Stop-loss strategies with serial correlation, regime switching, and transaction costs

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ABSTRACT

Stop-loss strategies are commonly used by investors to reduce their holdings in risky assets if prices or total wealth breach certain pre-specified thresholds. We derive closed-form expressions for the impact of stop-loss strategies on asset returns that are serially correlated, regime switching, and subject to transaction costs. When applied to a large sample of individual U.S. stocks, we show that tight stop-loss strategies tend to underperform the buy-and-hold policy in a mean-variance framework due to excessive trading costs. Outperformance is possible for stocks with sufficiently high serial correlation in returns. Certain strategies succeed at reducing downside risk, but not substantially.

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1. Introduction

Many investors attempt to limit the downside risk of their investments by using stop-loss strategies, the most common of which is the stop-loss order, a standing order to liquidate a position when a security's price crosses a pre-specified threshold. By closing out the position, the investor is hoping to avoid further losses.

If prices follow random walks, any price movement in the past has no bearing on future returns—as long as the risky asset has a positive risk premium, the investor's portfolio will have a higher expected return by staying invested in the asset rather than liquidating it after its price reaches a particular limit. In this case, Kaminski and Lo (2014) have shown that the stop-loss strategy tends to underperform a buy-and-hold strategy. However, there is extensive evidence that financial asset prices do not follow random walks (e.g., Lo and MacKinlay, 1988; Poterba and Summers, 1988; Jegadeesh and Titman, 1993).
A natural question is whether these departures from randomness can be exploited using a dynamic investment strategy, including stop-loss policies.

In this article, we focus on simple dynamic strategies incorporating stop-loss rules to determine how they compare to static buy-and-hold strategies. We provide closed-form expressions for the returns of a large class of these strategies and derive conditions under which they underperform or outperform buy-and-hold. Assuming that prices follow a first-order autoregressive process, we prove that the log-returns of “tight” stop-loss strategies—strategies with price triggers that are close to the asset’s current price—are approximately linear in the interaction term between autocorrelation and volatility, providing an explicit relation between the profitability of a stop-loss policy, return predictability, and volatility. This expression yields bounds on how large return autocorrelation and volatility must be to beat a buy-and-hold strategy after accounting for trading costs.

We extend our theoretical analysis by simulating various return processes and by comparing the performance of stop-loss and buy-and-hold policies in a mean-variance framework. We consider two general processes—an AR(1) and a regime-switching process—and vary the underlying parameters for each. In the first case, with a high enough serial correlation and volatility, the stop-loss strategy provides superior risk-adjusted returns in comparison to the buy-and-hold strategy. In the regime-switching case, the stop-loss strategy gives better performance in a few cases, and this outperformance comes from a large reduction in volatility rather than an improvement in raw returns. We also look at the tail performance of the strategy, as measured by skewness and maximum drawdown. We find that if a longer horizon for past returns is used to make the decision whether to stop out or not, downside risk tends to improve over the buy-and-hold.

To illustrate the practical relevance of stop-loss strategies, we perform a detailed empirical analysis of the performance of these strategies using a large sample of U.S. stock returns from 1964 to 2014. To derive realistic measures of performance, we incorporate transaction costs in our backtests by using bid-ask spreads, as well as historical estimates when such spreads are missing. Our empirical findings are most relevant to short-term traders, who usually employ tight stop-loss policies and frequently change their positions. We find that the performance of tight stop-loss strategies is closely related to the realized return autocorrelation over the investment period, which supports the common trading adage: “The trend is your friend.” However, such strategies require a lot of trading, leading to high transaction costs. As a result, tight stop-loss strategies are able to outperform the buy-and-hold strategies only when asset returns are significantly serially correlated.

Of course, a stop-loss rule alone does not fully define an entire investment strategy since, after exiting a risky investment, the investor must decide when to re-enter. We consider several simple re-entry policies as part of our definition of a stop-loss rule and demonstrate that it is usually beneficial to re-invest soon after being stopped out in the case of tight stops. Another aspect that must be considered is where cash is invested after a stop-out. Assuming that cash is immediately invested in a risk-free asset, we show that the risk-free rate has a significant impact on the effectiveness of a stop-loss strategy, and this impact reconciles some of the inconsistencies among existing empirical studies of stop-loss strategies.

From a broader perspective, the use of stop-loss strategies can correct for the tendency of investors to hold on to losers too long and to sell winners too early, a behavioral bias known as the “disposition effect” first documented by Shefrin and Statman (1985). The presence of behavioral biases such as this has been well documented in the finance literature. While most of this research has focused on the empirical evidence for these biases and the theoretical models to explain them, few studies have proposed methods for investors to actively avoid or protect against such biases. Stop-loss strategies are an important first step in this direction.

2. Literature review

Kaminski and Lo (2014) lay out the first general framework for analyzing stop-loss strategies. They start with analytical results for the performance of a stop-loss policy and consider three cases for the return process of the risky asset. For a simple random walk, the policy always produces lower expected returns. For an AR(1) process, the policy improves performance in the case of momentum, but hurts performance in the case of mean reversion. For a two-state Markov regime-switching model, the strategy sometimes gives better performance, since it tends to outperform the buy-and-hold strategy in the low-mean state.

