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TRC Networks and Systemic Risk

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The financial crisis of 2007–2009 has led to a heightened awareness of the interconnectedness of various asset classes, institutions, and transactions in the global financial system. This, in turn, has motivated the search for and development of useful measures to anticipate future systemic disruptions, which have taken many forms (Billio et al. [2012]; Bisias et al. [2012]). Two classes of analysis that have emerged for characterizing systemic importance are network analysis and portfolio-referent measures of risk contribution.

Network analysis has proven particularly useful in providing visual representation of linkages, typically in the form of counterparty exposures, between multiple financial entities. Although network analysis has been used extensively in other fields,[1] its use in finance is relatively new.[2] There is little financial theory to guide the analytics, thus rendering network analysis less attractive to financial economists. A more practical impediment to the use of network analysis, however, is the heavy reliance on reference data and detailed mappings of counterparty relationships between entities. These data can be difficult to collect and rationalize both for operational and business reasons (Stein [2013]). Furthermore, although network representation provides great flexibility in visualizing and exploring linkages between institutions, it can be overly general in describing the risk exposures of one entity to another as these may be numerous but are often not systemically important.

Portfolio-referent measures of risk contribution, which have been adapted from the portfolio risk management literature, provide a more focused view of key systemic risks and offer richer financial intuition. These techniques originally evolved to allow portfolio managers to identify those positions in their portfolios that contribute most to losses when such losses are much higher than their statistical expectation. In other words, these techniques highlight which positions contribute the most to rare but very large portfolio losses. The transformation of such approaches from portfolio management tools to methods for measuring systemic risk typically involves viewing the entire financial system as a single portfolio and then seeking to identify those firms that contribute most to losses in extreme portfolio events (see, e.g., Acharya et al. [2010] and Adrian and Brunnermeier [2010]). An attractive feature of risk attribution techniques is their amenability to implementation using aggregate, often publicly available, data. Although such measures provide explicit indications of firms that are likely to drive systemic distress, they may miss important linkages between financial institutions (FIs) through which distress may be propagated.

In this article, we introduce a new mechanism for integrating these two approaches to provide information on key linkages between institutions, which may become relevant during periods of systemic stress. Our goal is to illustrate new insights that are only available through this integration. The approach we introduce, which we call TRC (tail-risk contribution) networks, is primarily a visualization technique that allows researchers and analysts to filter typical network visualizations to highlight the key entities and relationships that are most important in times of system-wide distress.

By way of example, we highlight the potential exposure of 2a-7 money market funds (MMFs) to some key FIs during the summer of 2011. Because we use historical data from mid-2011 to demonstrate our approach, the analysis in our example is no longer directly actionable. Instead, we hope that regulators and risk managers will find the approach useful when applied to their own datasets. Importantly, all of the analysis we show herein can be performed using public data (albeit in some cases with a disclosure time lag).

PORTFOLIO-REFERENT MEASURES OF RISK

In this section we review the basic machinery of portfolio-referent risk measures such as TRC. We also provide an example of how these measures may be transformed into measures of systemic risk and demonstrate their use by applying this approach to a set of large FIs with substantial debt exposure.

Portfolio-referent risk measures such as TRC were first developed for credit portfolio management. Our approach is similar to those of Acharya et al. [2010] and Adrian and Brunnermeier [2010]. These authors used market value-at-risk (VaR) and related measures to proxy for the systemic importance of individual institutions within the financial system. A firm's solvency is measured directly with respect to the volatility of its equity (or credit default swap [CDS] spreads or other market observables) and the relationship between movements in these values to those of other firms. The goal of these approaches is to indirectly measure the likelihood of joint distress of systemically important financial institutions (SIFIs) and to then identify the firms that contribute most to joint losses across many firms. Such rare instances of high loss across many firms can be implicitly considered to be crisis events.

Unlike earlier authors, however, our approach draws directly on economically motivated structural models of firm-level default (Merton [1974]) and the manner in which common factors may drive default across firms. These models have been well established in the finance literature and widely used for corporate credit risk management for over a decade by a number of FIs (Kealhofer and Bohn [1998]), and they can be implemented using existing credit risk management tools with some modest modifications. Furthermore, in contrast to some other approaches, the structural approach that we adopt endogenizes each firm's capital structure and leverage, providing a straightforward and economically intuitive link between the default likelihood of individual firms and aggregate systemic risk in the financial system. In addition to its economic intuition and ease of implementation by virtue of its similarity to models used by academics and practitioners, this approach enjoys the benefit of an extensive literature in both of these communities (Bohn and Stein [2009]).

In the remainder of this section, we briefly review the basic structural model of default developed by Merton [1974], its generalization to a portfolio context, and the use of the portfolio formulation for calculating credit tail-risk contribution. We then describe how this approach can be extended to measure systemic risk.

A Structural Model of Default

Recall that under the Merton [1974] model, the assets of a firm evolve stochastically. A key insight of the model is that the firm's equity has the same payoff as a call option on the firm's assets with a strike price equal to the face value of debt. Thus, at debt maturity, the equity holders may elect either to pay back the debt or to default, thus forfeiting future claims on the cash flows of the enterprise. Equity holders have an incentive to pay back debt when the value of the firm is greater than the value of the debt (i.e., when the option is in the money) and to default when the value of the firm is less than the value of the debt. In this framework, default occurs when the value of the firm's assets falls below the face value of its debt at maturity.

More formally, consider a firm with a balance sheet containing an amount D of zero coupon debt maturing at time T and amount E of common equity. The total assets of the firm, A, can then be written as:

$$A = D + E. \tag{1}$$

Assume that the market value of the firm's assets follow a geometric Brownian motion:

$$dA = \mu_A \, Adt + \sigma_A \, Adz, \tag{2}$$

where μ_A is the drift of the firm's assets and σ_A is the asset volatility. The governing differential equation is then:

$$\frac{1}{2}\frac{\partial^2 E}{\partial A^2}\sigma_A^2 A^2 + \frac{\partial E}{\partial A} rA + \frac{\partial E}{\partial t} - rE = 0, \tag{3}$$

where r is the risk-free rate.

Finally, the volatility of the market value of the firm's assets, σ_A, is related to the volatility of the firm's equity, σ_E, through the firm's leverage (A/E) and the delta ($\partial E/\partial A$) of the firm's equity with respect to its assets:

$$\sigma_E = \frac{\partial E}{\partial A}\frac{A}{E}\sigma_A. \tag{4}$$

To derive a probability of default, PD, note again that under the model, equity holders only have an incentive to pay debt when a residual value remains after the debt is satisfied. Thus, the probability of the firm's assets being below the face value of debt exactly coincides with the probability that the firm will be insolvent. It can be shown that:

$$\begin{aligned} \text{PD} &= \Pr(A \le D) \\ &= \Phi\left(-\frac{\ln\left(\frac{A}{D}\right) + \left(\mu_A - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}} \right), \end{aligned} \tag{5}$$

where $\Phi(\cdot)$ is the cumulative normal distribution function. Note that in practice, a number of modifications are made to both the model and its estimation.[3]

Portfolios under the Structural Approach

Because of common factors of production and the impact of market and other dynamics, two firms may exhibit default correlation to varying degrees if their asset values are correlated due to a common dependence on the same factors. Thus, two firms whose asset values are similarly driven by a key market factor may exhibit high correlation, whereas two firms that do not share common factors may exhibit lower correlation. This dynamic, motivated by the capital asset pricing model (CAPM), has formed the basis of a number of widely used credit portfolio construction and risk management approaches.

