What Happened To The Quants In August 2007?: Evidence from Factors and Transactions Data*

Amir E. Khandani† and Andrew W. Lo‡

First Draft: April 15, 2008
Latest Revision: October 23, 2008

Abstract

During the week of August 6, 2007, a number of quantitative long/short equity hedge funds experienced unprecedented losses. It has been hypothesized that a coordinated deleveraging of similarly constructed portfolios caused this temporary dislocation in the market. Using the simulated returns of long/short equity portfolios based on five specific valuation factors, we find evidence that the unwinding of these portfolios began in July 2007 and continued until the end of 2007. Using transactions data, we find that the simulated returns of a simple marketmaking strategy were significantly negative during the week of August 6, 2007, but positive before and after, suggesting that the Quant Meltdown of August 2007 was the combined effects of portfolio deleveraging throughout July and the first week of August, and a temporary withdrawal of marketmaking risk capital starting August 8th. Our simulations point to two unwinds—a mini-unwind on August 1st starting at 10:45am and ending at 11:30am, and a more sustained unwind starting at the open on August 6th and ending at 1:00pm—that began with stocks in the financial sector and long Book-to-Market and short Earnings Momentum. These conjectures have significant implications for the systemic risks posed by the hedge-fund industry.

*The views and opinions expressed in this article are those of the authors only, and do not necessarily represent the views and opinions of AlphaSimplex Group, MIT, any of their affiliates and employees, or any of the individuals acknowledged below. The authors make no representations or warranty, either expressed or implied, as to the accuracy or completeness of the information contained in this article, nor are they recommending that this article serve as the basis for any investment decision—this article is for information purposes only. We thank Paul Bennett, Kent Daniel, Pankaj Patel, Steve Poser, Li Wei, Souheang Yao, and participants at the 2008 Q-Group Conference, the JOIM 2008 Spring Conference, the NBER 2008 Risk and Financial Institutions Conference, the NY Stern School Finance Seminar, and the 2008 QWAFAFEW Boston Meeting for helpful comments and discussion. Research support from AlphaSimplex Group and the MIT Laboratory for Financial Engineering is gratefully acknowledged.

†Graduate Student, Department of Electrical Engineering and Computer Science, and Laboratory for Financial Engineering, MIT.
‡Harris & Harris Group Professor, MIT Sloan School of Management; director, MIT Laboratory for Financial Engineering; and Chief Scientific Officer, AlphaSimplex Group, LLC. Please direct all correspondence to: Andrew W. Lo, MIT Sloan School of Management, 50 Memorial Drive, E52–454, Cambridge, MA 02142.
Contents

1 Introduction and Summary ........................................... 1

2 Literature Review .................................................. 4

3 The Data
   3.1 Compustat Data .................................................. 6
   3.2 TAQ Transactions Data ......................................... 7

4 Factor Portfolios .................................................. 8
   4.1 Factor Construction ............................................. 11
   4.2 Market Behavior in 2007 ....................................... 13
   4.3 Evidence from Transactions Data ............................. 16

5 Measures of Market Liquidity ...................................... 19
   5.1 Marketmaking and Contrarian Profits ......................... 19
   5.2 Market Liquidity: 1995 to 2007 ............................. 28
   5.3 Market Liquidity in 2007 ..................................... 31
   5.4 Determining the Epicenter of the Quake .................... 39

6 Conclusions ......................................................... 45

A Appendix
   A.1 Expected Profits for Stationary Returns ..................... 48
   A.2 Expected Profits for a Linear Factor Model ................ 50
   A.3 Extreme Movers on August 6, 2007 ........................ 52

References ......................................................... 55
1 Introduction and Summary

During the first half of 2007, events in the U.S. sub-prime mortgage markets affected many parts of the financial industry, setting the stage for more turmoil in the fixed-income and credit world. Apart from stocks in the financial sector, equity markets were largely unaffected by these troubles. With the benefit of hindsight, however, signs of macro stress and shifting expectations of future economic conditions were apparent in equity prices during this period. In July 2007, the performance of certain well-known equity-valuation factors such as Fama and French’s Small-Minus-Big (SMB) market-cap and High-Minus-Low (HML) Book-to-Market factors began a downward trend, and while this fact is unremarkable in and of itself, the events that transpired during the second week of August 2007 have made it much more meaningful.