There are a few other analytical studies of stop-loss strategies. Glynn and Iglehart (1995) derive an optimal strategy by demonstrating that the expected value of the stock price at the time of exit satisfies a relatively simple ordinary differential equation (ODE). They also present an example of a utility function with a very heavy penalty on losses, which would lead the investor to set up a finite stop-loss limit. This contrasts with the case of constant relative risk aversion (CRRA) utility, where it is optimal to not use a stop-loss (Merton, 1969). Glynn and Iglehart’s ODE approach was later applied to derive the optimal selling rule in more complicated settings for the return distribution, including for a regime-switching process (Zhang, 2001; Pemy, 2011) and a mean-reverting process (Zhang and Zhang, 2008; Ekström et al., 2011). Besides the ODE approach, Abramov et al. (2008) analyze the trailing stop strategy in a discrete time framework, while Esipov and Vaysburd (1999) present a partial differential equation approach for analyzing stop-loss policies.

1 For transaction costs prior to 1993, we use the Hasbrouck (2009) dataset.
2 For surveys of behavioral biases, see Hirshleifer (2001) and Shefrin (2010).
With respect to the empirical literature on stop-loss strategies, Kaminski and Lo (2014) consider the strategy of investing in the S&P 500, using monthly frequency for the historical returns and U.S. long-term bonds as the “safe” asset. Erdestam and Stangenberg (2008) and Snorrason and Yusupov (2009) study the strategies applied to stocks in the OMX Stockholm 30 Index. Lei and Li (2009) use daily data for individual U.S. stocks (with the S&P 500 and the U.S. 30-day T-bill considered as the “safe” assets) and conclude that traditional stop-loss strategies are able to reduce losses for some stocks, but not for others. Trailing stop-loss strategies are found to consistently reduce investment risk.

There has been little attention paid to the transaction costs associated with stop-loss strategies. Two papers that do address this issue are Macrae (2005) and Detko et al. (2008). They note that in many cases, the associated hidden costs, such as slippage, result in lower strategy returns.

Finally, stop-loss strategies may also benefit investors by implicitly correcting for some of their behavioral biases. One such bias is the disposition effect, where investors tend to hold losers for too long and sell winners too early, as documented by Shefrin and Statman (1985); Ferris et al. (1988), and Odean (1998). Wong et al. (2006) find evidence for the disposition effect in an experimental setting and propose using stop-loss orders to offset this bias. However, studies exploring this possibility have yielded mixed and inconclusive results. For example, Garvey and Murphy (2004) investigate a sample of trading records for professional traders in the U.S. and find that, while traders tend to use stop-loss orders and avoid large losses, they still exhibit the disposition effect. Nevertheless, Richards et al. (2011) find that retail investors who employ stop-loss strategies exhibit the disposition effect to a smaller extent than those who do not.

We build on the existing literature in several important ways. The first is an advanced theoretical formula approximating stop-loss strategy performance when returns follow an AR(1) process. We link the conclusions from this formula to historical stop-loss performance. The second direction is that we rigorously incorporate transaction costs into our analysis of simulated and historical strategy performance. The sample of assets we consider is individual U.S. stocks, which has different dynamics than the S&P 500 futures considered in Kaminski and Lo (2014). The third piece is that we consider downside risk by looking at skewness and maximum drawdown as performance metrics. Finally, we perform extensive simulations to gain insights on how the performance of the strategies is related to their specifications and the parameters of the underlying returns process.

3. Analytical results

We consider the stop-loss strategy introduced by Kaminski and Lo (2014) and later generalized by Erdestam and Stangenberg (2008). We invest 100% in the risky asset at the start of the period. If its cumulative return over \(J\) consecutive periods drops below a specified threshold \(\gamma\), we liquidate our position and invest in the risk-free asset; otherwise we stay fully invested. To buy the asset again, the cumulative return over \(J\) periods drops below a specified threshold \(\delta\).

Denote by \(r_t\) the log return on the risky asset at time \(t\). Define the cumulative log return \(R_t(N)\) over \(N\) consecutive periods as:

\[
R_t(N) \equiv \sum_{j=1}^{N} R_{t-j+1}
\]

Let \(s_t\) be the proportion of wealth allocated to the risky asset at the start of period \(t\). We define the stop-loss strategy as:

**Definition 1.** A fixed rolling-window policy \(S(\gamma, \delta, J, l)\) is a dynamic asset allocation rule \(\{s_t\}\) between the risky asset \(Q\) and the safe asset \(F\), such that:

\[
s_t = \begin{cases} 
1 & \text{if } R_{t-j}(J) > \log(1 + \gamma) \quad \text{and} \quad s_{t-1} = 1 \quad \text{(stay in)} \\
0 & \text{if } R_{t-j}(J) \leq \log(1 + \gamma) \quad \text{and} \quad s_{t-1} = 1 \quad \text{(exit)} \\
0 & \text{if } R_{t-j}(l) < \log(1 + \delta) \quad \text{and} \quad s_{t-1} = 0 \quad \text{(stay out)} \\
1 & \text{if } R_{t-j}(l) \geq \log(1 + \delta) \quad \text{and} \quad s_{t-1} = 0 \quad \text{(re-enter)}
\end{cases}
\]

The strategy can be implemented in practice as follows. During each day we track the log cumulative return over the past \(J\) days, where \(J\) is specified by the strategy. The return over the current day is also included in the calculation. As we approach the close of the day, if the cumulative return drops below the specified threshold, we sell the asset. We thus assume that the asset price does not move significantly just prior to the close.

Since selling the asset right before the close may sometimes be problematic, we propose a modified stop-loss strategy that is more realistic to implement. At the start of each day, if the cumulative return over the previous \(J\) days (not counting the current day) is below a specified threshold, we submit a market-on-close order to sell the asset at the end of the day. We call this strategy the delayed fixed rolling window policy, \(S^d(\gamma, \delta, J, l)\); the formal definition of this policy is given in the Appendix.