In the single-factor model, two firms with a correlation ρ are related through their dependence on a single common factor $Z_t \sim \mathcal{N}(0,1)$. It can be shown that:

$$r_{it} = \sqrt{\rho}Z_{it} + \sqrt{1-\rho}\varepsilon_{it}, \tag{6}$$

where $r_{it} \sim \mathcal{N}(0,1)$ is the continuously compounded asset return of firm i at time t; $\varepsilon_{it} \sim \mathcal{N}(0,1)$ is an idiosyncratic shock; $\text{Corr}(\varepsilon_{it}, \varepsilon_{jt}) = 0$; $i \neq j$; and $\text{Corr}(Z_{it}, \varepsilon_{jt}) = 0$. Multifactor analogs replace the single common factor realization, Z_{it}, with a vector of common factor realizations. This also permits heterogeneous correlations among firms (Bohn and Stein [2009]).

The factor model representation permits analysis of the portfolio loss distribution. In special cases, this may be calculated analytically (Vasicek [1987]), but more typically, simulation techniques are required. In the case of computationally intensive methods such as Monte Carlo simulation, the factor representation also benefits from much faster computation time than would be the case using pairwise asset correlations.

One common credit portfolio metric in the context of credit risk management is the level of a portfolio's VaR, which is defined as the amount of economic capital that is required, under a portfolio model, to protect the portfolio from losses in α of all model outcomes in which $1 - \alpha$ is a small value, often on the order of 1% to 0.05%. Put differently, the 99.5% VaR ($\text{VaR}_{99.5}$) is the dollar value of portfolio losses beyond which losses occur less than 0.5% of the time.

Portfolio-Referent Measures of Firm Risk Contribution

VaR is sensitive to the variance of portfolio losses. The variance of the loss distribution is, in turn, sensitive to correlation among the firms whose securities are represented in the portfolio, these firms' default probabilities, and the exposure weights of the portfolio securities. Portfolios with higher variance, all else equal, will require greater capital since the probability of large losses will be greater for these portfolios than it would be for portfolios with lower variance. Portfolio variance is increased because of higher levels of correlation among firms' asset values, which suggests that they will default (or not default) together more frequently. Similarly, concentrated positions in the portfolio will also increase variance.

The converse is also true: If an individual exposure is highly correlated with many other exposures in the portfolio (or is very large relative to other exposures), it will tend to default under conditions in which the portfolio experiences large losses and it will tend not to default under conditions in which portfolio losses are low. This exposure will increase the portfolio's VaR because, when losses are high in the portfolio in general, it is more likely that the position will have defaulted, and capital will be required to cover its loss as well as the other losses in the portfolio. The TRC of an exposure describes the amount of capital that the exposure contributes to the overall capital of the portfolio at a given VaR level.

Mathematically, for the i^{th} exposure in the portfolio, the tail risk contribution, TRC_i, is defined as:

$$\text{TRC}_i = E[L_i | L_P = \text{VaR}_\alpha] = \frac{\partial \text{VaR}_\alpha}{\partial \omega_i}, \qquad (7)$$

where L_i and L_P are the losses on position i and on the entire portfolio P, respectively, and ω_i is the portfolio weight of exposure i.

Systemic Risk Measures

The transition from portfolio credit risk measures to systemic risk measures is straightforward. Rather than the portfolio of interest being an individual institution's loan or fixed income portfolio, we take the portfolio to be the entire volume of corporate liabilities for all key firms in the financial system (or for some subset of the financial firms in the global financial markets). We view these firms as *positions* in a portfolio, with each firm's portfolio weight proportional to its total liabilities. We then calculate the portfolio's VaR and the TRC of each firm in the portfolio. To the extent that a firm is among the largest contributors to the system portfolio, this firm may be systemically important since it will experience large losses in circumstances in which many other major FIs are also under duress.

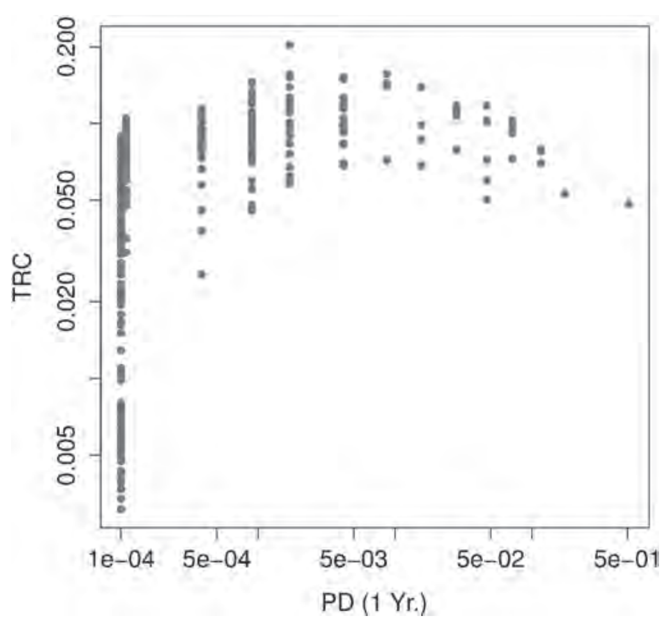
Importantly, although firms of lower credit quality are more likely to default in general, it is not always the case that firms with the lowest credit quality (i.e., those with the highest PD) are the most systemically important. Since TRC is a portfolio-referent measure, importance in the systemic portfolio depends on correlation and size as well as credit quality. For example, Exhibit 1 shows the relationship between the PD and the TRC of each firm in a sample portfolio (x- and y-axis, respectively; log scales). Not surprisingly, there is a positive relationship between PD and TRC: Firms that are more likely to default are generally also more likely to default during periods of portfolio stress. This relationship, however, is not a very strong one. In fact, a fairly large number of very high quality (low PD) firms have relatively high TRCs in the portfolio due to high correlation and/or large size. Thus, even very high credit quality firms may become caught up in a crisis.

TRC NETWORK VISUALIZATION

TRC is useful in identifying which institutions are more likely to fail in times of crisis. From a systemic perspective, however, the failure of individual institutions

EXHIBIT 1

Systemic TRC is not PD. The PD (x-axis) is Plotted against Systemic TRC (y-axis) for Each Institution on Log Scales



Note: Although there is a generally positive relationship between PD and TRC, a fairly large number of very low PD firms have relatively high TRCs in the portfolio owing to high correlation and/or large size.

is seldom the primary concern of policy makers and market participants. Instead, the impact that such failure may have on other institutions within the broader financial system is of concern.