Starting on Monday, August 6th and continuing through Thursday, August 9th, some of the most successful equity hedge funds in the history of the industry reported record losses. But what made these losses even more extraordinary was the fact that they seemed to be concentrated among quantitatively managed equity market-neutral or “statistical arbitrage” hedge funds, giving rise to the monikers “Quant Meltdown” and “Quant Quake” of 2007.

In Khandani and Lo (2007), we analyzed the Quant Meltdown of 2007 by simulating the returns of a specific equity market-neutral strategy—the contrarian trading strategy of Lehmann (1990) and Lo and MacKinlay (1990)—and proposed the “Unwind Hypothesis” to explain the empirical facts (see also Goldman Sachs Asset Management, 2007, and Rothman 2007a–c). This hypothesis suggests that the initial losses during the second week of August 2007 were due to the forced liquidation of one or more large equity market-neutral portfolios, primarily to raise cash or reduce leverage, and the subsequent price impact of this massive

\[1\]

For example, the Wall Street Journal reported on August 10, 2007 that “After the close of trading, Renaissance Technologies Corp., a hedge-fund company with one of the best records in recent years, told investors that a key fund has lost 8.7% so far in August and is down 7.4% in 2007. Another big fund company, Highbridge Capital Management, told investors its Highbridge Statistical Opportunities Fund was down 18% as of the 8th of the month, and was down 16% for the year. The $1.8 billion publicly traded Highbridge Statistical Market Neutral Fund was down 5.2% for the month as of Wednesday... Tykhe Capital, LLC—a New York-based quantitative, or computer-driven, hedge-fund firm that manages about $1.8 billion—has suffered losses of about 20% in its largest hedge fund so far this month...” (see Zuckerman, Hagerty, and Gauthier-Villars, 2007), and on August 14, the Wall Street Journal reported that the Goldman Sachs Global Equity Opportunities Fund “...lost more than 30% of its value last week...” (Sender, Kelly, and Zuckerman, 2007).
and sudden unwinding caused other similarly constructed portfolios to experience losses. These losses, in turn, caused other funds to deleverage their portfolios, yielding additional price impact that led to further losses, more deleveraging, and so on. As with Long Term Capital Management (LTCM) and other fixed-income arbitrage funds in August 1998, the deadly feedback loop of coordinated forced liquidations leading to deterioration of collateral value took hold during the second week of August 2007, ultimately resulting in the collapse of a number of quantitative equity market-neutral managers, and double-digit losses for many others.

This Unwind Hypothesis underscores the apparent commonality among quantitative equity market-neutral hedge funds and the importance of liquidity in determining market dynamics. We focus on these twin issues in this paper by simulating the performance of typical mean-reversion and valuation-factor-based long/short equity portfolios, and by using transactions data during the months surrounding August 2007 to measure market liquidity and price impact before, during, and after the Quant Meltdown. With respect to the former simulations, we find that during the month of July 2007, portfolios constructed based on traditional equity-valuation factors (Book-to-Market, Earnings-to-Price and Cashflow-to-Market) steadily declined, while portfolios constructed based on “momentum” metrics (Price Momentum and Earnings Momentum) increased. With respect to the latter simulations, we find that intra-daily liquidity in U.S. equity markets declined significantly during the second week of August, and that the expected return of a simple mean-reversion strategy increased monotonically with the holding period during this time, i.e., those marketmakers that were able to hold their positions longer received higher premiums. The shorter-term losses also imply that marketmakers reduced their risk capital during this period. Together, these results suggest that the Quant Meltdown of August 2007 began in July with the steady unwinding of one or more factor-driven portfolios, and this unwinding caused significant dislocation in August because the pace of liquidation increased and because liquidity providers decreased their risk capital during the second week of August.