We next present a theoretical analysis of the performance of the stop-loss strategy when the underlying returns follow an AR(1) process. We give an explicit expression for the returns of the strategy after accounting for the dependence on the
risk-free rate, as well as transaction costs. This enables us to analyze when the stop-loss strategy beats the buy-and-hold strategy in terms of raw returns.

### 3.1. Strategy returns for an AR(1) process

Suppose we are investing over a period of length T and hold the risky asset on the first day. The return on the safe asset is assumed to be constant and equal to $r_f$ in each period, while trading costs (as a percentage of capital) are also assumed to be constant at $c$ per period in which a transaction on the risky asset is made.\(^3\)

The log returns $\{r_t\}$ on the risky asset follow an AR(1) process:

$$r_t = \mu + \rho (r_{t-1} - \mu) + \epsilon_t, \quad \epsilon_t \sim WN(0, \sigma^2), \quad (3)$$

where $\rho \in (-1, 1)$ is a constant.

Consider the stop-loss strategy $S(\gamma, \delta, J, I)$. We restrict ourselves to cases where $\gamma$ and $\delta$ are small, while $J = I = 1$. This corresponds to a tight stop-loss/start-gain strategy in which we exit or re-enter the risky asset if its one-day return is too low or too high, respectively.

Let $a \equiv \log(1 + \gamma)$, $b \equiv \log(1 + \delta)$. In Proposition 1 we present an approximation for the performance of the strategy absent any trading costs:

**Proposition 1.** Assume $|\rho|$ is not too large. If $b \geq a$, the expected log-return of the stop-loss strategy $S(\gamma, \delta, 1, 1)$ is approximated by:

$$E[R_{sp}] \approx \pi \left[ 1 + (T - 1) \left( \phi \left( \frac{\mu - b}{\sigma} \right) + p_1 \left( \phi \left( \frac{b - \mu}{\sigma} \right) - \phi \left( \frac{a - \mu}{\sigma} \right) \right) \right) + Tr_f \right. $$

$$+ \left. \frac{\hat{\sigma}}{\sqrt{2\pi}} (T - 1) \left[ \exp \left( - \frac{(\mu - a)^2}{2\sigma^2} \right) + p_1 \left( \exp \left( - \frac{(a - \mu)^2}{2\sigma^2} \right) - \exp \left( - \frac{(b - \mu)^2}{2\sigma^2} \right) \right) \right], \quad (4)$$

where:

$$p_1 = \frac{P(r_{t-1} \geq b, \ a < r_t < b)}{P(r_{t-1} \geq b, \ a < r_t < b) + P(r_{t-1} \leq a, \ a < r_t < b)}, \quad \sigma^2 = \frac{\sigma^2}{1 - \rho^2}.$$

If $b < a$, then the expected return is approximately:

$$E[R_{sp}] \approx \pi \left[ 1 + (T - 1) \left( \phi \left( \frac{\mu - a}{\sigma} \right) + p_2 \left( \phi \left( \frac{a - \mu}{\sigma} \right) - \phi \left( \frac{b - \mu}{\sigma} \right) \right) \right) + Tr_f \right. $$

$$+ \left. \frac{\hat{\sigma}}{\sqrt{2\pi}} (T - 1) \left[ \exp \left( - \frac{(\mu - a)^2}{2\sigma^2} \right) + p_2 \left( \exp \left( - \frac{(b - \mu)^2}{2\sigma^2} \right) - \exp \left( - \frac{(a - \mu)^2}{2\sigma^2} \right) \right) \right], \quad (5)$$

where:

$$p_2 = \frac{P(r_{t-1} \leq a, \ b \leq r_t \leq a)}{P(r_{t-1} \leq a, \ b \leq r_t \leq a) + P(r_{t-1} \geq b, \ b \leq r_t \leq a)}, \quad \sigma^2 = \frac{\sigma^2}{1 - \rho^2}.$$

The first part in (4) is the return contributed from the mean, $\mu$, and is similar to the random walk case. However, in this case we have another part that depends on the autocorrelation coefficient, $\rho$, and volatility, $\sigma$. In fact, for small values of $|\rho|$, $\hat{\sigma}$ does not depend too much on $\rho$ and as a result the second part of (4) is close to linear in $\rho \sigma$.

To get an idea of how much the serial correlation adds to the return, we consider the case when $\mu \approx 0$ and $a = b = 0$. Here, we use a very tight stop-loss/start-gain strategy and assume low daily returns on the risky asset. We then have:

$$E(R_{sp}) \approx \pi \left[ 1 + \frac{1}{2} (T - 1) \right] + \rho \frac{\hat{\sigma}}{\sqrt{2\pi}} (T - 1) + Tr_f. \quad (6)$$

Assuming as before a risk-free rate of 0, and no trading costs, in order to beat the expected log-return of the buy-and-hold strategy, we need to have:

$$\pi \left[ 1 + \frac{1}{2} (T - 1) \right] + \rho \frac{\hat{\sigma}}{\sqrt{2\pi}} (T - 1) + Tr_f > \mu T \Leftrightarrow \rho > \frac{\pi \sqrt{2\pi}}{2\hat{\sigma}}. \quad (7)$$

\(^3\) Thus $c$ includes both the cost of one transaction on the risky asset and one transaction on the risk-free one, since whenever we sell one of the assets, we immediately invest in the other.
Assuming $\rho$ is small, we have $\frac{\sigma}{\bar{\sigma}} \approx 1$, and as a result, an approximate lower bound on $\rho$ is:

$$\rho > \sqrt{\frac{2\pi}{2\sigma^2}} \approx 1.25 \frac{\pi}{\sigma} \approx 1.25 \frac{\mu}{\sigma},$$

in order to beat the buy-and-hold strategy. For daily U.S. stock data over the 1964–2014 period, the ratio of daily return to standard deviation is 5.62%, implying that on average a serial correlation of around 7.0% or higher is necessary to beat the buy-and-hold strategy.