To provide additional transparency into this aspect of systemic stress, we introduce a new approach that combines network analysis with portfolio analytics. Specifically, as described earlier, we examine the systemic portfolio representing the major institutions in the financial system. Under our approach, we use network techniques to represent various systemically important financial entities (*nodes* or *vertices*) as well as the relationships among these entities (*links* or *edges*) on the graph. By populating the network with TRC-weighted values rather than standard accounting values, we arrive at a richer visualization. In addition, the network representation also permits the application of a number of measures of network topology that are useful in summarizing the state of the financial system.

We begin by discussing the traditional application of network analysis to systemic risk. We then show how to extend this approach by combining it with TRC analysis to derive a more informative and actionable view of risks to financial stability.

Accounting-Based Systemic Network Analysis

In a typical financial network analysis, each node of a network represents an FI, and a link between two nodes represents some sort of counterparty exposure between them. A basic property of network topology is the *node degree*, which, in directed networks, is decomposed into the *node-in degree* and *node-out degree* (Newman [2006]; Cohen and Havlin [2010]). The node degree is often used to measure the importance of the node in the network, which is referred to as *degree centrality*. Other classical measures of importance include *closeness centrality* (related to the node's distance from all other nodes in the network), *betweenness centrality* (related to the number of shortest paths between node pairs going through the given node), and *eigenvector centrality* (related to the node's influence on the entire network) (Newman [2010]).

In the examples that follow, however, we focus only on *bipartite networks*, that is, those networks with nodes that can be partitioned into two disjoint sets and whose links are only between nodes of different sets (i.e., no links occur between nodes within each set). For example, nodes can represent borrowers and lenders, in which case a link might represent a debt contract between borrower and lender (hence there would be no links between two borrowers or two lenders).

For such networks, node centrality is a less meaningful measure. In some cases, such as our context, the nodes' size corresponds to the size of the total exposures of the members of the network to a specific FI, whereas the weight of the links represents the size of the exposures. For example, in a network representation of CDS exposure, the nodes for firms with the largest aggregate CDS exposures would be the largest, but nodes for firms with minimal exposure would be quite small. Similarly, the link between two nodes would have a larger value (and perhaps be shown more prominently in the visualization) if the CDS exposure of one entity to the other were large. Such an analysis can be useful in answering questions about the overall connectivity of institutions and about the impact the failure of one or more specific institutions will have on the overall financial system. This approach to applying network analysis might be thought of as an accounting-based one that describes the current state of the system.

For weighted networks, the analog to the node degree is a generalization called the *weighted node degree*, which in directed weighted networks can similarly be decomposed into the node *weighted in-degree* and node *weighted out-degree* (also known as *node strength*; Barrat et al. [2004]; Boccaletti et al. [2006]; Newman [2010]; Opsahl, Agneessens, and Skvoretz [2010]).

For every node i, the weighted in-degree is defined as:

$$k^{win}(i) = \sum_{j}^{n} w(i,j), \tag{8}$$

where $w(i,j)$ denotes the ij^{th} cell in the weighted adjacency matrix, representing the weighted links between every node pair (i,j). In some settings, it is convenient to normalize the weights such that $\Sigma_i \Sigma_j w(i,j) = 1$, $i \neq j$.

The weighted out-degree is defined as:

$$k^{wout}(j) = \sum_{i}^{n} w(i,j) \tag{9}$$

and the total degree of each node, k, is the sum of the two:

$$k^{w}(i) = k^{win}(i) + k^{wout}(i). \tag{10}$$

In addition to the (weighted) node degree, we also make use of the the normalized Herfindahl index (Rhoades [1993]), H^*, to compare network topologies:

$$H = \frac{H_{raw} - 1/N}{1 - 1/N}, \tag{11}$$

where

$$H_{raw} = \sum_{i=1}^{N} s_i^2 \tag{12}$$

and s is the percentage share of the aggregate measure ($\sum_1^N s_i = 1$). For weighted networks, we estimate H separately for both the nodes and edges of the networks.

Combining Network and Portfolio Analysis

Accounting-based networks provide an overview of the state of the financial system and describe the various counterparty relationships between FIs. In addition to the overall connectedness of the entities, examining the degree of each node permits an assessment of which entities are most active in the market and most connected. Systemic risk analysis, however, also involves the behaviors of these systems in times of stress. Accordingly, we would ideally also like to understand how the network is stressed in times of crisis and which institutions and relationships are most important in a crisis state. In this section, we introduce one method for doing so.

In contrast to accounting-based networks, we make use of systemic TRC information for FIs to create a weighted adjacency matrix. Conceptually, being an important node during normal states of the economy is substantially different from being an important node during times of crisis. Our approach explicitly contemplates distressed states of the world—as characterized by high levels of credit distress across the financial system—and we use this information to construct the network. Although our example involves a bipartite network, the approach can easily be extended to more general network structures.

We define two types of FIs: *issuers* and *investors*. Issuers generate credit-risky securities, whereas investors purchase these securities and hold them in investment portfolios. This simple structure describes, to a first approximation, a number of types of relationships found in the capital markets, such as those between large banks and pension funds, or, as in the example to follow, between large FIs and MMFs.

To construct a systemic TRC network for a set of FIs (issuers) and their counterparties (investors), we used the following procedure:

1. For each issuer i in our sample, we calculated TRC_i, the dollar-valued TRC, and the TRC percentage $trc_i = \frac{TRC_i}{L_i}$, where L_i is the total liabilities of the issuer.
2. For each investor j, we calculated e_{ij}, the exposure of the investor to issuer i's debt (commercial article, demand deposits, corporate bonds, etc.). We did this for each issuer.
3. We created the weighted adjacency matrix and defined $w_{ij} = e_{ij} \times trc_i$ for all i, j.
4. We calculated each investor's weighted in-degree, $k^{win}(j)$, from Equation (8).
5. We calculated each issuer's weighted out-degree, $k^{wout}(i)$, from Equation (9).

This approach creates a network that highlights the relationships between financial entities and then filters the network to show only those nodes that would be largest in times of crisis, during which many firms may be simultaneously in distress. The issuers that are most likely to be sources of distress in a crisis are the nodes with the largest values. Similarly, the investors that are the most likely to be net holders of distressed assets during a period of financial crisis are represented as the largest investor nodes. Finally, the positions that are most likely to be distressed are shown with the heaviest weight.

AN EXAMPLE: MONEY MARKET FUNDS

To demonstrate our approach, we considered the potential exposure of U.S. 2a-7 MMFs to non-U.S. FIs in times of crisis. To illustrate the differences between TRC networks and the more traditional accounting-based network approach, we present each TRC network alongside an accounting-based analog (which we will refer to as a *normal network*) using the same data. The normal network was calculated using the algorithm described previously, except that in step 3 we defined the elements of the weighted adjacency matrix in the traditional way, that is, as $w_{ij} = e_{ij}$ for all i, j, where e_{ij} is simply the par value of the exposure between FIs i and j.