If correct, these conjectures highlight additional risks faced by investors in long/short equity funds, namely “tail risk” due to occasional liquidations and deleveraging that may be motivated by events completely unrelated to equity markets. Such risks also imply that long/short equity strategies may contribute to systemic risk because of their ubiquity, their
importance to market liquidity and price continuity, and their impact on market dynamics when capital is suddenly withdrawn.

As in Khandani and Lo (2007), we wish to acknowledge at the outset that the hypotheses advanced in this paper are speculative, tentative, and based solely on indirect evidence. Because the events surrounding the Quant Meltdown involve hedge funds, proprietary trading desks, and their prime brokers and credit counterparties, primary sources are virtually impossible to access. Such sources are not at liberty to disclose any information about their positions, strategies, or risk exposures, hence the only means for obtaining insight into these events are indirect. However, in contrast to our earlier claim in Khandani and Lo (2007) that “...the answer to the question of what happened to the quants in August 2007 is indeed known, at least to a number of industry professionals who were directly involved...”, we now believe that industry participants directly involved in the Quant Meltdown may not have been fully aware of the broader milieu in which they were operating. Accordingly, there is indeed a role for academic studies that attempt to piece together the various components of the market dislocation of August 2007 by analyzing the simulated performance of specific investment strategies like the strategies considered in this paper and in Khandani and Lo (2007).

Nevertheless, we recognize the challenges that outsiders face in attempting to understand such complex issues without the benefit of hard data, and emphasize that our educated guesses may be off the mark given the limited data we have to work with. We caution readers to be appropriately skeptical of our hypotheses, as are we.

We begin in Section 2 with a brief review of the literature. The data we use to construct our valuation factors and perform our strategy simulations are described in Section 3. The factor definitions and the results of the factor-based simulations are contained in Section 4. In Section 5, we use two alternate measures of market liquidity to assess the evolution of liquidity in the equity markets since 1995, and how it changed during the Quant Meltdown of 2007. Using these tools, we are able to pinpoint the origins of the Meltdown to a specific date and time, and even to particular groups of stocks. We conclude in Section 6.
2 Literature Review

Although the focus of our study is the Quant Meltdown of August 2007, several recent papers have considered the causes and inner workings of the broader liquidity and credit crunch of 2007–2008. For example, Gorton (2008) discusses the detail of security design and securitization of sub-prime mortgages and argues that lack of transparency arising from the interconnected link of securitization is at the heart of the problem. Brunnermeier (2008) argues that the mortgage-related losses are relatively small. For example, he indicates that the total expected losses are about the same amount of wealth lost in a non-so-uncommon 2% to 3% drop in the U.S. stock market. Starting from this observation, he emphasizes the importance of the amplification mechanism at play, and argues that borrowers’ deteriorating balance sheets generate liquidity spirals from relatively small shocks. Once started, these spirals continue as lower asset prices and higher volatility raise margin levels and lower available leverage. Adrian and Shin (2008) document a pro-cyclical relationship between the leverage of U.S. investment banks and the sizes of their balance sheets and explore the aggregate effects that such a relationship can have on asset prices and the volatility risk premium. This empirical observation increases the likelihood of Brunnermeier’s (2008) margin and deleveraging spiral. Allen and Carletti (2008) provide a more detailed analysis of the role of liquidity in the financial crisis and consider the source of the current “cash-in-the-market” pricing, i.e. market prices that are significantly below what plausible fundamentals would suggest.

Following the onset of the credit crunch in July 2007, beginning on August 6th, many equity hedge funds reported significant losses and much of the blame was placed on quantitative factors, or the “Quants”, as the most severe losses appear to have been concentrated among quantitative hedge funds. The research departments of the major investment banks were quick to produce analyses, e.g., Goldman Sachs Asset Management (2007) and Rothman (2007a,b,c), citing coordinated losses among portfolios constructed according to several well-known quant factors, and arguing that simultaneous deleveraging and a lack of liquidity were responsible for these losses. For example, the study by Rothman (2007a)—which was first released on August 9, 2007—reports the performance of a number of quant factors and attributes the simultaneous bad performance to “a liquidity based deleveraging phenomena”.