### 3.2. Impact of trading costs

We now incorporate trading costs into this framework. Let $C_{sp}$ be the log of cumulative transaction costs incurred over the period. Then the following result holds:

**Proposition 2.** If $|\rho|$ is not too large and $b \geq a$, the expected log transaction costs incurred are approximated by:

$$\mathbb{E}(C_{sp}) \approx c \mathbb{P}(r_{t-1} \leq a) + c(T-2) \left[ \mathbb{P}(r_{t-1} \leq a, \ r_t \geq b) + \mathbb{P}(r_{t-1} \geq b, \ r_t \leq a) \right] + p_1 \mathbb{P}(a < r_{t-1} < b, \ r_t \leq a) + (1 - p_1) \mathbb{P}(a < r_{t-1} < b, \ r_t \geq b),$$

where $p_1$ is defined as in Proposition 1. If $b < a$, then the expected transaction costs are approximately:

$$\mathbb{E}(C_{sp}) \approx c \mathbb{P}(r_{t-1} \leq a) + c(T-2) \left[ \mathbb{P}(r_{t-1} < b, \ r_t \geq b) + \mathbb{P}(r_{t-1} > a, \ r_t \leq a) \right] + p_2 \mathbb{P}(b \leq r_{t-1} \leq a, \ r_t \leq a) + (1 - p_2) \mathbb{P}(b \leq r_{t-1} \leq a, \ r_t \geq b),$$

where $p_2$ is defined as in Proposition 1.

To validate these results, we estimate the expected log return on stop-loss strategies using simulations for various values of the parameters in the model. We then compare the simulation estimates with the approximations obtained using Propositions 1 and 2; Tables A.4 and A.5 in the Appendix report the results. The approximations are very good, with the deviation between the simulated and the approximated values not exceeding 0.8% per year in each case, and not exceeding 0.4% in most cases.

As in the random walk case, we can derive conditions under which the stop-loss strategy beats the buy-and-hold strategy. Suppose we employ a tight stop-loss/start-gain policy and consider the case in which $a = b = 0$. Using (4) and (10), we obtain an approximate lower bound on the serial correlation $\rho$ in order to outperform the buy-and-hold strategy:

$$\rho > \frac{\mu + c(\mathbb{P}(r_{t-1} \leq 0, \ r_t \geq 0) + \mathbb{P}(r_{t-1} \geq 0, \ r_t \leq 0))}{\sigma}.$$  \hspace{1cm} (11)

It is clear that $\rho$ has to be positive. When $\rho > 0$ and $\mu = 0$, we have:

$$\mathbb{P}(r_{t-1} \leq 0, \ r_t \geq 0) + \mathbb{P}(r_{t-1} \geq 0, \ r_t \leq 0) \leq \frac{1}{2},$$

As a result, an approximate lower bound for $\rho$ is:

$$\rho > \frac{\mu + c}{\sigma}.$$  \hspace{1cm} (13)

For U.S. stocks over the 1964–2014 period, the daily mean is, on average, equal to 5.62% of volatility; however trading costs, as a fraction of volatility, are much higher. Assuming transaction costs of 0.2%, for $\sigma = 1\%$ the lower bound on $\rho$ becomes 32.0%, which is very high. It is evident that for a realistic scenario, we need to have not only a high serial correlation, but also a high volatility. For example, for a daily volatility of $\sigma = 4\%$, the lower bound on autocorrelation is 13.3%. This is still high, but serial correlation of this magnitude is not unrealistic, as we will see later in our empirical results.

### 4. Simulation analysis

To develop intuition for our theoretical results, we simulate the performance of the stop-loss strategy for various return-generating processes with various parameters and compare its performance to that of a simple buy-and-hold strategy. The comparison is made in terms of raw returns, certainty equivalent (CE) in a mean-variance framework, skewness, and maximum drawdown. While we vary the return process for the risky asset, we always assume the risk-free asset yields a 0% return.

We consider two different cases for the strategy depending on the horizon used to measure the cumulative return. In each case, we set the start-gain level at 0% and vary the stop-loss level. The first strategy type is a one-day stop-loss, where

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4 We use the fact that $\frac{\sqrt{2\pi}}{2} \approx 1.25$. 
\( I = J = 1 \), and the stop level ranges from \(-6\%\) to \(0\%\). The second is a two-week stop-loss, so that \( I = J = 10 \); here the stop level varies from \(-14\%\) to \(0\%\).

We also incorporate transaction costs into our simulations. We assume a level of 0.2\% per trade, which is approximately half of the average spread between the closing bid and ask prices over all stocks in our sample on all days in 2013 and 2014. We use the average over these two years instead of over the full sample period from 1964 to 2014 because trading costs have declined significantly in recent years, and these levels seem more practically relevant than higher historical averages.

