gives you the following two equations to solve:

$$\begin{aligned} 0 &= 31a + 1.01b \\ 2 &= 24a + 1.01b \end{aligned}$$

The solution is $a = -0.2857$ and $b = 8.7694$. So the value fo the replication portfolio is $29 \times (-0.2857) + 8.7694 = 0.484$. Notice that is this case the current exercise value of the option is 0. So even if the option was American, it would not be optimal to exercise it early. This means that the value of the put option is $max(0.484, 0) = 0.484$.

Now, consider the other case that the stock move down to \$23 in the first month. Again, you need to setup a similar portfolio but the equations are a little different since the put payoff and final stock price are different. In this case, you need to have:

$$\begin{aligned} 0 &= 26a + 1.01b \\ 5 &= 21a + 1.01b \end{aligned}$$

The solution is $a = -1$ and $b = 25.7426$ and the value is of the replicating portfolio is $23 \times (-1) + 25.7426 = 2.743$. For European option, this is equal to the value of the option. For American option, one can receive \$3 by exercising the option early. So the value of the American option is $max(2.743, 3) = 3$.

Since we did not specify in this question if the option is American of European, you would get full credit either way. Both solutions are given below:

European Option:

Now, go back to the first portfolio. You need to select a and b in this case such that the value of portfolio is equal to the prices we just calculated in each of the possible two outcomes. The equations for this case are:

$$\begin{aligned} 0.484 &= 29a + 1.01b \\ 2.743 &= 23a + 1.01b \end{aligned}$$

The solution is $a = -0.3765$ and $b = 11.2896$ and the put price is $25 \times (-0.3765) + 11.2896 = 1.88$

American Option:

Everything stays the same except that the value of the replicating portfolio must be 3 in the case that the stock moves to 23 in the first month. So the equations are:

$$\begin{aligned} 0.484 &= 29a + 1.01b \\ 3 &= 23a + 1.01b \end{aligned}$$

The solution is $a = -0.4193$ and $b = 12.5195$ and the put value is $25 \times (-0.4193) + 12.5195 = 2.04$

(b) This derivative will have the following payoff:(up,up)=\$6, (up,down)=\$4, and (down,up)=\$1. Again, set up the replicating portfolios. If price moves to \$29, we have:

$$\begin{aligned} 6 &= 31a + 1.01b \\ 4 &= 24a + 1.01b \end{aligned}$$

Solution $a = 0.2857$, $b = -2.8289$ and the option price is $29 \times 0.2857 - 2.8289 = 5.456$.

If price moves to \$23, we have:

$$\begin{aligned} 1 &= 26a + 1.01b \\ 0 &= 21a + 1.01b \end{aligned}$$

Solution is $a = 0.2$, $b = -4.1584$ and the option price is $23 \times 0.2 - 4.1584 = 0.4416$. The initial portfolio must satisfy the following equations:

$$\begin{aligned} 5.456 &= 29a + 1.01b \\ 0.4416 &= 23a + 1.01b \end{aligned}$$

Solution is $a = 0.8357$, $b = -18.5943$ and the option price is $25 \times 0.8357 - 18.5943 = 2.30$

31. Yes, let's consider price difference between a call and a put. This price difference is the same cashflow (today) as buying a call and writing a put. The future cash-flow of this combination is the same as being long the stock and having to pay \$124 (the strike price). By arbitrage, the current price should be also the same. Hence, the current price of the structure should be $\$120 - PV(\$124) = \$120 - \$124/1.0609 = \$3.11$. Hence, the current price of a call should be higher.

32. From Black-Scholes, we have:

$$C(S, K, T) = SN(d_1) - KR^{-T}N(d_2)$$

where

$$d_1 = \frac{ln(S/KR^{-T})}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

From Put-Call parity, we have:

$$P(S, K, T) = C(S, K, T) - S + KR^{-T}$$

This simplifies to:

$$\begin{aligned} P(S, K, T) &= S(N(d_1) - 1) - KR^{-T}(N(d_2) - 1) \\ &= S(-N(-d_1)) - KR^{-T}(-N(-d_2)) \\ &= KR^{-T}N(-d_2) - SN(-d_1) \end{aligned}$$

In this case, we have:

$$\begin{aligned} d_1 &= \frac{ln(50/(50 \times (1 + 10\%)^{-1/4}))}{30\% \times \sqrt{1/4}} + \frac{1}{2}30\% \times \sqrt{1/4} \\ &= \frac{ln(1.1^{1/4})}{0.15} + \frac{0.15}{2} \\ &= 0.234 \\ d_2 &= d_1 - 30\% \times \sqrt{1/4} \\ &= 0.084 \end{aligned}$$

Putting this into the put price formula given above you get:

$$\begin{aligned} P(50, 50, 1) &= 50 \times (1 + 10\%)^{-1/4}N(-0.084) - 50 \times N(-0.234) \\ &= 50 \times 1.1^{-1/4}0.467 - 50 \times 0.407 \\ &= 2.4 \end{aligned}$$

What if there is a dividend?

Now, if there is a dividend of \$1.5 expected in two months, you know that the price of the stock will fall by that amount after the dividend has been paid. One way to think about this is by thinking about the current stock as two separate claims. One part of the price is due the expected dividend of \$1.5 in two months and this part is worth $1.5(1 + 10\%)^{-1/6} = 1.48$ now. The second part is due to the claim on the remaining assets of the firm after the divided has been paid. This second part must be worth $50 - 1.48 = 48.52$ now.

The option is effectively an option on the value of the second component in three months. Hence it is an option with a strike price of \$50 on an asset that is worth now \$48.52. The only other component you need to put into the Black-Scholes option pricing formula is the volatility of the asset. But the volatility of the first component, the dividend in two months, is zero. Hence the volatility of the second component must be 30% annualized. Now you have all the component to use the Black-Scholes formula again. So the only difference is that you need to reduce the current price of the stock by the present value of the expected dividend.

33. Let C_t, P_t, B_t denote the current prices of European call, put and discount bond with maturity t. From put-call parity, we have

$$C_t + KB_t = P_t + S$$

where K is the strike price for both the call and the put and S is the current stock price. For the three-month put and call with strike $K = \$60$, we have $C_{1/4} = P_{1/4} = \$10$. Put-call parity then gives:

$$10 + KB_{1/4} = 10 + S \quad \text{or} \quad S = KB_{1/4}.$$

For the six-month put and call, we have

$$C_{1/2} + KB_{1/2} = P_{1/2} + S = P_{1/2} + KB_{1/4}$$

or

$$C_{1/2} = P_{1/2} + K(B_{1/4} - B_{1/2}).$$

Note that $B_{1/4} > B_{1/2}$. We have $C_{1/2} > P_{1/2}$.

Answer: A six month call is more valuable than a similar maturity put.

2.6 Risk & Portfolio Choice

1. (a) False: As long as asset returns are not perfectly positively correlated, diversification will reduce risk.

(b) True: If returns are completely uncorrelated, then all risk is non-systematic risk that can be diversified away. So the expected return for all assets should be the same - the risk-free rate of return.

2. False. Standard deviation includes both systematic and idiosyncratic risks. An asset that appears dominated may be correlated with other assets in such a way that makes it useful (i.e., variance reducing) in a diversified portfolio.

3. True. Only a few securities are needed to reduce a significant portion of idiosyncratic risks. Benefits to diversification become smaller afterwards. Suppose we have an equally-weighted portfolio of n securities that have identical expected return and variance (σ^2), but are uncorrelated with each other. The portfolio variance is then $\frac{\sigma^2}{n}$, which is a decreasing and convex function of n.

4. 30% portfolio standard deviation $\Rightarrow \frac{30\%}{40\%} = 0.75$ in risky and 0.25 in riskfree $\Rightarrow$ portfolio expected return is $(0.25)(5\%) + (0.75)(12\%) = 10.25\%$.

5. (a) False. Diversification cannot eliminate systematic risk.

(b) False. Diversification works whenever asset returns are not perfectly correlated.

(c) True. The contribution of a stock to portfolio risk depends on its systematic risk, not total risk.

(d) False. Diversification may reduce portfolio risk while keeping expected return constant (e.g., a portfolio of many securities with identical return/risk but imperfect correlation).

6. (a) False. The correlation structure between Stock A and other securities in the market may make it useful as part of a diversified portfolio.

(b) False. The optimal diversification depends on the return and risk of each security, as well as the correlation structure between securities.