4
Goldman Sachs Asset Management (2007) provide additional evidence from foreign equity markets (Japan, U.K., and Europe-ex-U.K.), indicating that the unwinds involved more than just U.S. securities. In a follow-up study, Rothman (2007b) called attention to the perils of endogenous risk; in referring to the breakdown of the risk models during that period, he concluded that: “By and large, they understated the risks as they were not calibrated for quant managers/models becoming our own asset class, creating our own contagion”.\(^2\) Using TASS hedge-fund data and simulations of a specific long/short equity strategy, Khandani and Lo (2007) hypothesized that the losses were initiated by the rapid “unwind” of one or more sizable quantitative equity market-neutral portfolios. Given the speed and price impact with which this occurred, we argued that it was likely the result of a forced liquidation by a multi-strategy fund or proprietary-trading desk, possibly due to a margin call or a risk reduction. These initial losses then put pressure on a broader set of long/short and long-only equity portfolios, causing further losses by triggering stop/loss and deleveraging policies. A significant rebound of these strategies occurred on August 10th, which is also consistent with the unwind hypothesis (see, also, Goldman Sachs Asset Management, 2007, and Rothman, 2007c).

In its conclusion, the Goldman Sachs Asset Management (2007) study suggests that “…it is not clear that there were any obvious early warning signs… No one, however, could possibly have forecasted the extent of deleveraging or the magnitude of last weeks factor returns”. Our analysis suggests that the dislocation was exacerbated by the withdrawal of marketmaking risk capital—possibly by high-frequency hedge funds—starting on August 8th. This highlights the endogenous nature of liquidity risk and the degree of interdependence among market participants, or “species” in the terminology of Farmer and Lo (1999). The fact that the ultimate origins of this dislocation were apparently outside the long/short equity sector—most likely in a completely unrelated set of markets and instruments—suggests that systemic risk in the hedge-fund industry has increased significantly in recent years.

In this paper, we turn our attention to the impact of quant factors before, during, and after the Quant Meltdown, using a set of the most well-known factors from the academic “anomalies” literature such as Banz (1981), Basu (1983), Bahandari (1988), and Jegadeesh

\(^2\)See also Montier (2007).
and Titman (1993). Although the evidence for some of these anomalies is subject to debate,\(^3\) nevertheless they have resulted in various multi-factor pricing models such as the widely cited Fama and French (1993) three-factor model. We limit our attention to five factors: three value-factors similar to those in Lakonishok, Shleifer, and Vishny (1994), and two momentum factors as in Chan, Jegadeesh and Lakonishok (1996), and describe their construction in Sections 3 and 4.

3 The Data

We use three sources of data for our analysis. Annual and quarterly balance-sheet information from Standard & Poor’s Compustat database is used to create various valuation factors for the members of the S&P 1500 index in 2007. To study market microstructure effects, we use the Trades and Quotes (TAQ) dataset from the New York Stock Exchange (NYSE). In addition, we use daily stock returns and volume from the University of Chicago’s Center for Research in Security Prices (CRSP) to calculate the daily returns of various long/short portfolios and their trading volumes. Sections 3.1 and 3.2 contain brief overviews of the Compustat and TAQ datasets, respectively, and we provide details for the CRSP dataset throughout the paper as needed.