We first consider the AR(1) process, and find that high serial correlation and volatility leads to outperformance of the stop-loss strategy. We also consider the regime-switching process of Kaminski and Lo (2014). Our results are consistent with theirs, namely that outperformance is quite rare. For both processes, the two-week stop-loss strategy leads to a more positive skewness and less negative maximum drawdown than the buy-and-hold strategy.

### 4.1. AR(1) process

Recall the specification of an AR(1) process:

\[
\epsilon_t = \mu + \rho(\epsilon_{t-1} - \mu) + \epsilon_t, \quad \epsilon_t \sim WN(0, \sigma^2).
\]

There are three parameters: the mean, \( \mu \); the volatility, \( \sigma \); and the serial correlation coefficient, \( \rho \). In our simulations, we vary the annualized unconditional volatility from 20\% to 50\% (an empirically plausible range for individual U.S. stocks) and serial correlation from \(-20\%\) to \(20\%\). While the serial correlation in daily U.S. returns is close to 0 historically, it can take on more extreme values over short periods. We fix the mean at 10\% per year for simplicity. For each unique set of values for these parameters, we run 100,000 simulations over a 252-day horizon. We feel this number of simulations is sufficiently large; standard errors of our estimates are reported in Section A.5 in the Appendix.

Fig. 1 shows the performance statistics for the one-day stop-loss strategy relative to the buy-and-hold strategy (i.e., we subtract the corresponding statistic for the buy-and-hold strategy from that of the stop-loss strategy). Fig. 2 displays the comparable results for the two-week strategy.

We see that returns and CE depend positively on serial correlation and volatility, and outperformance occurs only for high values of these two parameters, which is consistent with our model. The magnitude of relative performance is quite dramatic, even for a two-week strategy that uses a longer horizon to make decisions and hence does not trade as frequently. For a \(-20\%\) serial correlation, the stop-loss strategy can lose up to 30\% per year relative to the buy-and-hold strategy.

It is interesting to compare the two types of strategies in terms of skewness and maximum drawdown. The one-day strategy has lower skewness than the buy-and-hold strategy in most cases. One explanation is that using a one-day return to get out of the risky asset dampens the potential upside and hence reduces the right tail of the return distribution, even if
there is improvement in the left tail. In contrast, the two-week strategy yields higher skewness in almost all situations because the effect on the right tail is quite marginal, whereas the downside risk is cut, especially if serial correlation is high.

The one-day strategy improves maximum drawdown only in cases of positive serial correlation; the two-week strategy does so for quite a few values of negative correlation as well. This difference is due to the fact that while the one-day strategy cuts downside risk, it also incurs high transaction costs and produces poor returns when serial correlation is negative.

In conclusion, the returns and CE of stop-loss strategies depend heavily on serial correlation and volatility, outperforming the buy-and-hold strategy only for high values of these parameters. For an investor with preferences for positive skewness and a lower drawdown, the two-week strategy is quite attractive since it is able to reduce downside risk consistently without incurring too much in trading costs while maintaining most of the upside potential.

4.2. Regime-switching process

We now consider a Markov regime-switching (MRS) process for daily returns:

\[ r_t = \mu_i + \sigma_i \epsilon_t, \epsilon_t \sim WN(0, 1), \]

where \( i_t \in \{1, 2\} \) is the regime indicator that evolves according to a discrete Markov chain with transition probability matrix \( P \):

\[
P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix},
\]

so that \( p_{ij} = P(i_{t+1} = j | i_t = i) \). In the “bull market” regime, the risky asset’s returns are distributed as \( N(\mu_1, \sigma_1^2) \), and in the “bear market” regime, its returns are distributed as \( N(\mu_2, \sigma_2^2) \), where \( \mu_2 < \mu_1 \).

There are three sets of parameters: the means, \( \mu_1, \mu_2 \), the volatilities, \( \sigma_1, \sigma_2 \), and the transition probability matrix, \( P \). We vary each of them to see how the stop-loss and buy-and-hold strategies perform relative to each other. Table 1 lists the parameter values considered, which were chosen to capture a representative range of empirical characteristics for U.S. equities. The more extreme negative values of \( \mu_2 \) and \( \sigma_2 \) represent stock-market crashes. Both transition probability matrices we consider imply that these negative regimes are not rare but occur much less frequently than the positive regimes (20% and 14% of the time for \( P_1 \) and \( P_2 \), respectively). For each unique set of parameter values, we run 100,000 simulations over a 252-day horizon; standard errors are provided in the Appendix.

Figs. 3 and 4 show the performance statistics for the one-day and two-week stop-loss strategies relative to the buy-and-hold strategy. The stop-loss strategy produces lower returns in almost all cases; it is able to outperform in terms of CE when
volatility is high in the bear regime. Performance is better when the expected return in the bear regime is low and when there is little switching between regimes. The intuition behind this is that for the stop-loss strategy to outperform, it must switch out of the risky asset during negative regimes. Further, the negative regime should last for a considerable amount of time so that the relative gain of investing in the risk-free asset will offset transaction costs.

The improvement in CE in situations of high volatility in the bear regime happens for two reasons. First, when volatility is very high, the stop-loss strategy is more likely to be triggered, correctly divesting when expected returns are negative. Second, the high volatility in the negative regime results in a low CE for a mean-variance investor, so the risk reduction due to holding the risk-free asset produces significant benefits relative to the buy-and-hold strategy.