7. (c) is the only correct choice. (a) one can never eliminate the systematic risk - it is the un-diversifiable risk by definition. (b) you can keep the expected return at the desired level and reduce risk. Diversification gives you extra flexibility to reduce risk but can never hurt. (d) We have seen in many examples that even have two securities can reduce risk substantially, for example if the two securities are perfectly negatively correlated.

8. The portfolio R can not be on the Markowitz efficient frontier. It is dominated by a weighted portfolio W of Q and S, with weights $\frac{2}{3}$ on Q and $\frac{1}{3}$ on S. Indeed:

$$E[R_W] = \frac{2}{3}E[R_Q] + \frac{1}{3}E[R_S] = 10.5\% = E[R_R]$$

So W has the same expected return as R. We now look at the variance of W, which depends on the correlation ρ between Q and S.

$$Var[R_W] = \left(\frac{2}{3}\right)^2 Var[R_Q] + \left(\frac{1}{3}\right)^2 Var[R_S] + 2\frac{2}{3}\frac{1}{3}\rho\sqrt{Var[R_Q]Var[R_S]}$$

We want to show that W has a smaller variance than R, for any possible value of ρ ($-1 \leq \rho \leq 1$). The variance of W is maximized for $\rho = 1$. So:

$$Var[R_W] \leq \left(\frac{2}{3}\right)^2 0.15^2 + \left(\frac{1}{3}\right)^2 0.185^2 + 2\frac{2}{3}\frac{1}{3}(0.15 \times 0.185)$$

This gives $Var[R_W] \leq 0.0261$. The standard deviation is the square root of the variance, so:

$$SD[R_W] \leq \sqrt{0.0261} \leq 16.2\% < 16.5\% = SD[R_Q]$$

W has the same expected return as R and a strictly lower standard deviation, so R can not be on the efficient frontier.

9. Let x be the proportion of B. The expected portfolio return is $11x + 15(1-x) = 12$, which gives $x = \frac{3}{4} = 0.75$. So you should invest 75% of your wealth in B and 25% in A. The variance is then

$$\text{var}\,[(1-x)A + xB] = (1-x)^2\sigma_A^2 + x^2\sigma_B^2 + 2x(1-x)\sigma_{AB}.$$

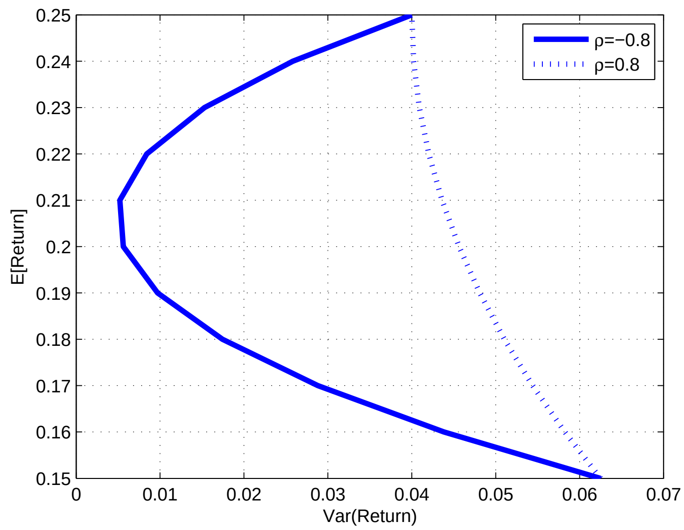
$$\sigma^2 = 0.25^2 \times 0.04 + 0.75^2 \times 0.032 + 2 \times 0.25 \times 0.75 \times 0.025 = 0.030.$$

So the standard deviation is 0.173.

10. (a) 0%: E(r_P) = 15%; $\sigma(r_P)$ = 25%.
10%: E(r_P) = 16%; $\sigma(r_P)$ = 24.13%.
20%: E(r_P) = 17%; $\sigma(r_P)$ = 23.32%.
30%: E(r_P) = 18%; $\sigma(r_P)$ = 22.59%.
40%: E(r_P) = 19%; $\sigma(r_P)$ = 21.93%.
50%: E(r_P) = 20%; $\sigma(r_P)$ = 21.36%.
60%: E(r_P) = 21%; $\sigma(r_P)$ = 20.88%.
70%: E(r_P) = 22%; $\sigma(r_P)$ = 20.50%.
80%: E(r_P) = 23%; $\sigma(r_P)$ = 20.22%.
90%: E(r_P) = 24%; $\sigma(r_P)$ = 20.06%.

100%: $E(r_P) = 25\%$; $\sigma(r_P) = 20\%$.

(b) 0%: $E(r_P) = 15\%$; $\sigma(r_P) = 25\%$.
10%: $E(r_P) = 16\%$; $\sigma(r_P) = 20.93\%$.
20%: $E(r_P) = 17\%$; $\sigma(r_P) = 16.97\%$.
30%: $E(r_P) = 18\%$; $\sigma(r_P) = 13.20\%$.
40%: $E(r_P) = 19\%$; $\sigma(r_P) = 9.85\%$.
50%: $E(r_P) = 20\%$; $\sigma(r_P) = 7.50\%$.
60%: $E(r_P) = 21\%$; $\sigma(r_P) = 7.21\%$.
70%: $E(r_P) = 22\%$; $\sigma(r_P) = 9.18\%$.
80%: $E(r_P) = 23\%$; $\sigma(r_P) = 12.37\%$.
90%: $E(r_P) = 24\%$; $\sigma(r_P) = 16.07\%$.
100%: $E(r_P) = 25\%$; $\sigma(r_P) = 20\%$.



(c) In (b), the two securities are negatively correlated, so their risks cancel out, leading to much lower portfolio variance than that in (a).

11. (a) Mean $= 0.9(0.08)+0.1(0.02) = 7.4\%$; standard deviation $= 0.9(0.2) = 18\%$.

(b) Mean $= 0.8(0.08)+0.2(0.08) = 8\%$; standard deviation $= (0.8)^2(0.2)^2 + (0.2)^2(0.4)^2 = 17.89\%$.

(c) Q is better than P. It has a higher mean, but a lower standard deviation.

(d) One would hold B for diversification purposes. Notice that though A dominates B, Q dominates A. However, Q contains some B.

12. Let w be the weight on A. w solves $0.10w+0.15(1-w) = 0.12$. $w = 0.6$. The portfolio standard deviation is 24.33% (use the following equation: $\sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_Aw_B\sigma_{AB}$).

13. If you want an expected rate of return of 18%, the required weight on fund A (w_A) is given by

$$w_A(.20) + (1 - w_A)(.15) = .18 \quad (6)$$
$$w_A = .6 \quad (7)$$
$$1 - w_A = w_B = .4 \quad (8)$$

Given these portfolio weights, the portfolio standard deviation is given by

$$\sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_Aw_B\sigma_{AB} \quad (9)$$
$$= (.6^2)(.36) + (.4^2)(.1225) + 2(.6)(.4)(.0840) = .1895 \quad (10)$$
$$\sigma_p = .4353. \quad (11)$$

14. Let w_A, w_B, w_C be portfolio weights. Solve the following problem:

$$\text{Min}_{w's}\ \sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 \quad (12)$$
$$+2w_Aw_B\sigma_{AB} + 2w_Aw_C\sigma_{AC} + 2w_Bw_C\sigma_{BC} \quad (13)$$

$$s.t. \quad w_A + w_B + w_C = 1 \quad (14)$$
$$w_A(.20) + w_B(.15) + w_C(.10) = .16 \quad (15)$$

The two constraints can be reduced to

$$w_B = 1.2 - 2w_A \quad (16)$$
$$w_C = w_A - 0.2 \quad (17)$$

Plugging the constraints and variances into the expression for portfolio variances, we get the following simplified problem:

$$\text{Min}_{w_A}\ \sigma_p^2 = (.5065)w_A^2 - (.2294)w_A + .1453 \quad (18)$$

First order conditions gives the optimal solution of $w_A = \frac{.2294}{2(.5065)} = .2265$. Hence, $w_B = .7471$, $w_C = .0264$, $\sigma_p^2 = .1193$, and $\sigma_p = .3454$.

Another way to solve the optimization problem is to use the "solver" function in a spreadsheet. The basic steps are to minimize the cell containing the formula for portfolio variance by changing the cell containing portfolio weight, subject to 2 constraints (expected return and total weight).

15. (a) XYZ may have high idiosyncratic risk that can be diversified away in a portfolio of securities.