Money Market Fund Holdings Data

Network analysis requires data on specific relationships between financial firms. In general, these data can be difficult to obtain because of issues of confidentiality and conformity. Thus, it is desirable to make use of publicly available data whenever possible.

Although regulators are often able to obtain detailed data on institutional portfolios, many market participants do not have such access. To demonstrate our approach, we obtained an anonymized but linkable (Stein [2013]) dataset from Moody's Investor's Service that provides a snapshot of the holdings of a subset of MMFs as of July 2011. The data contain information on the issuer, debt type, and exposure size of each security in the fund portfolio, as well as information on the investor, including the individual exposure-level holdings of each fund along with the exposure size.

Because of the way in which the anonymization was done, we were able to link this holdings information to other data on the FIs that issued the securities. In particular, we were able to link the holdings information to the TRC results we estimated based on a portfolio of all non-U.S. FIs to which the funds had exposure; hence, we were able to use our TRC results for large financial firms. (See the discussion at the end of this article for caveats about this selection method.) We focused our analysis on 2a-7 prime MMFs. The raw data on the 2a-7 MMF holdings are now publicly available (with a delay) by virtue of regulatory reporting requirements.

For each MMF, we aggregated individual portfolio exposures by issuer. These issuer-level holdings were then aggregated along corporate family trees to arrive at a single measure for each major FI. For example, if a fund held \$10MM of bonds issued by Bank ABC-U.S. and \$10MM of bonds issued by Bank ABC-France, and both of these banks were units of Bank ABC-Switzerland, assuming these were the only related exposures, we would record this as \$20MM of exposure to Bank ABC-Switzerland.

Network Visualization

We began by generating a normal network using the standard accounting-based approach. Exhibit 2 shows an example applied to our MMF data, where the investors are represented by the black circles (right-hand side of the exhibit running from about 2 o'clock to 6 o'clock), and the issuers are represented by the gray circles around the rest of the perimeter. The weight of the links between the funds and FIs is proportional to e_{ij}. Note that due to substantial size differences between the typical FI and the typical fund, we scaled the size of the FI nodes relative to all FIs and the MMF nodes relative to all MMFs.

Next, we used the same underlying data to generate a TRC network, shown in Exhibit 3. As in the normal network, the MMFs are represented by the black circles, and the FIs are represented by the gray circles. The weight of the links between the funds and the FIs is proportional to $w_{ij} = e_{ij} \times trc_i$. Again, we scaled the size of the FI nodes relative to all FIs and the MMF nodes relative to all MMFs, weighted in both cases by the TRC value.

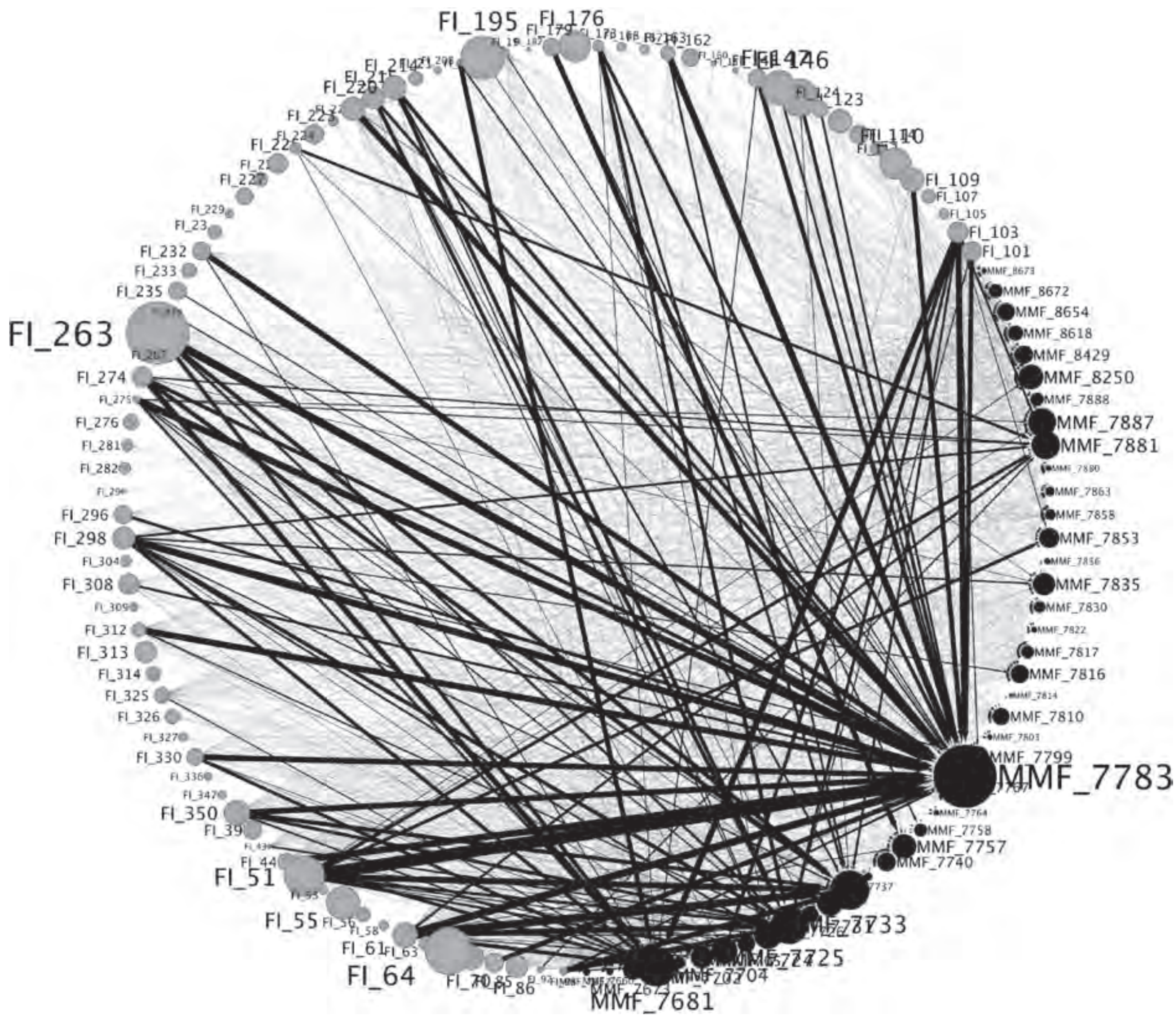
The differences between the TRC-network representation (Exhibit 3) and the normal-network representation (Exhibit 2) are both conceptual and practical. The accounting-based representation provides a wealth of information about every exposure of every MMF to every financial firm. Its richness and completeness are useful for cataloging the relationships between various entities and the overall density of the network. This richness, however, is also a limitation in some settings. It is not always clear, from a normative perspective, how to interpret these relationships or which entities are most susceptible to shocks that could cause a cascade of distress events through the financial system.