When it comes to skewness and kurtosis, the results are similar to those of the AR(1) process. The one-day strategy gives lower skewness, while the two-week strategy gives higher skewness in comparison to the buy-and-hold strategy. Furthermore, while the one-day strategy improves maximum drawdown over the buy-and-hold strategy in about half the cases (again, for high volatility in the bear regime), the two-week strategy does so in all situations. We can conclude that it does a good job of managing downside risk.

In summary, the stop-loss strategy beats the buy-and-hold strategy when volatility in the negative regime is high, when the returns in the negative regime are low, and when there is little switching between the two regimes. Using a wider stop results in more frequent outperformance, since stops are then more likely to occur during negative regimes. Finally, the two-week stop-loss strategy offers a consistent reduction in maximum drawdown and a more positive return skewness, whereas this occurs in fewer cases for the one-day strategy.

**Table 1**

Parameter Values of MRS Process Simulations. Table 1 lists the parameter values of the simulations for the MRS process. The means, $\mu_1\mu_2$, and standard deviations, $\sigma_1\sigma_2$, are annualized. The higher values of $\sigma_2$ and the more extreme low values of $\mu_2$ capture stock market crashes. There are two cases for the transition probability matrix $P$: $P_1$, when there is little switching out of regimes, and $P_2$, when there is frequent switching.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values Considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\mu_1, \mu_2)$</td>
<td>(10%, −10%), (15%, −20%), (20%, −30%)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>20%, 30%</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>40%, 80%</td>
</tr>
<tr>
<td>$P$</td>
<td>$P_1 = \begin{bmatrix} 0.99 &amp; 0.01 \ 0.04 &amp; 0.96 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0.96 &amp; 0.04 \ 0.25 &amp; 0.75 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

Fig. 3. One-Day Stop-Loss Strategy Performance for MRS Process. Performance statistics for stop-loss strategy with $I = J = 1$, start level of 0%, and varying stop levels. All statistics are measured relative to the buy-and-hold strategy. Mu Case corresponds to one of the three possible pairs of means listed in Table 1; $P$ Case corresponds to one of the two transition matrices listed in the same table.
5. Empirical analysis

To gauge the practical relevance of stop-loss strategies, we document their performance when applied to individual U.S. stocks over the 1964–2014 sample period. The raw returns on the strategies are very poor due to excessive transaction costs. When it comes to the CE in a mean-variance framework, the results look better due to a reduction in volatility when using a stop-loss strategy in comparison to the buy-and-hold strategy. Finally, we connect the empirical results to our model and demonstrate that historical returns on the strategies exhibit heavy dependence on the interaction term between volatility and serial correlation.

5.1. Data and methodology

Stop-loss strategy performance is calculated on a yearly basis. At the start of every year from 1964 to 2014 we use the CRSP database to identify all stocks listed on the NYSE, AMEX, and NASDAQ. We exclude shares of non-U.S. companies, Americus trust components, exchange-traded funds, closed-end funds, and real estate investment trusts. We also remove all stocks with a closing price below $5 on the first trading day of the year, and stocks with less than 250 active trading days before that day. We consider the daily returns of the remaining stocks, adjusted for dividends and stock splits; any missing returns are replaced with zero. Stop-loss strategies are then applied to each of these stocks.

Table A.6 in the Appendix contains summary statistics for the daily returns in the stock sample. In addition to the statistics for the entire 1964–2014 period, we also provide statistics for consecutive 5-year subperiods (the last subperiod from 2009 to 2014 contains 6 years). As we have posited in Propositions 1 and 2, serial correlation is a very important factor in explaining the performance of tight stop-loss strategies. During the 1964–1988 period, average serial correlation tends to be close to zero or positive. However, from 1989 to 2014, this average is negative over all subperiods. Since average serial correlation is slightly negative over the full period, we expect the historical returns on stop-loss strategies to be inferior to buy-and-hold returns.

We consider two different rates of return on the safe asset. The first is simply 0% all the time; the second is the U.S. 30-day T-bill return. Monthly returns on the T-bill are obtained from Ibbotson Associates and converted to daily returns assuming continuous compounding.

Finally, we incorporate trading costs by assuming that the investor pays one-half of the bid-ask spread whenever a stock is traded, where the spread is calculated with end-of-day closing bid and ask prices from CRSP. However, prior to 1993 these data are missing for a large portion of the stocks. We resort to using estimates from closing prices over the 1964–1992 period from Joel Hasbrouck’s website\(^5\) obtained using a Gibbs sampling approach described in Hasbrouck (2009).

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\(^5\) [http://people.stern.nyu.edu/jhasbrou/](http://people.stern.nyu.edu/jhasbrou/).
5.2. Strategy performance

We analyze stop-loss strategies with the stop and start horizons ranging from one day to two weeks. We restrict the start-gain level, $\delta$, to values between 0% and 1.5%, and vary the stop level, $\gamma$, between 0% and $-20\%$. The safe asset is assumed to be the U.S. 30-day T-bill. Fig. 5 shows strategy performance for the different parameter values.

Tight stop-loss strategies have very poor returns, with the one-day strategy employing a 0% stop losing more than $-20\%$ per year for all start levels considered, in comparison to the $+15.2\%$ annual gains for the buy-and-hold strategy. This is not surprising since tight stop-loss strategies require a great deal of trading. Strategies with a wider stop-loss limit provide better performance but still underperform the buy-and-hold strategy in terms of raw returns.