(b) $r_P = 18\%$; $\sigma_P = 17.09\%$. (Note: used the following equation: $\sigma_p^2 = w_{ABC}^2\sigma_{ABC}^2 + w_{XYZ}^2\sigma_{XYZ}^2 + 2w_{ABS}w_{XYZ}\sigma_{ABC,XYZ}$).

(c) Let w be the weight of ABC in the portfolio. w solves $0.20w + 0.15(1-w) = 0.195$. $w = 0.9$. The portfolio has a standard deviation of 18.66%.

16. (a) Recall that the tangency portfolio maximizes the Sharpe Ration; i.e. it is the portfolio with the higher expected return divided by the standard deviation. It is defined by $(w^*, 1-w^*)$ such that:

$$w^* \equiv \arg\max_w \frac{0.2w + 0.15(1-w) - 0.05}{\sqrt{0.2^2w^2 + 0.25^2(1-w)^2 + 2w(1-w)(0.2)(0.25)(0.2)}}.$$

Maximizing using Excel Solver yields $w^* = 0.7922$.

(b) The tangency portfolio has an expected return of 18.8506% and standard deviation of 17.4834%. To obtain an expected return of 19.5%, we should choose a portfolio with the riskfree security and the tangency portfolio (weights $= (w, 1-w)$) such that $0.05w + 0.188506(1-w) = 0.195$. $w = -0.04689$.

(c) The portfolio in (b) has a standard deviation of $1.0469 \times 0.174834 = 0.1830$, which is less than that of the portfolio in (c) of the last question. The reduction in portofio risk is the result of an additional risk-free security.

17. Let's call D the portfolio of equal investament of each stock.

$$E[R_D] = \frac{0.11 + 0.145 + 0.09}{3} = 11.5\%$$

$$Var[R_D] = \frac{Var[R_A] + Var[R_B] + Var[R_C]}{9} + \frac{2}{9}\left(Cov(R_A, R_B) + Cov(R_C, R_B) + Cov(R_C, R_A\right.$$

We use $Cov(R_i, R_j) = Cor(R_i, R_j)SD[R_i]SD[R_j]$ We get $Var[R_D] = 0.068$, so $SD[R_D] = 26.08\%$

Note: The Standard deviation of the portfolio is lower than the standard deviation of any of the stocks. This is an example of the benefits of diversification.

18. (a) Linearity of expected returns requires: $w_a \times 8\% + (1-w)a \times 20\% = 8\%$. Solving for w_a yields 28.6% in the risk free asset and 71.4% in the S&P. Sigma is given by $0.714 \times 20\% = 14.28\%$.

(b) Even though the Emerging Markets fund has the same return and a higher standard deviation, its correlation with the S&P may lead to achieve lower levels of risk for the portfolio as a whole. If the correlation is sufficiently low, we may be able to diversify away some of the risk. We should certainly consider the Emerging Fund.

(c) Portfolio A's expected return is 10%. Portfolio A's standard deviation is calculated as follows

$$\sigma^2 = 0.82 \times 202 + 0.22 \times 302 + 2 \times 0.8 \times 0.2 \times 0 \times 20 \times 30$$

$$\sigma^2 = 292$$

$$\sigma = 17.09\%$$

(d) The weights will be the same as in part a, i.e., 28.6% in the risk free asset and 71.4% in Portfolio A. The standard deviation of the new portfolio will be $0.7143 \times 17.09\% = 12.2\%$, which is lower than for the portfolio in part a. We are clearly better off: now we can achieve the same return with a lower level of risk.

19. $\mathrm{E}(r_P) = w_A E[r_A] + w_B E[r_B] + w_C E[r_C]$.
$\mathrm{var}(r_P) = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + 2w_Aw_B\mathrm{cov}_{AB} + 2w_Aw_C\mathrm{cov}_{AC} + 2w_Bw_C\mathrm{cov}_{BC}$.

(a) $w_A = w_B = w_C = \frac{1}{3}$; $\mathrm{E}(r_P) = 15\%$; $\mathrm{var}(r_P) = 0.06127$.

(b) $w_A = \frac{105}{105+40+75} = 0.4773, w_B = \frac{40}{105+40+75} = 0.1818, w_C = \frac{75}{105+40+75} = 0.3409$; $\mathrm{E}(r_P) = 14.32\%$; $\mathrm{var}(r_P) = 0.05862$.

(c) $w_A = \frac{50}{50+30+27} = 0.4673, w_B = \frac{30}{50+30+27} = 0.2804, w_C = \frac{27}{50+30+27} = 0.2523$; $\mathrm{E}(r_P) = 13.93\%$; $\mathrm{var}(r_P) = 0.05878$.

20. (a) For IBM, $\beta_{IBM} = \frac{\rho_{IBM,M}\sigma_{IBM}\sigma_M}{\sigma_M^2} = 0.9$. By plugging into CAPM, we get $r_{IBM} = 0.121$. Similarly, $\beta_{GM} = 1$ and $r_{GM} = 0.13$.

(b) The variance is $(0.16)(0.09)+(0.16)(0.0625)+(2)(0.16)(0.4)(0.3)(0.25) = 0.034$. The beta is $(0.4)(0.9) + (0.4)(1) = 0.76$.

(c) The efficient portfolio will be on the efficient frontier, with $\sigma = \sqrt{0.034} = 0.1844$. Given that $\sigma_M = \sqrt{0.04} = 0.20$ for the market, the portfolio has $0.1844/0.20 = 0.922$ or 92.2% in the market portfolio and the remainder in the safe asset. Its expected return is $(0.922)(0.13 - 0.04) + 0.04 = 0.123$.

21. (a) $E(R_B) = r_f + \beta(E(R_m) - r_f) = 4\% + (12\% - 4\%) = 12\%$
$\beta = \frac{Cov(R_A,R_B)}{Var(R_A)} \Rightarrow Cov(R_A, R_B) = Var(R_A) = 0.04$
$Cov(R_B, R_C) = 0$ (by definition).

(b) Expected return = 12% for all cases as both assets have an expected return of 12%.

The standard deviation equals 40% if w = 0 as the whole portfolio is in asset B.

The standard deviation equals 20% if w = 1 as the whole portfolio is in asset A.

The standard Deviation (portfolio return) if w = 1/2 equals

$$\sqrt{(1/2)^2 Var(R_A) + (1/2)^2 Var(R_B) + 2(1/2)(1/2)Cov(R_A, R_B)} = 26.5\%.$$

(c) The portfolios in order of preference are w = 1, w = 0.5, and w = 0.

Since all of the portfolios have the same return, we can rank them from lowest standard deviation to highest standard deviation.

(d) $E(R_D) = \frac{1}{2}E(R_B) + \frac{1}{2}r_f = 6\% + 2\% = 8\%.$
$Var(R_D) = Var(\frac{1}{2}R_B) = \frac{1}{4}Var(R_B) = 0.04.$
$StdDev(R_D) = 0.2.$

(e) Suppose we invest α in A and $1 - \alpha$ in C.
$StdDev(\alpha R_A) = \alpha StdDev(R_A) = 0.2\alpha$

Since we want a portfolio with a standard deviation of 0.2, $\alpha = 1$.

The expected return of the portfolio is 12%.

(f) Any combination of A and B has an expected return of 12%.

However, including B in the portfolio would only increase portfolio variance.

Thus, A and C are mean-variance efficient while B is not.

(g) $\alpha R_A + (1 - \alpha)R_C = 10\%.$

$12\alpha + 4(1 - \alpha) = 10.$

$\alpha = 0.75.$

Thus, hold 75% in A and 25% in C.

22. (a) The expected return of the equally weighted portfolio is $025 \times 11 + 0.25 \times 14.5 + 0.25 \times 9 + 0.25 \times 11.5 = 11.5\%$.
The variance is $\sum_i x_i^2 \sigma_i^2 + 2\sum_i \sum_{j<i} x_i x_j \sigma_{i,j}$ with $x_i = 0.25$ for all i.
The variance is 0.044, and the standard deviation is 21%.

(b) Using Excel Solver, we mimize portfolio standard deviation, subject to a given expected return:

Portfolio return	10%	12%	14%	17%	20%
Portfolio std	20.4%	21.3%	27.9%	42.7%	59.3%

(c) Maximize the Sharpe ratio using Excel Solver. The result is to invest 37% in A, 42% in B, -2% in C and 24% in D. The Sharpe ratio in this case is 0.33.