In contrast, the TRC network representation provides a very skewed (though very important) perspective. Rather than capturing the relationships in steady state, it highlights those firms and relationships that carry the most risk in times of market distress. For example, in comparing Exhibit 3 to Exhibit 2, one is struck by the relatively less cluttered web of relationships highlighted in Exhibit 3, the TRC representation. This is partially so because, at the time these data were collected, most of the relationships between MMFs and FIs were not especially important from a systemic perspective. The TRC-based representation filters out these less central relationships, emphasizing only those that are most relevant during times of stress.

Less obvious from casual inspection is the fact that the relative importance of the financial entities

E X H I B I T 2

Normal (accounting-based) Network Representation of MMF Exposures to Individual FIs



Notes: Node size is calculated based on the total exposure of an MMF to an FI while link size is proportional to the total size of all exposures from a specific FI to a specific MMF. The data shown here only represent a subset of the total exposures. FIs were selected based on the ease of computing all relevant statistics.

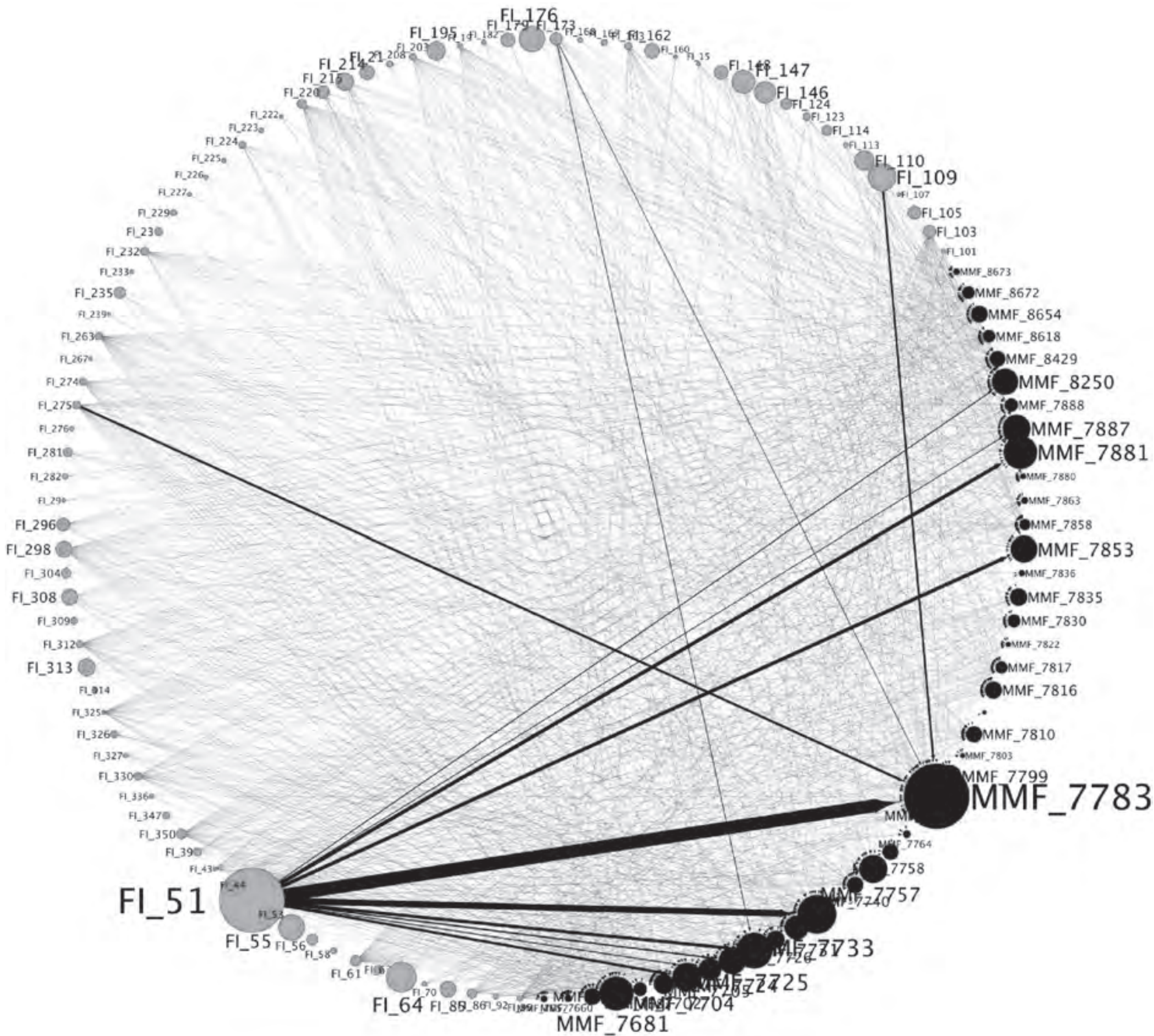
themselves is also represented differently. Consider the firm labeled FI_{51}, in the lower left-hand portion of both exhibits. In the normal network representation, the firm's node is slightly larger than average, suggesting a relatively moderate exposure by MMFs to the firm. FI_{51} is neither particularly important nor unimportant in this representation. In contrast, in the TRC network representation, this node is the largest shown, suggesting that in times of system-wide stress, exposures to this firm are likely to be affected substantially and that impact may be transmitted to key MMFs. Conversely, FI_{263} (upper left at about 10 o'clock) is the largest node in the normal network because of the large volume of exposures that MMFs have to it; however, because of the credit quality of the issuer and its correlation with other issuers, under the TRC-based analysis, FI_{263} is only marginally important during times of stress.

Network Comparison

TRC networks can provide valuable visualizations and descriptive quantitative information on the state of

Exhibit 3

TRC Network Representation of MMF Exposures to Individual FIs



Notes: Node size is calculated based on the total TRC-weighted exposure of an MMF to an FI (or vice versa) while link size is proportional to the total size of all TRC-weighted exposures from a specific FI to a specific MMF. The data shown here only represent a subset of the total exposures. FIs were selected based on the ease of computing all relevant statistics.

the system. To further explore the added information the TRC network provides, we analyzed some of their topological properties. Both the normal network and TRC network are directed, weighted networks, with $N = 401$ nodes (358 FI + 43 MMF) and $K = 877$ links connecting FIs with MMFs (recall that we are only examining U.S. 2a-7 MMFs that have exposure to non-U.S. FIs; thus, the full populations of MMFs, FIs, and MMF exposures, respectively, are all much larger than in our example). As both networks have the same number of nodes and links, the main differences between them lie in the weighted-in and -out degree and in the weighted-node size. Furthermore, as both networks are in fact bipartite networks, most classical centrality measures and other network topology measures do not apply.

Therefore, we focused on measures of similarity between the networks. As an initial naive measure of similarity, we began by measuring the rank correlation between all node degrees of the normal and TRC networks. The correlation turns out to be relatively high ($\hat{r}$ = 0.81), suggesting that the two approaches rank FIs similarly.