3.1 Compustat Data

Balance-sheet information is obtained from Standard & Poor’s Compustat database via the Wharton Research Data Services (WRDS) platform. We use the “CRSP/Compustat Merged Database” to map the balance-sheet information to CRSP historical stock returns data. From the annual Compustat database, we use:

- **Book Value Per Share** (item code BKVLPS)
- **Basic Earnings Per Share Excluding Extraordinary Items** (item code EPSPX)
- **Net Cashflow of Operating Activities** (item code OANCF)
- **Fiscal Cumulative Adjustment Factor** (item code ADJEX,F)

We also use the following variables from the quarterly Compustat database:

\(^3\)See, for example, Fama and French (2006), Lewellen and Nagel (2006), and Ang and Chen (2007).
• Quarterly Basic Earnings Per Share Excluding Extraordinary Items (item code EPSPXQ)
• Cumulative Adjustment Factor by Ex-Date (item code ADJEX)
• Report Date of Quarterly Earnings (item code RDQ)

There is usually a gap between the end of the fiscal year or quarter and the date that the information is available to the public. We implement the following rules to make sure any information used in creating the factors is, in fact, available on the date that the factor is calculated. For the annual data, a gap of at least 4 months is enforced (for example, an entry with date of December 2005 is first used starting in April 2006) and to avoid using old data, we exclude data that are more than 1 year and 4 months old, i.e., if a security does not have another annual data point after December 2005, that security is dropped from the sample in April 2007). For the quarterly data, we rely on the date given in Compustat for the actual reporting date (item code RDQ, Report Date of Quarterly Earnings) to ensure that the data is available on the portfolio construction date. For the handful of cases that RDQ is not available, we employ an approach similar to that taken for the annual data. In those cases, to ensure that the quarterly data is available on the construction date and not stale, the quarterly data is used with a 45-day gap and any data older than 135 days is not used (for example, to construct the portfolio in April 2007, we use data from December 2006, and January or February 2007, and do not use data from April or March 2007).

3.2 TAQ Transactions Data

The NYSE Trade and Quote (TAQ) database contains intra-day transactions and quotes data for all securities listed on the NYSE, the American Stock Exchange (AMEX), the National Market System (NMS), and SmallCap issues. The dataset consists of the Daily National Best Bids and Offers (NBBO) File, the Daily Quotes File, the Daily TAQ Master File, and the Daily Trades File. For the purposes of this study, we only use actual trades as reported in the Daily Trades File. This file includes information such as the security symbol, trade time, size, exchange on which the trade took place, as well as a few condition and correction flags. We only use trades that occur during normal trading hours (9:30am to 4:00pm). We also discarded all records that have a Trade Correction Indicator field entries
other than “00”\textsuperscript{4} and removed all trades that were reported late or reported out of sequence, according to the Sale Condition field.\textsuperscript{5} During the 63 trading days of our sample of TAQ data from July 2, 2007 to September 28, 2007, the stocks within the universe of our study—the S&P 1500—yielded a total of approximately 805 million trades, ranging from a low of 4.9 million trades on July 3, 2007 to a high of 23.7 million trades on August 16, 2007. The cross-sectional variation of the number of trades was quite large; for example, there were approximately 11 million trades in Apple (AAPL) during our sample period while Lawson Products (LAWS) was only traded 6,830 times during the same period. On average, we analyzed approximately 11.3 million trades per day to develop our liquidity measures.

Using transactions prices in the Daily Trades File, we construct 5-minute returns within each trading day (no overnight returns are allowed) based on the most recent transactions price within each 5-minute interval, subject to the filters described above. These returns are the inputs to the various strategy simulations reported in Section 4 and 5. For the estimation of price-impact coefficients in Section 5.3, transactions prices are used, again subject to the same filters described above.

### 4 Factor Portfolios

To study the Quant Meltdown of August 2007, we use the returns of several long/short equity market-neutral portfolios based on the kinds of quantitative processes and factors that might be used by quant funds. For example, it is believed that the value premium is a proxy for a market-wide distress factor (see Fama and French, 1992).\textsuperscript{6} Fama and French (1995) note that the typical value stock has a price that has been driven down due to financial distress. This observation suggests a direct explanation of the value premium: in the event of a credit crunch, stocks in financial distress will do poorly, and this is precisely when investors are

\textsuperscript{4}According to the TAQ documentation, a Trade Correction Indicator value of “00” signifies a regular trade which was not corrected, changed or canceled. This field is used to indicate trades that were later signified as errors (code “07” or “08”), canceled records (code “10”), as well as several other possibilities. Please see the TAQ documentation for more details.