23. Since Y and Z have identical expected return and standard deviation, their correlations with X are all that matters. We can treat a (X, Y, Z) portfolio as a (X, (Y, Z)) portfolio, where (Y, Z) has an expected return of 10% (regardless of weights) and standard deviation greater than or equal to 11.62% (equal weight). The weights on X and (Y, Z) are fixed given a portfolio return, so the portfolio variance can only vary with the weights on Y and Z.

(a) Z. Since 10% < 12% < 15%, the frontier portfolio must have positive weights on both X and (Y, Z). The portfolio variance is lower when the weight on Z is higher, because corr(X,Z) < corr(X,Y), so the weight on Z must be higher when variance is minimized.

(b) Y. Since 9% < 10% < 15%, the frontier portfolio must have negative weights on X. Now, the portfolio variance is lower when the weight on Y is higher. Therefore, the weight on Y must be higher when variance is minimized.

(c) Z. Since 9% < 10% < 15%, the frontier portfolio must have positive weights on X and negative weights on (Y, Z). One of Y and Z must have a negative weight, and it must be Y in the minimum-variance portfolio because corr(X,-Y) < corr(X,-Z). Therefore, the weight on Z must be higher.

24. First, for 10 stocks:

$$Var[\frac{R_1 + \cdots + R_{10}}{10}] = \frac{Var[R_1]}{100} + \cdots \frac{Var[R_{10}]}{100} + \sum_{i=1}^{10} \sum_{j=i+1}^{10} \frac{2}{100} Cov(R_i, R_j)$$

We have $Cov(R_i, R_j) = 0.3(SD[R_i]SD[R_j]) = 0.3Var[R_i]$. The number of terms in the sum $\sum_{i=1}^{10} \sum_{j=i+1}^{10}$ is $\frac{10(10-1)}{2}$.

$$Var[\frac{\sum_{i=1}^{10} R_i}{10}] = \frac{10}{100} Var[R_i] + 0.3 \frac{90}{100} Var[R_i] = 0.0453$$

For 10 stocks, the standard deviation of the portfolio is 21.29%

For 100 stocks, the same calculus gives

$$Var[\frac{\sum_{i=1}^{100} R_i}{100}] = \frac{100}{10000} Var[R_i] + 0.3 \frac{9900}{10000} Var[R_i] = 0.0376$$

so a standard deviation of 19.39%.

For 1000 stocks, the same calculus gives

$$Var[\frac{\sum_{i=1}^{1000} R_i}{1000}] = \frac{1000}{1000000} Var[R_i] + 0.3\frac{999000}{1000000} Var[R_i] = 0.0368$$

so a standard deviation of 19.19%.

25. (a) The maximum Sharpe ratio is (13%-5%)/8% = 0.5 Therefore for sd = 24% you can expect max return = $5\% + 0.5 \times 24\% = 17\%$.

(b) To achieve this you need to invest 150% in the market and -50% (i.e., short) the riskless asset.

26. A (expected return, standard deviation) pair is feasible if it is below the Capital Market Line (CML); it is efficient if it is on the CML. Equivalently, a pair is feasible if it has a lower Sharpe ratio than that of the market portfolio and efficient if it has the same Sharpe ratio as the market portfolio. The market Sharpe ratio is

(a) Sharpe ratio = 1.0375. It is not feasible.

(b) Sharpe ratio = 1.1667. It is not feasible.

(c) Sharpe ratio = 0.5000. It is feasible and efficient.

(d) Sharpe ratio = 0.7667. It is not feasible.

(e) Sharpe ratio = 0.0000. It is feasible but not efficient.

(f) Sharpe ratio = 1.1667. It is not feasible.

27. Since Ms Belle can borrow and invest at the 2% risk-free rate, she will hold a mix of the risk free portfolio and the optimal risky portfolio. This optimal risky portfolio can be found by maximizing the Sharpe ratio: $S = \frac{\mu - r_f}{\sigma}$.

Let x be the proportion of Mycronics Corp. in the optimal risky portfolio. Then

$$\sigma^2 = x^2 0.4^2 + (1-x)^2 0.18^2 + 2x(1-x)(0.4)(0.18)(0.3)$$

$$\max_x S^2 = \frac{\mu - r_f}{\sigma} = \frac{(0.25x + 0.095(1-x) - 0.02)^2}{x^2 0.4^2 + (1-x)^2 0.18^2 + 2x(1-x)(0.4)(0.18)(0.3)}$$

Taking the derivative gives:

$$0.0008747 - 0.00012168x - 0.00398784x^2 = 0$$

This equation is easy to solve. Alternatively, you can do the maximization numerically using the excel Solver Add-in. Both methods give $x = 45.3\%$, and Sharpe ratio of 0.63.

28. Samantha should buy some e.Coli shares. Her optimal equity portfolio is a mix of 22.5% e.Coli and 77.5% index fund. Again, Samantha should maximize the Sharpe Ratio of her portfolio. This is equivalent to maximizing the square of the SR. Let's call x the fraction of e.Coli (and $1 - x$ the fraction of the index fund).

$$E[R] - r_f = (0.12 - 0.055)(1 - x) + (0.25 - 0.055)x$$

$$(E[R] - r_f)^2 = 0.004225 + 0.0169x + 0.0169x^2$$

$$Var[R] = 0.16^2(1 - x)^2 + 0.5^2x^2 + 2 \times 0.4 \times 0.5 \times 0.16x(1 - x)$$

$$Var[R] = 0.0256 + 0.0128x + 0.2116x^2$$

You then want to maximize the squared Sharpe Ratio as a function of x.

$$SR^2 = \frac{(E[R] - r_f)^2}{Var[R]}$$

To find the maximum, you calculate the derivative, and find x such that the derivative is equal to 0. This gives $x = 22.54\%$.

29. (a) I would not recommend C because it is not as well diversified as B: it has the same return as B but higher variance. If one wants to invest everything in one fund, B is a better choice than C.

(b) I would recommend A because Keith Richards is old and wealthy, so there is no point putting all of his money in a risky fund like B or C for the higher expected return.

(c) B or C may be a better choice if they are correlated with his other investments in such a way that reduces overall portfolio risk.

30. Given the optimality of (70%, 20%, 10%), the tangency portfolio must be (L, S) $= \left(\frac{20\%}{20\%+10\%}, \frac{10\%}{20\%+10\%}\right) = \left(\frac{2}{3}, \frac{1}{3}\right)$. The weights for the young executive's portfolio should then be $\frac{2}{3}(100\% - 10\%) = 60\%$ on L and $\frac{1}{3}(100\% - 10\%) = 30\%$ on S.

31. The investor should choose portfolio A. If the investor is only going to invest in one of those portfolios (and some risk free Treasury bills), he should compare the portfolios using the Sharpe Ratio:

$$SR_i = \frac{E[R_i] - r_f}{SD[R_i]}$$

where r_f is the risk free rate, which is here the Treasury bill rate.

$$SR_A = 0.65$$

$$SR_B = 0.625$$

$$SR_C = 0.56$$

Portfolio A is the one with the highest Sharpe Ratio.

Note: If the investor wants more or less risk than portfolio A, he can adjust with the amount of Treasury bills.

32. (a) The market portfolio has an expected return of 12.50% and standard deviation of 22.75%. The Sharpe ratio is 0.3296.

(b) There are many ways of improving the portfolio's Sharpe ratio. The highest possible Sharpe ratio is 0.3511, and it is achieved with portfolio (0.2823, 0.2747, 0.2367, 0.2063). To find this, follow the usual numerical optimization in excel.

2.7 CAPM

1.

$$
\begin{aligned}
E\ (r_p) &= r_f + \beta\left[E\left(r_m\right) - r_f\right] \\
18 &= 6 + \beta\left(14 - 6\right) \\
\beta &= 12/8 = 1.5
\end{aligned}
$$

2. The appropriate discount rate for the project is:

$$r_f + \beta\left[E\left(r_m\right) - r_f\right] = 8 + 1.8(16 - 8) = 22.4\%.$$

Using this discount rate,

$$
\begin{aligned}
NPV &= -40 + 15\left[\frac{1}{.224} - \frac{1}{(.224)(1.224^{10})}\right] \\
&= 18.09.
\end{aligned}
$$

Using Excel, we find that the breakeven discount rate is .353. The highest value that beta can take therefore is determined by

$$
\begin{aligned}
35.73 &= 8 + \beta(16 - 8) \\
\beta &= 27.73/8 = 3.467.
\end{aligned}
$$

3. (a) False. $\beta = 0$ implies $E(r) = r_f$, not zero.