Because the scales of MMFs and FIs are different, we repeated this analysis separately for FIs alone. As expected, this substantially reduced the correlation; however, because many of the small nodes remain small in both networks, the Spearman correlation for the FI nodes is still modest ($\hat{\rho}$ = 0.66).

In measuring systemic risk, we are often primarily concerned with the most important nodes in the financial network, and the correlation measures we have been using do not highlight this well. When we examine only the very largest nodes, however, we see much more differentiation between the TRC and normal networks. In fact, only three of the 10 largest normal-network nodes are included in the list of the 10 largest TRC-network nodes.

For example, consider nodes FI_{51}, FI_{263}, and FI_{109}. The weighted node degree of FI_{51} increases from the eighth largest in the normal network (out of 263 nodes) to the largest (rank = 1) in the TRC network. In contrast, the rank of FI_{263} is 1 in the normal network, but it drops to 116 in the TRC network. More revealingly, the rank of node FI_{109} moves from 27 in the normal network to 3 in the TRC network.

The same type of reordering of importance can be observed with respect to specific exposures between counterparties in the network. For example, of the 877 counterparty relationships shown in our example, the investment by MMF_{7725} in FI_{101} is ranked 85 in the regular network, which puts it in the 9.7th percentile. In contrast, in the TRC network, the rank of that counterparty relationship drops to 569, relegating it to about the 65th percentile. Conversely, the counterparty relationship between MMF_{7733} and FI_{109} is ranked in about the 30th percentile at 267 in the normal network, but it jumps to the 8.5th percentile in the TRC network with a rank of 75.

Our estimates of H for each network reflect the tendency of TRC networks to differentiate more sharply between FI risk profiles. For example, we found H to be almost twice as large for the node degree of the TRC network than for the node degree of the normal network ($\hat{H}_{norm}$ = 0.03 and $\hat{H}_{TRC}$ = 0.06). This increased value of $\hat{H}$ suggests greater differentiation between nodes and relationships in the TRC network.

EXHIBIT 4
Summary Statistics for Normal and TRC Networks (# nodes = 401, # edges = 877)

	Normal	TRC
Node Degree Heterogeneity		
H	0.03	0.06
Skewness	1.91	5.47
Kurtosis	5.77	41.64
Between Network Correlation		
Node Degree Correlation (all)		0.81
Node Degree Rank Correlation (FIs)		0.66

Finally, we can characterize this heterogeneity in systemic importance in terms of the distribution of node degrees across the two networks. The skewness (σ^4) of the distribution of network node degrees in the normal network is $\hat{\sigma}^4_{norm}$ = 1.91, whereas the skewness of the TRC network, $\hat{\sigma}^4_{TRC}$ = 5.47, is much larger. The kurtosis (σ^3) of the distribution of network node degrees also suggests much more heterogeneity for the TRC network than the normal network: $\hat{\sigma}^3_{norm}$ = 5.77 and $\hat{\sigma}^3_{TRC}$ = 41.64.

Taken as a whole, these results suggest that the TRC network differentiates more dramatically between "more important" and "less important" nodes and links. Exhibit 4 summarizes the statistics we have been discussing.

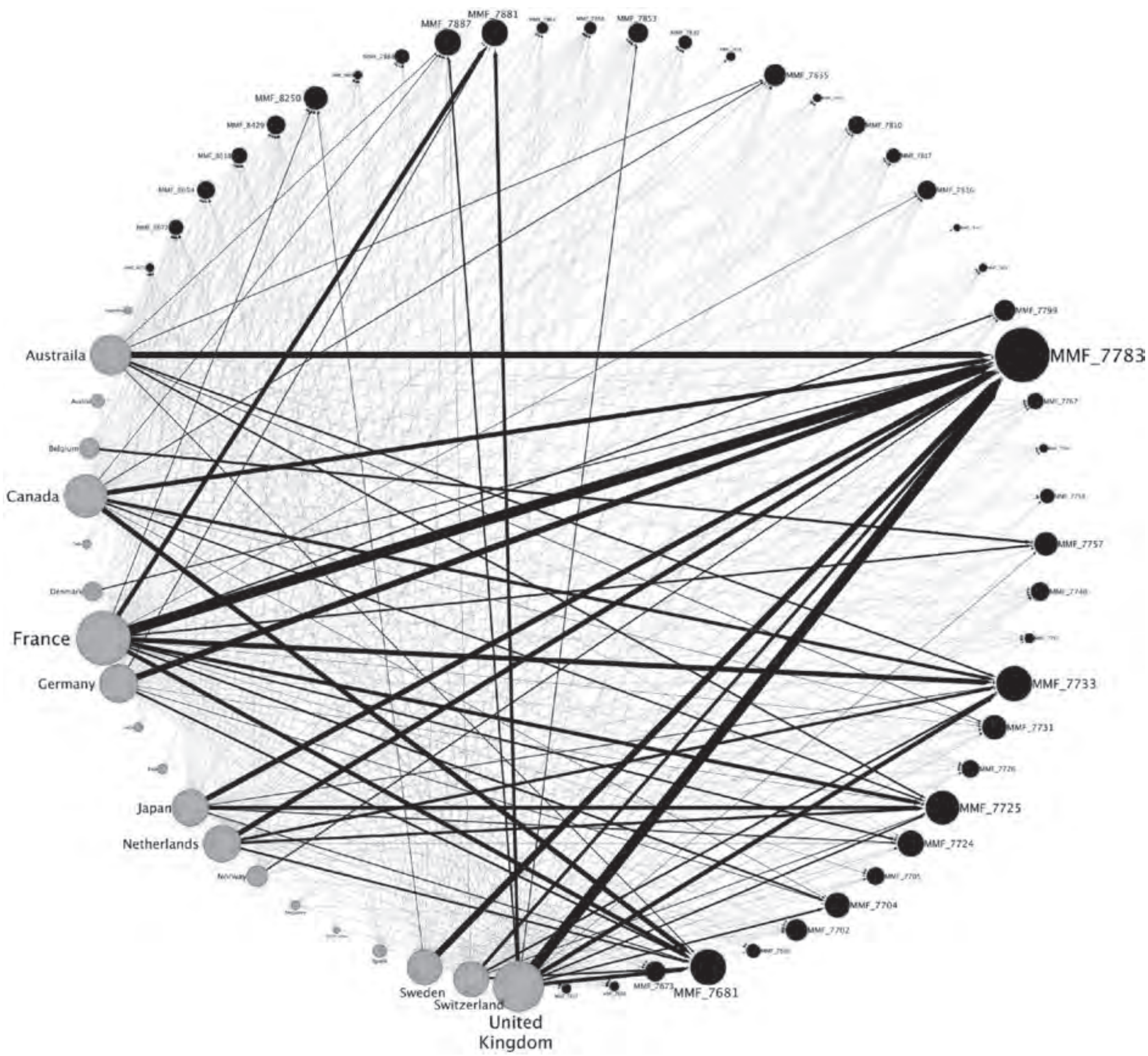
DISCUSSION

The example in the previous section demonstrates one approach to applying TRC networks for systemic risk analysis. In this section, we suggest possible extensions and discuss some caveats and implementation issues.