Stop-loss strategies do better when we use CE as the basis of comparison. While most strategies still underperform due to worse returns and higher transaction costs, most strategies employing a wide stop limit do as well as the buy-and-hold strategy, and even a little better. In particular, two-week strategies with a start gain level between 0.5% and 1.5%, and a stop level between 12% and 16% tend to do best. For these strategies, the CE value is between $-14.9\%$ and $-16.3\%$, in comparison to $-18.3\%$ for the buy-and-hold strategy. The favorable results stem from the volatility reduction upon employing a stop-loss strategy.

For a mean-variance investor, the buy-and-hold strategy is not necessarily optimal. Such an investor would tend to allocate only a portion of their wealth to the risky asset, based on the perceived mean and variance of the asset, as well as the individual risk aversion. At the same time, the buy-and-hold strategy is widely used in practice and therefore serves as a natural benchmark when looking at stop-loss strategy performance.

Finally, strategies with wider stops have lower skewness, usually worse than the buy-and-hold strategy, whereas strategies with tight stops have higher skewness and outperform. This is because upside is reduced in all cases. However, performance for tight stop-loss strategies is very poor, leading to a significant shift in expected returns and a seemingly shorter left tail. There are quite a few cases when stocks suffer significant intraday losses, and during those cases it is common to observe very high bid-ask spreads (as large as 50% of stock price). This, of course, leads to very poor returns, and in fact quite a few that are worse than $-90\%$ over the year, implying a very fat left tail. When it comes to drawdowns, we are able to get improvement over the buy-and-hold strategy, especially when a two-week strategy is used, because the downside is reduced without incurring too much in trading costs in the process.

Since many proprietary trading strategies employ tight stops, from now on we focus on strategies with small values for the stop and start levels. We take a “typical” tight stop-loss strategy, $S(-2\%, 0\%, 1, 1)$ (using the U.S. 30-day T-bill as the safe asset), and regress its returns in excess of the buy-and-hold strategy on the statistical properties of stock log returns. We

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6 In the Appendix, we include results when assuming a 0% risk-free rate of return.
control for the time effect as follows. Each “observation” corresponds to the return on the stop-loss strategy applied to a particular stock in a particular year. We employ indicator variables for each of the 51 years in the sample, excluding the last year to avoid collinearity.

We also control for the firm size effect. On the first day of each of the years in the sample, we compute the market capitalization for each of the available stocks, and split the resulting values into deciles. This way we assign a decile to each stock in each year, and we use indicator variables for each of the ten deciles, again excluding the last one.

To summarize, the regression is:

\[ R_{it}^S - R_{it}^{BH} = \alpha + \beta x_{it} + \text{controls} + \epsilon, \]

where \( R_{it}^S \) is the return on the stop-loss strategy for stock \( i \) in year \( t \) and \( R_{it}^{BH} \) is the return on the buy-and-hold strategy; \( x_{it} \) are various functions of stock returns; controls are the time and size controls defined earlier; and \( \epsilon \) is a random error term.

Table 2 contains the results of the regression. The mean (along with time and size controls) is able to explain a significant portion of return variation, around 41.7%. Adding volatility and the interaction between volatility and mean increases the \( R^2 \) by about 6.0%. Adding the remaining regressors boosts \( R^2 \) by another 14.1%, a significant increase.

We next consider a set of regressors motivated by our model for the performance of stop-loss strategies when log-returns follow an autoregressive process. Following Propositions 1 and 2, we only include the mean, \( \mu \), and the interaction term, \( \rho(1) \times \sigma \), between the AR(1) coefficient and volatility as the explanatory variables. Table 2 shows that these two terms explain more than 50% of the variation in returns. Furthermore, the estimated regression coefficients are in line with our model. The coefficient for \( \mu \) of \(-0.43\) implies highly negative dependence on the expected return of the underlying process, since higher returns hurt relative performance. The coefficient for the interaction term \( \rho(1) \times \sigma \) of \(2.2\) implies significant dependence on serial correlation and volatility. A more detailed discussion of why these coefficients are in line with our model is provided in the Appendix.

If we include all eight summary statistics as regressors, the \( R^2 \) only improves marginally, suggesting that our model is able to explain the relative performance of tight stop-loss strategies very well, and that this performance has a close to linear dependence on the product of serial correlation and volatility.

To further investigate the dependence of strategy returns on autocorrelation, in each year we divide the sample of stocks into seven groups based on their realized serial correlation for the year. In each group, we compute the return on the \( S(-2\%, 0\%, 1, 1) \)
strategy in excess of the buy-and-hold strategy using the U.S. 30-day T-bill as the “safe” asset. Table 3 contains the results of this procedure. We find a drastic difference in performance across the realized serial correlation groups. The tight stop-loss strategy applied to stocks with autocorrelation exceeding 15% outperforms the buy-and-hold strategy by 5.5% per year, while it dramatically underperforms by 57% per year for stocks with autocorrelation less than –10%. The overall patterns in this table show that higher autocorrelation leads to significantly better returns. It should be noted that trading costs and positive equity risk premia can cause a tight stop-loss strategy to underperform the buy-and-hold strategy even for stocks with positive serial correlation. The tight stop policy outperforms buy-and-hold only for the stocks in the highest autocorrelation group, and even then only during some of the subperiods of the entire 1964–2014 sample.

Table 3 also contains the excess returns on the strategy in different autocorrelation groups over time. Throughout all of the subperiods, higher autocorrelation gives much better relative performance. The pattern of severe underperformance for stocks with low serial correlation and outperformance for stocks with high serial correlation holds for most subperiods as well.