(b) False. Investors require a risk premium only for bearing systematic risk (undiversifiable or market) risk.

(c) False. 75% of your portfolio should be in the market, and 25% in bills. Then,

$$\beta_p = .75 \times 1 + .25 \times 0 = .75.$$

4. Since the stock's beta is equal to 1.2, its expected rate of return should be equal to $6 + 1.2(16 - 6)$ or 18%.

$$
\begin{aligned}
E(r) &= \frac{D + P_1 - P_0}{P_0} \\
.18 &= \frac{6 + P_1 - 50}{50} \\
P_1 &= \$53.
\end{aligned}
$$

5. Using the SML: $4 = 6 + \beta(16 - 6)$, we get that

$$\beta = -2/10 = -.2.$$

This asset has a negative market beta, i.e. its return is negatively correlated with that of the market. This asset can thus serve as an insurance against systematic risk. Instead, the return on the risk-free asset is simply uncorrelated with the market return.This is why the stock has a lower expected return.

6. **a.** Since the market portfolio by definition has a beta of 1, its expected rate of return is 12%.

b. $\beta = 0$ means no systematic risk. Hence, the portfolio's fair return is the risk-free rate, 5%.

c. Using the SML, the *fair* rate of return of a stock with $\beta = -0.5$ is:

$$E(r) = 5 + (-.5)(12 - 5) = 1.5\%.$$

The *expected* rate of return, using the expected price and dividend of next year:

$$E(r) = 44/40 - 1 = .10, \text{ or } 10\%.$$

Because the expected return exceeds the fair return, the stock must be underpriced.

7. (a) False. A risky asset whose return is negatively correlated with the market has a negative β and negative risk premium.

(b) False. The stocks' movements are amplified with a factor two with respect to the market's movements

(c) False. It is overvalued. A smaller risk premium is paid for a particular beta than would be expected from the CAPM.

8. Intercept: r_F and slope: $\overline{r_M} - r_F$

9. Slope: β

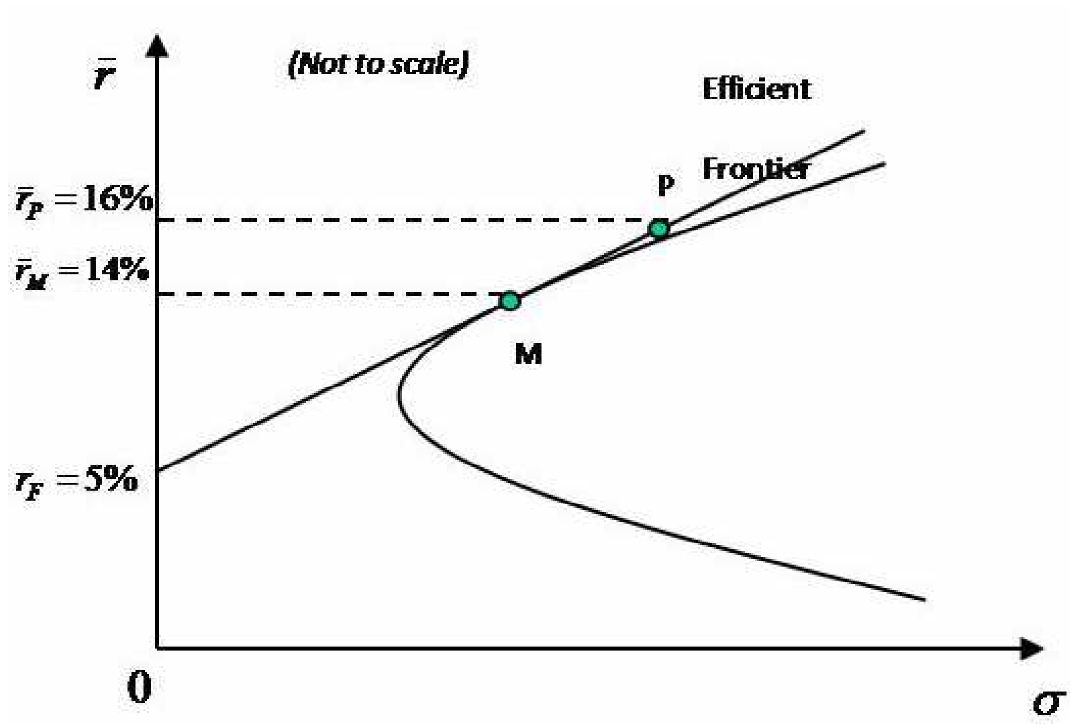
10. Here is the answer:

(a) CAPM: $\overline{r_p} - r_F = \beta_p(\overline{r_M} - r_F)$

Thus, $\beta_p = \frac{\overline{r_p} - r_F}{\overline{r_M} - r_F}$, or $\beta_p = \frac{0.16 - 0.05}{0.14 - 0.05} = 1.22$

(b) Portfolio P is on the efficient frontier. In the risk-return space, in presence of a risk-free asset and under the CAPM assumption, the efficient frontier is the half-line starting from the risk-free point and going through the market portfolio point.

Therefore, we can write: $\widetilde{r_p} = \alpha \cdot r_F + (1 - \alpha) \cdot \widetilde{r_M}$
Note that $\alpha < 0$, since $\overline{r_p} > \overline{r_M}$.



Taking the expected value and variance of the above equation, we obtain respectively: $\overline{r_p} = \alpha \cdot r_F + (1-\alpha)\cdot \overline{r_M}$
${\sigma_p}^2 = (1-\alpha)^2 {\sigma_M}^2$. From the above, we see that $(1-\alpha) = \beta_p$. Then, ${\sigma_p}^2 = {\beta_p}^2 {\sigma_M}^2$ or $\sigma_p = \beta_p \sigma_M$, since $\beta_p > 0$.

$\Rightarrow \sigma_p = 1.22 \times 25\% = 30.5\%$

(c) It is clear that $\rho_{r_p, r_M} = 1$. Another way to see that is to go back to the definition of beta:

$\beta_p = \frac{cov(\widetilde{r_p}, \widetilde{r_M})}{cov(\widetilde{r_M}, \widetilde{r_M})} = \frac{\sigma_p \sigma_M \rho_{r_p, r_M}}{{\sigma_M}^2} = \frac{\sigma_p \rho_{r_p, r_M}}{\sigma_M}$

11. (a) Total value of the portfolio P is $\$350,000$. The beta of the portfolio, β, is the linear combination of the asset betas: $\beta_p = \frac{50}{350} \times \beta_{r_F} + \frac{100}{350} \times \beta_{S\&P} + \frac{200}{350} \times \beta_{AD}$

$\Rightarrow \beta_p = \frac{50}{350} \times (0) + \frac{100}{350} \times (1.0) + \frac{200}{350} \times (1.5) = 1.14.$
CAPM: $\overline{r_p} = r_F + \beta_p(\overline{r_M} - r_F)$
$\Rightarrow \overline{r_p} = 6\% + 1.14 \times (12\% - 6\%) = 12.857\%$

(b) Assuming CAPM holds and the portfolio follows a random walk (no serial correlation, i.e., return outcomes from one year to another are independent), the forecasted portfolio value after five

years amounts to:
$P = \$350,000 \times (1+\overline{r_p})^5 = \$640,786$

(c) First, $\overline{r_M}$ and r_F can have varied over the past five years. Second and more importantly, there is a "statistical effect". Indeed, CAPM allows for determining expected returns. The observed returns can depart from the expected return on a particular time series. But at given $\overline{r_M}$ and r_F, this does not affect the expected return.

12. III can be described as a portfolio of 3 assets A, B, C (assuming the three subsidiaries form the entire III activity). The investment in A, B, C is respectively 40%, 40% and 20% of total portfolio value (III's market value).
Therefore, we can write:
$\beta_{III} = \frac{2}{5} \times \beta_A + \frac{2}{5} \times \beta_B + \frac{1}{5} \times \beta_C$

$\Rightarrow \beta_C = 5 \times 1.0 - 2 \times (0.8) - 2 \times (1.4) = 0.6$

$\overline{r_C} = r_F + \beta_C(\overline{r_M} - r_F) = 6\% + 0.6 \times 6\% = 9.6\%$

13. Stocks 1 and 2 have the same beta $\Rightarrow$ they have the same systematic risk. Hence, the fact that stock 2 is more volatile, i.e. $\sigma_1 < \sigma_2$ implies that the firm-specific risk of stock 2 is more important.
Remember from the course, an analytical way to see that is to consider the following equation:
$\overline{r} - r_F = \beta(\widetilde{r_M} - r_F) + \epsilon$

where $\overline{\epsilon} = 0$ and $cov\widetilde{\epsilon}, \widetilde{r_M} = 0$ (you can verify that the above equation is consistent with CAPM by taking the expected value of both terms of the equality). Hence:

$var(\widetilde{r}) = \sigma^2 = beta^2 var(\widetilde{r_M}) + var\widetilde{\epsilon}$

where $\beta^2 var(\widetilde{r_M})$ the systematic part of the stock volatility and $var(\widetilde{\epsilon})$ the firm specific part.

