Economics Letters 18 (1985) 351–353
North-Holland

A LARGE-SAMPLE CHOW TEST FOR THE LINEAR SIMULTANEOUS EQUATION

Andrew W. LO
University of Pennsylvania, Philadelphia, PA 19104, USA

Whitney K. NEWEY
Princeton University, Princeton, NJ 08540, USA

Received 15 January 1985

A simple large-sample Chow test for stability of coefficients in a linear simultaneous equation is proposed. It is shown that the appropriate test statistic may be formed conveniently from particular sums of squared residuals.

A useful model validity check for a single simultaneous equation may be obtained by extending Chow's (1960) coefficient stability test to the simultaneous equations case. A Wald version of such a stability test may be formulated as well.

Consider the following single simultaneous equation. Divide the sample of T observations into two subsamples, call them 1 and 2, each with T_1 and T_2 observations, respectively, where $T = T_1 + T_2$. Let y_i be a $T_i \times 1$ vector of observations of the left-hand side variable, X_i a $T_i \times q$ matrix of observations of right-hand side variables, and Z_i a $T_i \times K$ matrix of observations of instrumental variables, where rank $(Z_i) = K$ and $i = 1, 2$. Let the unrestricted version of the two subsample equations be given by the two equations

$$y_i = X_i \delta_i + \epsilon_i, \qquad i = 1, 2. \tag{1}$$

We make the following assumptions:

$$\begin{aligned}
&\operatorname*{plim}_{T_i \to \infty} Z_i' Z_i / T = Q_i, && && Q_i \text{ non-singular}, \quad i = 1, 2, \\
&\operatorname{rank}\Big(\operatorname*{plim}_{T_i \to \infty} Z_i' X_i / T\Big) = q, && Z_i'\epsilon_i/\sqrt{T_i} \overset{A}{\sim} \mathrm{N}\big(0, \sigma_i^2 Q_i^{-1}\big), && i = 1, 2, \\
&\operatorname*{plim}_{T \to \infty} \epsilon_1' \epsilon_2 / T = 0.
\end{aligned} \tag{2}$$

For $i = 1, 2$ define the projection matrices $P_i = Z_i(Z_i'Z_i)^{-1}Z_i'$ and the predicted values $\hat{X}_i = P_i X_i$, $\hat{y}_i = P_i y_i$. The 2SLS estimator of δ_i for each subsample i is $\hat{\delta}_i = (\hat{X}_i'\hat{X}_i)^{-1}\hat{X}_i' y_i = (\hat{X}_i'\hat{X}_i)^{-1}\hat{X}_i'\hat{y}_i$. Let $\hat{\sigma}_i^2 = (y_i - X_i\hat{\delta}_i)'(y_i - X_i\hat{\delta}_i)/(T_i - q)$ be estimators of the σ_i^2's. Then a Wald test-statistic for structural stability is given by

$$W_c = (\hat{\delta}_1 - \hat{\delta}_2)'\left[\hat{\sigma}_1^2(\hat{X}_1'\hat{X}_1)^{-1} + \hat{\sigma}_2^2(\hat{X}_2'\hat{X}_2)^{-1}\right]^{-1}(\hat{\delta}_1 - \hat{\delta}_2), \tag{3}$$

which is asymptotically distributed as chi-square with q degrees of freedom as *both* T_1 and T_2 increase.

We may use the results presented by Startz (1983) [see also Lo and Newey (1983)] to obtain a sums-of-squared-residuals version of W_c. Normalization by the estimated standard deviations and stacking yield

$$y^* = \begin{vmatrix} y_1/\hat{\sigma}_1 \\ y_2/\hat{\sigma}_2 \end{vmatrix} = \begin{vmatrix} X_1/\hat{\sigma}_1 & 0 \\ 0 & X_2/\hat{\sigma}_2 \end{vmatrix} \begin{vmatrix} \delta_1 \\ \delta_2 \end{vmatrix} + \begin{vmatrix} \epsilon_1/\hat{\sigma}_1 \\ \epsilon_2/\hat{\sigma}_2 \end{vmatrix} = X^*\delta^* + \epsilon^*. \tag{4}$$