Extensions

In our example, we have illustrated a micro-level representation: individual MMFs to individual FIs. This is one of many possible formulations. More macro-oriented investigators might prefer higher levels of aggregation. For example, Exhibits 5 and 6 make use of the same data as in that example, but in these exhibits, the FI data have been aggregated by the domicile of the ultimate corporate parent of the issuer. This representation gives some sense of the potential impact of banking system distress within a specific country (or, more generally, a downturn in that country's economy) on the MMF sector.

Exhibit 5
Normal (accounting-based) Representation of MMF Exposures to FIs by Domicile



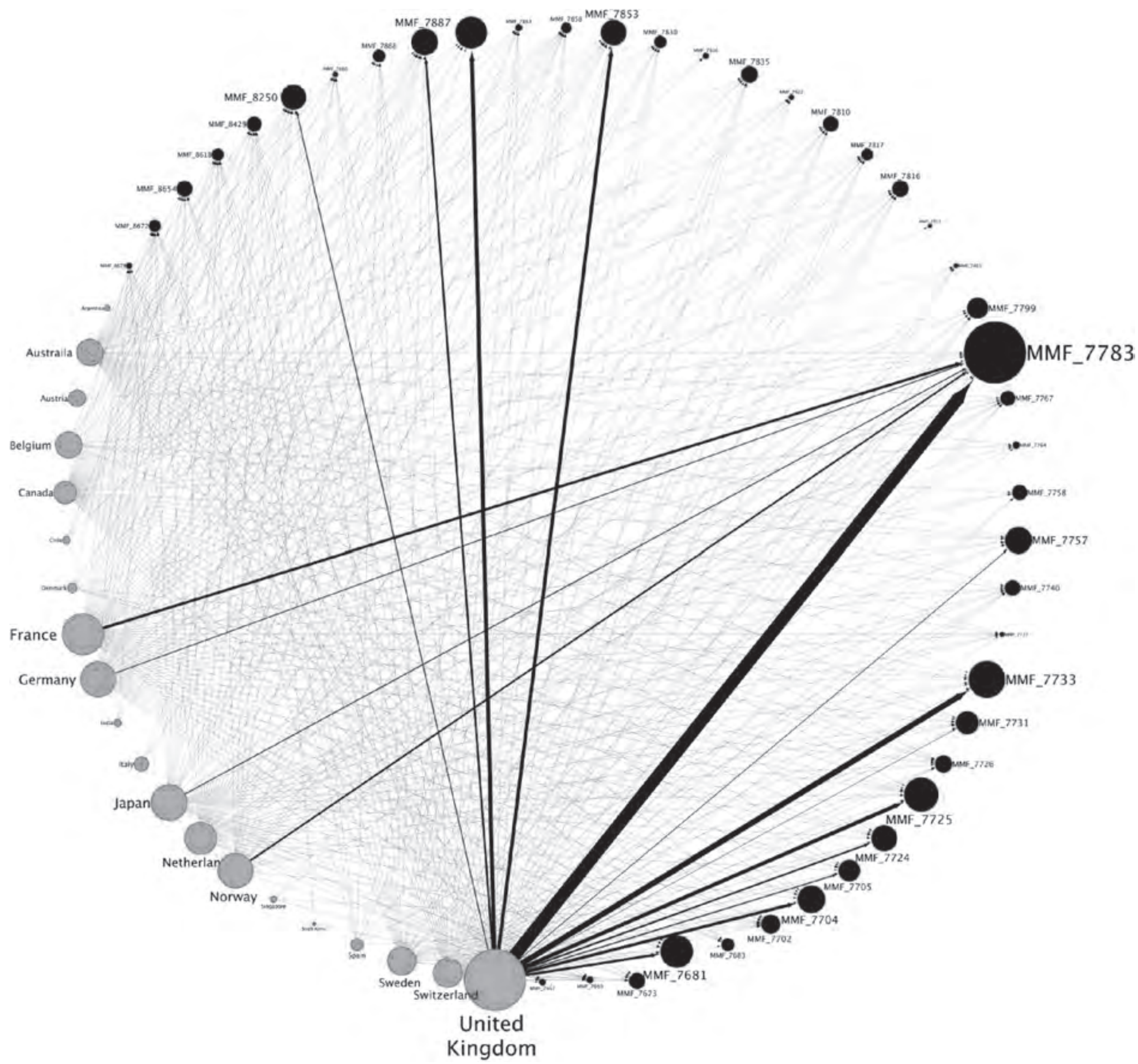
Notes: Node size is calculated based on the total exposure of an MMF to all FIs in a given domicile, while link size and color are proportional to the total size of all exposures from a specific FIs in a domicile to a specific MMF. The data shown here only represent a subset of the total exposures.

Here again, we get a more nuanced and re-prioritized picture of the "flash points" from the TRC-based rendering than from the more generic accounting-based one. As in the previous example, this intuition is supported by quantitative measures of the network topology. For example, in Exhibit 7 we present the top five domiciles, ranked by out-degree for normal and TRC networks, respectively. Two of the top five domiciles in the TRC network (Japan and the Netherlands) were not included in the top five of the normal network. For the full networks, the between-network rank correlation of domiciles was only 0.32.

As in the earlier example, we may calculate a number of descriptive statistics on the topology of the two networks. For node degree, $\hat{H}_{norm} = 0.05$ and $\hat{H}_{TRC} = 0.07$ for the normal and TRC networks, respectively (with $\hat{\sigma}^4_{norm} = 2.36$ and $\hat{\sigma}^4_{TRC} = 3.66$, and $\hat{\sigma}^3_{norm} = 8.99$ and $\hat{\sigma}^4_{TRC} = 18.13$, respectively). Exhibit 8 compares the

Exhibit 6

TRC Network Representation of MMF Exposures to FIs by Domicile



Notes: Node size is calculated based on the total TRC-weighted exposure of an MMF to all FIs in a given domicile, while link size and color are proportional to the total size of all exposures from a specific FI in a domicile to a specific MMF. The data shown here only represent a subset of the total exposures.

networks. This aggregation approach could naturally be extended along other dimensions of aggregation, such as asset class, institution type, sector, security type, and so forth, as well as by aggregating the investors by different classes of interest.

More generally, the topology can be extended in a number of ways. In our examples, we considered only bipartite networks. With sufficient information, however, this could be extended to allow richer representations. For instance, many FIs also use MMFs to hold their own and their clients' cash deposits. Even more broadly, it would be informative to consider the portfolios of the FIs themselves (perhaps in a nested fashion), and our representation could potentially accommodate this.[4]

In addition, a time-series analysis of the networks and their properties, as suggested by Billio et al. [2012], would likely provide much better benchmarking and

Exhibit 7

Domicile Ranking According to their Weighted Out-Degree in the Normal and TRC Networks. (Between network Spearman rank correlation = 0.32 for all domiciles)

Rank	Normal	TRC
1	France	UK
2	UK	France
3	Canada	**Japan**
4	Australia	Germany
5	Germany	**Netherlands**

Note: Bold typeface indicates a domicile in the top 5 in the TRC network but not in the normal network.