Therefore, if $\beta_1 = beta_2$ and $\sigma_2 > \sigma_1 \Rightarrow var(\widetilde{\epsilon_2}) > var(\widetilde{\epsilon_1})$.

Under CAPM, investors are well diversified and hold the market portfolio. The idiosyncratic risks of the stocks within the portfolio cancel out. Therefore, investors pay for the systematic risk (undiversifiable risk). The expected return on a stock (typically the opportunity cost of capital investors will use to discount cash flows) depends on the beta of the stock, following the CAPM equation. Since stock 1 and 2 have the same beta, they should have the same expected return.

14. Using the results of the previous problem, one can write:
$var(\widetilde{r}) = \beta^2 var(\widetilde{r_M}) + var(\widetilde{r_M})$

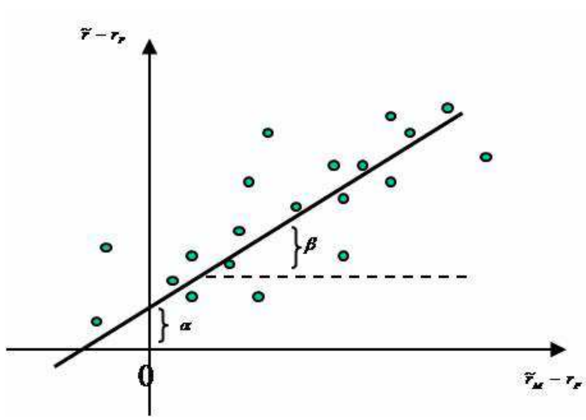
$\Rightarrow \frac{\beta^2 var(\widetilde{r_M})}{var(\widetilde{r}}$ represents the fraction of the total variance of a given stock which relates to market risk. Applying this equation to stocks A and B, respectively, we have:

$\frac{{\beta_A}^2 var(\widetilde{r_M})}{var(\widetilde{r_A})} = \frac{0.6^2 \times 0.25^2}{0.4^2} = 14\%$

$\frac{{\beta_B}^2 var(\widetilde{r_M})}{var(\widetilde{r_B})} = \frac{1.2^2 \times 0.25^2}{0.4^2} = 56.25\%$

Therefore, at a given volatility, the fraction of total risk that is attributable to market risk increases with the absolute value of the beta (high absolute value of beta means strong correlation to the market portfolio).

15. The statistics refer to the linear regression of the risk premium of the Ampersand Electric common stock with the market premium.



(a) In the given statistics, estimates the ordinate at origin of the linear regression. In CAPM, $\alpha = 0$ and the expected risk premium of stocks are aligned along the SML. $\widehat{\alpha} \neq 0$ suggests a departure from the CAPM. In the Ampersand Electric case, the alpha is positive and may indicate a higher-than-normal expected return. Typically, one could think that particular investors have different information on Ampersand Electric stock than the overall investors. However, you should note that the standard error on is much higher than the estimate itself. Therefore, you should consider that is not a relevant parameter in the linear regression and take it equal to 0. As a result, you would conclude that there is no higher-than-normal expected return.

(b) R^2 represents the amount of the total risk of the stock that is explained by the CAPM equation. A higher R^2 would not increase your confidence in the estimated β. The quality of the β estimate is given by the standard error. In this case, $\widehat{\beta} = 1.2$ with a standard error of 0.27. An even lower standard error relative to $\widehat{\beta}$ would increase your confidence in the estimated beta.

16. (a) $r_Q = 2 + 0.45 \times (9.5 - 2) = 5.375\%$
$r_R = 2 + 1.45 \times (9.5 - 2) = 12.875\%$
$r_S = 2 - 0.20 \times (9.5 - 2) = 0.5\%$

(b) $P_0 = (P_1 + D_1)/(1 + r)$
Therefore, $P_Q = (45 + 0.5)/1.05375 = \$43.18$
$P_R = (75)/1.12875 = \$66.45$
$P_S = (20 + 1)/1.005 = \$20.9$

17. Risk-free rate = 2% (from the previous question) Factor risk premiums are as follows (BM, p. 209):
Market risk premium = 5.2%
Size risk premium = 3.2%
Book-to-market risk premium = 5.4%

Using APT:

$r_Q = 2 + 0.45 \times 5.2 + 0.05 \times 3.2 + 0.14 \times 5.4 = 5.26\%$
$r_R = 2 + 1.45 \times 5.2 - 0.33 \times 3.2 - 0.22 \times 5.4 = 7.3\%$
$r_S = 2 - 0.2 \times 5.2 + 1.21 \times 3.2 + 0.64 \times 5.4 = 8.29\%$

18. (a) Using the regression formula:
$\alpha_i = (\overline{r_i} - r_F) - \beta_{estimated}(\overline{r_M} - r_F)$

Therefore:
$\alpha_T = 0.6 - 0.727 \times 1 = -0.127\%$
$\alpha_U = 1.1 - 0.75 \times 1 = 0.35\%$

Assuming $S\&P500$ as a proxy to the market.

(b) Using CAPM:
$r_T = 0.3 + 0.727 \times 0.6 = 0.7362\%$ per month
$r_T = 0.3 + 0.75 \times 0.6 = 0.752\%$ per month

(c) Under CAPM, the market portfolio is the optimal tangency portfolio. Since both T and U are included in the $S\&P500$ and assuming $S\&P500$ is a proxy to the market, invest all your money in the $S\&P500$.

19. (a) Inconsistent. Higher beta requires higher expected return.

(b) Inconsistent. Portfolio A lies above the CML. This would suggest that the market portfolio is inefficient.

(c) Inconsistent. Portfolio A lies above the CML. This would suggest that the market portfolio is inefficient.

(d) Inconsistent. Portfolio A does not lie on the SML.

(e) InConsistent. The implied risk-free rate would be negative if A lies on the SML.

20. (a) False. The returns given are for the market porfolio, with $\beta = 1$. The returns for the stocks given will have a $\beta > 1$.

(b) False. All portfolios on the SML have $\alpha = 0$

21. (a) $\beta_A = \frac{0.0525}{0.0625} = 0.84$

$\beta_B = \frac{0.0437}{0.0625} = 0.70$

(b) Stock A has the highest expected return, because it has the highest beta value.

22. (a) False. It depends on the correlation between the returns of stock A and other stocks in the portfolio and the market portfolio. Stock A might be an interesting pick in terms of reducing risk.

(b) False. Diversification means that one chooses an optimal combination of weights of the individual assets to eliminate idiosyncratic risk.

(c) True. The market premium is always positive.

(d) False. A security with a beta of zero offers the risk-free rate.

(e) False. CAPM gives a measure for the expected return of an asset, given a certain market premium and β.

(f) True

(g) False. $\beta = \frac{1}{3} \times 0 + \frac{2}{3} \times 1 = 0.66$

23. (a) $r_A = r_F + \beta \times (r_M - r_F)$ Thus,
$r_A = 0.03 + 0.8 \times 0.04 = 0.062 = 6.2\%$

(b) $E[S_A] = 50 \times (1 + 0.062) = \$53.10$

(c) $E[S_A] = 50 \times (1 + 0.062) - 1 = \$52.10$

24. (a) True. Beta is a measure for systematic risk.

(b) False. The two specific portfolios will have a higher beta than the historic market portfolio.

(c) False. All portfolios on the SML have $\alpha = 0$.

(d) False. All portfolios on the CML offer the highest possible return for a given risk

(e) False. An idiosyncratic risk-term can be added to the CAPM.

25. (a) False. $\overline{r_i} = r_F + \beta_{iM}(\overline{r_M} - r_F) = r_F$, for $\beta = 0$.