Define $\hat{\delta}^*$ and Z^* as

$$\hat{\delta}^* = \begin{vmatrix} \hat{\delta}_1 \\ \hat{\delta}_2 \end{vmatrix}, \qquad Z^* = \begin{vmatrix} Z_1/\hat{\sigma}_1 & 0 \\ 0 & Z_2/\hat{\sigma}_2 \end{vmatrix}. \tag{5}$$

Let P^* denote the projection matrix $P^* = Z^*(Z^{*\prime}Z^*)^{-1}Z^{*\prime} = \operatorname{diag}(P_1, P_2)$. Note that W_c may then be interpreted as a Wald statistic of the form given in eq. (12) of Startz (1983) with $\hat{\sigma}^2 = 1$, $\hat{\delta} = \hat{\delta}^*$, $(\hat{X}'\hat{X})^{-1} = \operatorname{diag}(\hat{\sigma}_1^2(\hat{X}_1'\hat{X}_1)^{-1}, \hat{\sigma}_2^2(\hat{X}_2'\hat{X}_2)^{-1})$, $R = [I_q - I]$, and $r = 0$. Let

$$X_0 = \begin{vmatrix} X_1/\hat{\sigma}_1 \\ X_2/\hat{\sigma}_2 \end{vmatrix}, \qquad \hat{X}_0 = P^*X_0 = \begin{vmatrix} \hat{X}_1/\hat{\sigma}_1 \\ \hat{X}_2/\hat{\sigma}_2 \end{vmatrix}. \tag{6}$$

Then the restricted 2SLS estimator $\tilde{\delta}^*$ of δ^* is $[\tilde{\delta}_1'\tilde{\delta}_2']'$ where $\tilde{\delta}_1 = \tilde{\delta}_2 = (\hat{X}_0'\hat{X}_0)^{-1}\hat{X}_0'y^*$. Eq. (12) of Startz (1983) then implies that a form of W_c based on sums of squared residuals is given by

$$\begin{aligned} W_c &= (y^* - X^*\tilde{\delta}^*)'P^*(y^* - X^*\tilde{\delta}^*) - (y^* - X^*\hat{\delta}^*)'P^*(y^* - X^*\hat{\delta}^*) \\ &= ((y_1 - X_1\tilde{\delta}_1)'P_1(y_1 - X_1\tilde{\delta}_1) - (y_1 - X_1\hat{\delta}_1)'P_1(y_1 - X_1\hat{\delta}_1))/\hat{\sigma}_1^2 \\ &\quad + ((y_2 - X_2\tilde{\delta}_2)'P_2(y_2 - X_2\tilde{\delta}_2) - (y_2 - X_2\hat{\delta}_2)'P_2(y_2 - X_2\hat{\delta}_2))/\hat{\sigma}_2^2. \end{aligned} \tag{7}$$

The appropriate test statistic has the same form as the standard linear regression case except the residuals here are first projected on to the instruments.

Note that the restricted estimator $(\hat{X}_0'\hat{X}_0)^{-1}\hat{X}'y^*$ is not equal to 2SLS on the pooled sample. One reason for this is that we have allowed the variance of the disturbance to differ across subsamples. However, even if the estimated variances were equal the restricted estimator would not be 2SLS on the pooled sample.

In order to obtain the sum of squared residuals form of the test it is necessary to use the same instruments for the restricted and unrestricted estimators. Therefore, even when the coefficients are constrained to be the same in the two sub-samples, it is appropriate to form the second stage regression with right-hand side variables which are the predicted values obtained using only the subsample to which the observation belongs.

We have not discussed the deficient observations case where $T_i < K$ or $T_i < q$ for some i. The asymptotic theory which we have appealed to above would most likely provide a poor approximation in this case. This case has been considered by Erlat (1983).

References

Chow, Gregory C., 1960, Tests of equality between subsets of coefficients in two linear regressions, Econometrica 28, 591–605.

Erlat, H., 1983, A note on testing for structural change in a single equation belonging to a simultaneous equations system, Economics Letters 13, nos. 2–3, 185–189.

Lo, A.W. and W.K. Newey, 1983, A residuals-based Wald test for the single linear simultaneous equation, Mimeo. (Harvard University, Cambridge, MA).

Startz, R., 1983, Computation of linear hypothesis tests for two-stage least squares, Economics Letters 11, nos. 1–2, 129–131.