To develop greater intuition for positive serial correlation in equity returns, we record the proportion of stocks in our sample that fall in a particular serial correlation group each year and the striking results are given in Table 3. During the first half of the 1964–2014 period, many stocks exhibited high autocorrelation: more than 20% of stocks in each subperiod exhibit correlation exceeding 10%, with as much as 43% of such stocks around 1980. However, over the most recent 25 years, there have been much fewer of these stocks: 15.2% in the 1989–1993 period, and less than that in subsequent periods, with only 5.2% of such stocks in 2009–2014.7

The pattern is reversed for mean-reverting stocks (i.e., those with low serial correlation). In the 1964–1983 period, less than 20% of stocks exhibit autocorrelation of less than –10%. This proportion jumped to 27% in 1984–1988, then to 41% in 1989–1993, and has stayed above 27% in each of the subsequent subperiods. The explanation for this pattern is beyond the

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7 We calculate these proportions by adding the percentages of stocks in the buckets for correlation between 10% and 15% and for correlation of 15% or more.
scope of this paper. However, as we have demonstrated, using tight stop-loss strategies provides a simple yet effective way to trade serial correlation; thus there is significant economic value in being able to forecast it.

5.3. Delayed stop-loss strategies

As documented in Section 5.1, daily U.S. stock returns exhibit slight negative autocorrelation over the 1964–2014 period, with most of it occurring during the past two decades. This suggests that stocks may have a tendency to revert in the short-term following large price movements. This anomaly has been well-documented in the finance literature (e.g., Bremer and Sweeney, 1991; Benou and Richie, 2003; Savor, 2012). As a result, the delayed stop-loss strategies, $S^d$, may provide superior returns to their non-delayed counterparts.

Recall that with a delayed stop-loss strategy we wait an extra day to trade out or trade into the risky asset. For example, if the past return over a certain horizon was below a specified threshold on day $t$, then the strategy would switch to the safe asset at the end of day $t + 1$ instead of day $t$.

Fig. 6 contains the performance metrics of the delayed stop-loss strategy relative to its non-delayed counterpart using the same specifications. As before, we consider three different past horizon pairs $(l, J)$, stop-loss levels ranging from 0% to −24%, and start-gain levels between 0% and 1.5%. We see that the delayed strategy provides an improvement in all cases for the return, CE, and maximum drawdown. The improvement is particularly drastic for the one-day strategy using tight stops, since this is the one for which one-day reversals would be most relevant. For example, one-day strategies using a stop of 0% and start-gain under 1% experience an improvement of 2% to 5% per year when using the delayed specification.

For strategies with wider stops performance gets better when using delays, but it is very marginal. Thus overall, their performance relative to the buy-and-hold strategy would not change drastically, and it would still look similar to Fig. 5. Finally, we note that delayed strategies usually have better skewness than the non-delayed ones—because upside is more preserved by capturing the positive returns on days following price declines. The only exception is very tight one-day strategies, where skewness decreases; however this is not due to reducing upside, but due to the significant shift in the average (and very negative) strategy return resulting from high transaction costs.

We conclude that using delayed stop-loss strategies marginally improves performance in most cases, and significantly improves returns for the case of tight stops. They are also easier to execute, since the investor can just submit a market-on-close order at the end of the next day rather than trying to submit one right before the close while tracking returns in real-time as with the original stop-loss strategies. Therefore, if an investor does decide to employ stop-loss strategies within our framework, it is generally beneficial to use the delayed specification.
6. Conclusion

From both analytical and empirical perspectives, stop-loss strategies can improve investment performance in certain circumstances. Our theoretical results show that the log return on a tight stop-loss strategy is close to linear in the interaction term between return volatility and autocorrelation when returns follow an AR(1) process. When returns follow a regime-switching process, wider stop-loss policies will outperform buy-and-hold only in the case when volatility is low in the bull regime and high in the bear regime. And, of course, transaction costs have a significant impact on performance, especially when a tight stop-loss level is used.

When applied to individual U.S. stock returns from 1964 through 2014, we find that stop-loss strategies employing a tight stop produce significantly lower returns in comparison to buy-and-hold strategies. Most of this poor performance stems from very high transaction costs. At the same time, the strategies offer a large reduction in volatility, which in a few cases leads to outperformance over the buy-and-hold strategy in terms of CE for a mean-variance investor.

We also explore how stop-loss strategies affect the skewness and maximum drawdown of returns. In our simulations, most two-week strategies are able to provide an improvement over buy-and-hold strategies because they successfully reduce downside risk while preserving upside potential. However, the empirical results using historical stock returns are not as good due to high transaction costs. Nevertheless, some strategies still perform on par with buy-and-hold strategies in terms of skewness and drawdowns.

A closer investigation of the historical performance of tight stop-loss strategies shows that the mean and the interaction term between volatility and serial correlation explain over 50% of the variation in returns on these strategies in excess of the buy-and-hold strategy, and that the estimated regression coefficients are consistent with our analytical results. The trading costs on these strategies are very high, and as a result, a high serial correlation (typically over 10%) is necessary to outperform the buy-and-hold strategy. We also find a striking pattern that over the first half of the 1964–2014 period, stocks tend to exhibit positive serial correlation, while over the second half this correlation turned significantly negative. The mean reversion in returns over the past two decades helps explain why tight stop-loss strategies have done poorly.

These results clarify the role that stop-loss strategies can play in modern portfolio management. While buy-and-hold portfolios are attractive low-cost passive vehicles for long-term investors, their performance can be improved if asset-price dynamics are more complex than the standard random walk model.

Appendix A. Supplementary material

Supplementary material associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jfinmar.2017.02.003.

References