(b) False. Suppose a portfolio consists of assets 1 and 2 with weights w_1 and w_2, then we have:
$\beta_{PM} = w_1\beta 1M + w_2\beta_{2M}$
The portfolio has a beta of 0.25.

(c) True. Similarly to CAPM factors, APT factors reflect only non-diversifiable risks.

26. (a) According to CAPM,

$$\overline{r_i} - r_F = \beta_{iM}(\overline{r_M} - r_F)$$
$$12 - 5 = 1(\overline{r_M} - 5))$$
$$\overline{r_M} = 12\%$$

(b) Expected rate of returns on stocks with $\beta = 0.5$ and 1.5

$$\overline{r_i} = r_F + \beta_{iM}(\overline{r_M} - r_F)$$
$$= 5 + 0.5 \times (12 - 5))$$
$$= 8.5\%$$
$$\overline{r_i} = r_F + \beta_{iM}(\overline{r_M} - r_F)$$
$$= 5 + 1.5 \times (12 - 5))$$
$$= 15.5\%$$

(c) gross return $= \frac{P_1+D_1}{P_0} = 1.07$ Because 7% is lower than 8.5%, the stock is overpriced.

27. (a) $\beta_{AM} = \frac{cov(r_A,r_M)}{{\sigma_M}^2} = \frac{0.0525}{0.0625} = 0.84$

$\beta_{BM} = \frac{cov(r_B,r_M)}{{\sigma_M}^2} = \frac{0.0427}{0.0625} = 0.68$

(b) $cov(r_P, r_M) = cov(w_1r_1 + w_2r_2, r_M)$

$cov(r_P, r_M) = w_1cov(r_1, r_M) + w_2cov(r_2, r_M)$

$\beta_PM = \frac{w_1cov(r_1,r_M)+w_2cov(r_2,r_M)}{{\sigma_M}^2}$

$\beta_PM = \frac{w_1cov(r_1,r_M)}{{\sigma_M}^2} + \frac{w_2cov(r_2,r_M)}{{\sigma_M}^2}$

$w_1\beta_{1M} + w_2\beta_{2M}$

$\beta = 0.6 \times 0.84 = 0.504$

(c) $\beta = 0.6 \times 0.84 + 0.4 \times 0.6832 = 0.777$

28. (a) First compute the stocks' market betas:

$$\begin{aligned} \beta_1 &= \frac{\sigma_{1M}}{\sigma_M^2} = \frac{\rho_{1M}\sigma_1\sigma_M}{\sigma_M^2} && (19) \\ &= \frac{(0.4)(0.2)(0.15)}{0.15^2} && (20) \\ &= 0.5333 && (21) \end{aligned}$$

$$\begin{aligned} \beta_2 &= \frac{(0.7)(0.3)(0.15)}{0.15^2} && (22) \\ &= 1.4. && (23) \end{aligned}$$

Then use CAPM to compute required rate of returns:

$$\begin{aligned} r_1 &= r_f + \beta_1(r_M - r_f) && (24) \\ &= 5\% + (0.5333)(10\% - 5\%) && (25) \\ &= 7.7\% && (26) \end{aligned}$$

$$\begin{aligned} r_2 &= 5\% + (1.4)(5\%) && (27) \\ &= 12\%. && (28) \end{aligned}$$

(b) For a portfolio of 40% in stock 1 and 60% in stock 2:

$$\begin{aligned} r_p &= x_1 r_1 + x_2 r_2 && (29) \\ &= (0.4)(7.7\%) + (0.6)(12\%) && (30) \\ &= 10.27\% && (31) \end{aligned}$$

$$\begin{aligned} \sigma_p^2 &= x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2 && (32) \\ &= (0.4^2)(0.2^2) + (0.6^2)(0.3^2) && (33) \\ &\quad +(2)(0.4)(0.6)[(0.5)(0.2)(0.3)] && (34) \\ &= 0.0532 && (35) \end{aligned}$$

$$\sigma_p = (0.0532)^{1/2} = 0.2307. \qquad (36)$$

(c) Let x_M be the weight on the market portfolio. We require:

$$r_p = 10.27\% = (1 - x_M)(5\%) + (x_M)(10\%)$$

or

$$10.27\% = 5\% + (x_M)(5\%).$$

Hence

$$x_M = \frac{10.27\% - 5\%}{5\%} = 1.054.$$

We should invest 105.4% in the market and −5.4% at the risk-free rate. The standard deviation to the return on this portfolio would be $1.054 \times 0.15 = 0.1581$.

(d) $r_p = 10.27\%$, as in part (b). However now:

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2 \quad (37)$$
$$= (0.4^2)(0.2^2) + (0.6^2)(0.3^2) + (2)(0.4)(0.6)(-0.7)(0.2)(0.3) \quad (38)$$
$$= 0.01864 \quad (39)$$
$$\sigma_p = 0.137 \quad (40)$$

Risk-return trade-off on the market is:

$$\frac{r_M - r_F}{\sigma_M} = \frac{0.10 - 0.05}{0.15} = 0.333.$$

Risk-return trade-off on the portfolio is:

$$\frac{r_p - r_F}{\sigma_p} = \frac{0.1027 - 0.05}{0.137} = 0.385.$$

There is a better trade-off with this portfolio, thus the market is not mean-variance efficient.

29. The beta of the firm is given by

$\beta_{rMfirm} = \frac{1}{3}\beta_x + \frac{1}{3}\beta_y + \frac{1}{3}\beta_z$

Hence:

$1.2 = \frac{1}{3}1.2 + \frac{1}{3}0.6 + \frac{1}{3}\beta_z$

or

$\beta_z = \frac{1.2 - (\frac{1}{3})(1.2) - (\frac{1}{3})(0.6)}{\frac{1}{3}}$

$\beta_z = 1.8$

The appropriate discount rate is:

$r_{rMproject} = r_f + \beta_z(r_M - r_f)$

$r_{rMproject} = 6\% + 1.8 \times (16\% - 6\%)$

$r_{rMproject} = 24\%$

30. (a) True - with the qualification that the information in question is about stock expected returns and covariances. Of course investors may have different non-stock-related information.

(b) False - low beta stocks have on average outperformed their CAPM expected returns over the long run (see figure on p. 100 Brealey and Myers)

31. (a) The realized rate of return over the period was different for each manager. The ranking would be Z, X, Y.

(b) Alpha in this regression gives a measure of risk-adjusted return (assuming that the source of risk is the market). This is the same as the excess return to what CAPM predicts, and it is often used as a performance measure. By this measure, Z and X perform well and roughly the same, but Y performs poorly. However, we must note that the standard errors for alphas estimated are not statistically significant. This fact does not mean that we cannot compare alphas at all; it simply means we should be very cautious trying to draw strong forward-looking inference from the alphas. They do provide some information about manager performance over the period given.

(c) The differences in beta for the portfolios indicates that the managers had different investment styles/strategies. In particular, we can see that fund Z invested in particularly high-beta stocks relative to X and Y. Since all 3 funds invest only in common stocks, we can make some inference that Z probably overweights in sectors such as hi-tech (or other high-beta sectors) whereas X and Y probably hold sectors in roughly the same proportion as the market.

(d) R-squared is a measure of the proportion of deviation (risk) attributable to the market relative to the total portfolio deviation. The most relevant inference is that portfolio Z was the least diversified fund, i.e. it held the most idiosyncratic risk. Y held less, and X held very little idiosyncratic risk.

(e) Sharpe Ratio Analysis - would be a replacement for sections b) and d).

Several steps are required to do the Sharpe ratio analysis correctly.