Exhibit 8

Summary Statistics for Domicile-Based Normal and TRC Networks (# nodes = 63, # edges = 385)

	Normal	TRC
Node Degree Heterogeneity		
H	0.05	0.07
Skewness	2.36	3.66
Kurtosis	8.99	18.18
Between Network Correlation		
Node Rank Correlation	0.32	

calibration of the approach than the single-period static analysis we demonstrate here. As a monitoring tool, variations in the levels of various summary statistics (e.g., H, σ^3, σ^4) would likely be of great interest.

Furthermore, given that the bulk of the processes underlying our analysis may be automated, the analytics we have discussed to summarize TRC networks may provide useful screening statistics that could be used as warning flags for those focused on systemic risk. Indeed, a variety of network representations, markets, and relationships may be useful in scanning for systemic events. Generating this analysis in an automated fashion would permit investigators to monitor markets and institutions more broadly, focusing in more detail on those situations in which a network summary statistic appears to be of interest. In fact, generation of the complete network graph may be less useful in many situations. To fully realize this benefit, however, more work is needed to refine these topological measures to accommodate the nature of TRC networks.

Finally, although beyond the scope of this article, we note that the TRC network approach can also be extended to accommodate other risk factors, including liquidity or credit measures such as CDS prices.

Qualifications

Although our example is illustrative, it is important to highlight some potential pitfalls in the application of our approach. One issue relates to the quality of the data currently available on various counterparty relationships. The main goal of this article has been to demonstrate the insights that are feasible using the proposed approach, rather than to perform a rigorous analysis of the MMF domain. In fact, because of the limited ability to link data (e.g., as a result of reporting issues in the cases of some funds, we were not always able to easily form family trees or rigorously enforce entity consistency in our dataset), our results may not always be actionable in their current form.

A second and more substantive issue in our example relates to the potential for large issuers to be omitted from the portfolio because their securities are not held by any of the funds in our sample (recall that we limited our MMF sample to U.S. prime 2a-7 funds). Although the MMFs are unaffected by the distress of those issuers whose securities they do not hold, the TRC calculations for the issuers of securities they do hold may well be affected by the omitted issuers, if those issuers are large.

For example, a 2a-7 fund may not hold debt of a large Asian financial insurance company (particularly if the insurance company tended to issue long-dated debt). This insurance company, however, may be highly correlated to other FIs and may thus affect the TRC of the issuers of the securities that the 2a-7 fund does hold. Thus, although omitting the large Asian insurance firm from the portfolio may make sense from a holdings perspective, it may not make sense when calculating the systemic TRC of the FIs held by the MMFs. Note, however, that because thousands of financial institutions borrow in the capital markets, it may be counterproductive to calculate TRC for all of these entities for each network. A reasonable remedy for this mismatch could be to first define a fixed number (e.g., 250) of the most systemically important institutions, for example, and then to proceed using this sample as the systemic portfolio.

In the most general sense, we view the TRC network approach as useful primarily for macroprudential risk measurement and topological analysis rather than for producing specific point predictions. Indeed, limitations on data as well as the structure and parameter estimates of the portfolio model will limit the degree to which a specific network view of systemic risk faithfully represents the underlying dynamics of the firms in the network (though no more so than many other approaches to systemic risk assessment).

CONCLUSION

We have introduced a new type of systemic risk analytic—TRC networks—that draws on insights from both the credit portfolio management and network analysis domains. The approach enjoys some of the economic intuition that accompanies the study of portfolios while also benefiting from the systems' perspective that underlies network frameworks. We view our approach as primarily a visualization tool at this point, although, with additional work, we expect that the quantitative measures of network topology will also provide useful screening tools. We presented one example of our approach using data on MMFs and large FIs, but our method can be implemented using publicly available data in many cases and is amenable to automation, allowing it to be scaled to examine a broad set of areas.

Combining credit portfolio analytics and network analysis is natural in that a central concern in systemic risk analysis is the impact of the failure of one or more SIFIs. The explicit focus of credit analysis is a firm's failure probability and the correlation of one firm's failure with the failures of other firms. Credit portfolio analysis provides an elegant framework for separating idiosyncratic failures from those that are more likely to occur in periods of system-wide stress.

We have found that network methods informed by the overlay of credit portfolio analytics provide a clearer, more concise, and potentially more actionable set of observations than either credit portfolio-referent measures or network-based representations alone. Of course, it would be naive to consider the credit TRC-based network analysis to be a complete one. Little guidance exists, for example, on how to think about issues such as liquidity or market seizures. More fundamentally, there is no reason to believe that simulations of a systemic portfolio based on historical data will provide a complete picture of the behavior of markets and institutions in times of extreme stress. Nonetheless, the approach can fill in details along a number of useful dimensions for which current approaches are less informative. In addition, the network representation provides a consistent framework for examining both portfolio-analytic and scenario-based evaluations of systemic risk.

More generally, aggregation along other dimensions of interest (e.g., instrument type, asset class, etc.) may be useful for different applications, as could representations involving alternative systemic portfolio risk measures such as marginal expected shortfall, principal components loadings (Billio et al. [2012]), or the credit absorption ratio (Reyngold, Shnyra, and Stein [2015]). Such a structure would also permit the use of additional network measures (e.g., measures of centrality).

Understanding systemic risk is crucial for maintaining financial stability, especially during times of crisis. TRC networks represent one special case of a much broader class of analytics that employ various types of portfolio-referent measures to inform and expand the relevance of network representations.

ENDNOTES

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[1]See, for example, Watts [2002]; Newman [2006]; Lee, Kim, and Jung [2006]; Boccaletti et al. [2006]; Ortega, Sola, and Pastor [2008]; Tumminello et al. [2011]; Kenett et al. [2011]; Madi et al. [2011]; Gao et al. [2011]; and Cohen and Havlin [2010].

[2]See Billio et al. [2012]; Tumminello, Lillo, and Mantegna [2010]; Kenett [2010]; Schweitzer et al. [2009]; Mantegna [1999].

[3]In this article, we used PDs provided by Moody's Analytics. These are calculated using the Vasicek-Kealhofer model (Kealhofer [2003]). Of particular interest is an extension that is estimated specifically to accommodate the unique business model of many financial firms. This business model creates a volatility profile that is different than many other corporations in that the firm volatility is driven both by the volatility of an institution's financial portfolios as well as the volatility of the income derived from the substantial services franchises (e.g., underwriting and lending, issuance letters of credit, etc.).

[4]Naturally, this representation introduces additional complexity in that the default of one issuer in period t could materially change the default probability of a counterparty in period $t + 1$ if the counterparty had a large exposure to the issuer. Such dynamic modeling creates a number of challenges that are beyond the scope of this article.

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