$R^2 = \beta_i, M^2 \frac{\sigma^2{}_M}{\sigma^2{}_i}$

From this equation we can solve for ${\sigma^2}_i$ in terms of ${\sigma^2}_M$:

$${\sigma^2}_X = 1.05^2 \frac{{\sigma^2}_M}{0.92} = 1.198{\sigma^2}_M$$

$${\sigma^2}_Y = 1.1^2 \frac{{\sigma^2}_M}{0.88} = 1.375{\sigma^2}_M$$

$${\sigma^2}_Z = 1.6^2 \frac{{\sigma^2}_M}{0.65} = 3.938{\sigma^2}_M$$

Taking the square-root:

$$\sigma_X = 1.095\sigma_M$$

$$\sigma_X = 1.173\sigma_M$$

$$\sigma_X = 1.985\sigma_M$$

Finally, the Sharpe Ratio is given by:

$$SR_i = \frac{r_i - r_F}{\sigma_i}$$

$$SR_X = \frac{0.098 - 0.025}{1.095\sigma_M} = \frac{0.0667}{\sigma_M}$$

$$SR_Y = \frac{0.09 - 0.025}{1.173\sigma_M} = \frac{0.0554}{\sigma_M}$$

$$SR_Z = \frac{0.134 - 0.025}{1.985\sigma_M} = \frac{0.0549}{\sigma_M}$$

$$SR_{S\&P500} = \frac{0.09 - 0.025}{\sigma_M} = \frac{0.065}{\sigma_M}$$

Thus, we can compare Sharpe ratios even though we are not given . On a risk-adjusted basis, we see that X performs best, followed by Y, and finally Z. In fact, only X outperforms the S&P500 on this basis. This time, the "risk-adjustment" is relative to total portfolio risk (rather than relative to market risk exposure as with interpreting alpha). Since the Sharpe ratio already accounts for diversification benefits, we do not have to worry about diversification that R-squared implies.

32. (a) Expected Return assuming CAPM

$$E(R_A) = 1.5\% + 1.2 \times (8\%) = 11.1\%$$

(b) Achieve 8% return optimally

If CAPM holds, we will hold the market and the risk-free security. In this case, our proxy for the market is the S&P500 Index Fund and the Money Market Fund is our risk-free security.

If w = weight on SP500 and (1-w) = weight on MM, then solving the following

$8\% = w \times (9.5\%) + (1 - w) \times (1.5\%)$

yields

$w = 81.25\%$ in SP500 and $(1 - w) = 18.75\%$ in MM

(c) What if alpha = .005 per month with s.e. = .0015

First, notice that 0.5% per month is an economically huge alpha (annualized 6%) and the t-stat = 3.33 is very large, indicating statistical significance.

If you believe in CAPM, then disregard this estimate of past over-performance since alpha is always zero in expectation for the future. Don't change your optimal portfolio.

If you think CAPM may be flawed, then the large and statistically significant alpha indicates that the manager has consistent high risk-adjusted returns. Your optimal risky portfolio should now include some New Economy Fund and some SP500 Fund. The weight in the MM Fund will decrease since we need less risky investments to achieve 8% expected return.

33. (a) Ms. Tortoise. Because the alpha of Mr. Hare = 34 - 3+4(11-3) = -1, but alpha of Ms. Tortoise = 12 - 3+1(11-3) =+1

(b) Mr. Hare. Because the alpha of him = 34 - 7+4(13-7) =+3, but alpha of Ms. Tortoise = 12 - 7+1(13-7) =-1

34. False. Beta measures systematic risk only, which is just one component of total risk.

35. True. If the CAPM holds, it must be true for all assets that $E[]r_i - r_F] = \beta_i E[r_M - r_F]$. It can be the case, however, that measured alpha is not zero and CAPM still holds, due to measurement error.

36. Long run returns are significantly related to beta (if we take evidence from 1931 to 2005), i.e., higher betas generated higher returns. However, high beta portfolios fall below SML, low beta portfolios land above

SML, and a line fitted to 10 portfolios ranked by their betas is flatter than SML. CAPM does not seem to work well over the last 30 years (the relation between beta and actual average return has been much weaker since the mid-1960's). Factors other than beta seem important in pricing assets, namely size and market-to-book ratios. In fact, small stocks have outperformed stocks with high ratios

37. (a) False. Sharpe ratio = (Expected Return - Risk free rate)/ Standard Deviation of return

(b) Maybe. The value(market cap)-weighted average of all assets is the market portfolio, which have a beta of 1. Therefore the value weighted average of all beta must equal to 1

On the other hand, the equal weighted average beta does not necessarily have to be 1. For example, small stocks may have higher beta then the EW average will be higher than 1.

38. (a) The maximum Sharpe ratio is (13%-5%)/8% = 0.5 Therefore for sd = 24% you can expect max return = 5% + 0.5*24% = 17%

(b) To achieve this you need to invest 150% in the market and -50% (ie short) the riskless asset.

2.8 Capital Budgeting

1. (a) Projects A,B, D, E all have payback period of more than 3 years. So you will only take C & F for a total NPV of 46.5

(b) No. Two possible answers:

- It is not he manager's role to avoid taking risk. They should seek to create maximum value from their projects, that is maximize NPV. Investor will adjust their portfolio to reflect their risk tolerance.
- All projects here have a cost-of-capital of 12%. If this is calculated correctly, all projects are equally risky in terms of systematic risk. The rest is diversifiable risk and investor can easily get rid of those by holding a well diversified portfolio.

(c) B, for higher NPV.

(d) No. I will still prefer B over A. Of course, I prefer the combination of A and AA over B, but that's not the question here.

(e) You basically have 2 choices: Take B, which use up all your capital, or invest in A+C+F. The latter give you higher NPV at 57+41+5.5 = 103.5.

(f) My answers to c and d stay the same. The borrowing cost does not reflect the true cost of capital. However, as long as the debt is issued at fair value, I can utilize it and expand my investment choice to 300m. Then I can take both projects A & B for a NPV of 121.

2.
$$\begin{aligned} PV &= 7000 \times 15 \times \left(\frac{1}{1.08} + \frac{0.94}{1.08^2} + \cdots + \frac{0.94^{11}}{1.08^{12}} \right) \\ &= 7000 \times 15 \times \left(\frac{1}{1.08 - 0.94} \right) \left(1 - \frac{0.94^{12}}{1.08^{12}} \right) \\ &= \$608,254.12 \end{aligned}$$

3. We should take the projects so that the total NPV from these projects is maximized. To achieve this, we should give priority to projects with high NPV/investment ratios:

Project	Investment in 2000	NPV	NPV/Investment
Q	10.5	5.5	0.52
R	2.0	0.5	0.25
S	6.0	2.5	0.42
T	7.5	2.0	0.27
U	1.5	1.0	0.67
V	3.0	1.0	0.33

U, Q and S have the highest NPV/investment ratios, and they happen to require \$18 million investment in total. These are the projects that should be undertaken, and the joint NPV is 1.0+5.5+2.5 = \$9 million.

4. The appropriate discount rate is $5\% + 1.2(12\% - 5\%) = 13.4\%$.

NPV $= -8.5 + \frac{1}{0.134-0.025} = \$0.67$ million.

DEF should undertake the expansion.

5. NPV $= -$Investment $+ (1-t)\times$ PV (Revenues) $+t\times$ PV (Depreciation) $-(1-t)\times$ PV (Costs)

Investment $= 50$

PV (Revenues) = PV annuity of @ 10% $= 25\times\frac{1}{0.1}\left(1-\frac{1}{1.1^5}\right) = \$\ 94.77$m

$(1-t)\times$ PV (Revenues) $= \$61.6$m

PV (Depreciation) = PV annuity of @ 10% $= 10\times\frac{1}{0.1}\left(1-\frac{1}{1.1^5}\right) = \$$ 37.9m

T $\times$ PV (Depreciation) $= \$13.27$m

PV (Costs) $= 300\times 20,000\times 5 = \$\ 30$m

$(1-t)\times$ PV (Costs) $= \$\ 19.5$m

NPV $= -50 + 61.6 + 13.27 - 19.5 = \$\ 5.37$ m

The firm should invest in the project.

6. (a) CF_0 = -600
for year 1-3, CF = 400 - 100 - (400-200-100)*30%(tax) = 270

(b) $NPV = -600 + \frac{270}{1+10\%} + \frac{270}{(1+10\%)^2} + \frac{270}{(1+10\%)^3} = 71.45$
Since NPV is larger than 0, we should take the project.

(c) Under the new scenario, notice that tax will stay the same because accounting expenditure stays at 300 each year.
Therefore, CF_0 = -900 and $CF_1 = CF_2 = CF_3 = 370$

(d) $NPV = -900 + \frac{370}{1+10\%} + \frac{370}{(1+10\%)^2} + \frac{370}{(1+10\%)^3} = 20.14$. NPV is still positive and you should take the project.

7. (a) Cash flows:

Year	0	1	2
Revenue		80	80
Cost		(25)	(25)
Depreciation		(50)	(50)
PBT		5	5
Tax		(1.5)	(1.5)
PAT		3.5	3.5
Cash Flow	(100)	53.5	53.5

(b) Halliburton should decline the project as it is negative NPV

$$NPV = -100 + \frac{53.5}{1.04} + \frac{53.5}{1.06^2} = -\$942,882.77.$$