\textsuperscript{5}These filters have been used in other studies based on TAQ data; see, for example, Christie, Harris and Schultz (1994) or Chordia, Roll and Subrahmanyam (2001). See the TAQ documentation for further details.

\textsuperscript{6}Kao and Shumaker (1999) document some intuitive links between macro factors and the return for value stocks. Of course, the problem can be turned on its head and stock returns can be used to predict future macro events, as in Liew and Vassalou (2000) and Vassalou (2003). Behavioral arguments are also used to explain the apparent premium for the value factors (see Lakonishok, Shleifer, and Vishny, 1994).
least willing to put money at risk.\textsuperscript{7}

![Cumulative Return Graph](image)

**Figure 1:** The cumulative daily returns for the Market, Small Minus Big (SMB), High Minus Low (HML), Momentum factors as well as the contrarian strategy of Lehmann (1990) and Lo and MacKinlay (1990) for January 3, 2007 to December 31, 2007. Data for Market, SMB, HML and Momentum factors were obtained from Kenneth French’s website (please see footnote 8 for details). The contrarian strategy was implemented as in Khandani and Lo (2007) using daily return for stocks listed in the S&P 1500 on January 3, 2007. Volume data was obtained from the NYSE website.

By simulating the returns of a portfolio formed to highlight such factors, we may be able to trace out the dynamics of other portfolios with similar exposures to these factors. An example of this approach is given in Figure 1, which contains the cumulative returns of the Fama and French SMB and HML portfolios as well as a Price-Momentum factor portfolio during 2007.\textsuperscript{8} Trading volume during this period also shows some unusual patterns, giving some support to the Unwind Hypothesis mentioned above. During the week of July 23, 2007, volume began building to levels well above normal.\textsuperscript{9}

\textsuperscript{7}One should note that the “distress” of an individual firm cannot be treated as a risk factor since such distress is idiosyncratic and can be diversified away. Only aggregate events that a significant portion of the population of investors care about will result in a risk premium.

\textsuperscript{8}Data was obtained from the data library section of Kenneth French’s web site:

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

Please refer to the documentation available from that site for further details.

\textsuperscript{9}The first day with extremely high volume is June 22, 2007, which was the re-balancing day for all Russell
weeks of July 23, July 30, August 6, and August 13 reached record levels of 2.9, 3.0, 3.6, and 3.1 billion shares, respectively, before finally returning to a more normal level of 1.9 billion shares during the week of August 20.

Figure 1 also displays the cumulative return of Lehmann’s (1990) and Lo and MacKinlay’s (1990) short-term mean-reversion or “contrarian” strategy which was used by Khandani and Lo (2007) to illustrate the Quant Meltdown of 2007. The sudden drop and recovery of this strategy during the week of August 6th, following several weeks of lower than expected performance, captures much of the dislocation during this period.

To develop some intuition for this dislocation, consider the underlying economic motivation for the contrarian strategy. By taking long positions in stocks that have declined and short positions in stocks that have advanced over the previous trading day, the strategy actively provides liquidity to the marketplace. By implicitly making a bet on daily mean reversion among a large universe of stocks, the strategy is exposed to any continuation or persistence in the daily returns, i.e., price trends or momentum. Broad-based momentum across a group of stocks can arise from a large-scale liquidation of a portfolio that may take several days to complete, depending on the size of the portfolio and the urgency of the liquidation. In short, the contrarian strategy under-performs when the usual mean reversion in stock prices is replaced by a momentum, possibly due to a sizable and rapid liquidation. We will elaborate on this theme in Section 5.1.

In Section 4.1, we describe five specific factors that we propose for capturing the events of August 2007, and in Section 4.2 we present the simulations for these factor portfolios before, during, and after the Quant Meltdown.

---

10 Components of the S&P 1500 as of January 3, 2007 are used. Strategy holdings are constructed and the daily returns are calculated based on the **Holding Period Return** from the CRSP daily returns file. See Khandani and Lo (2007) for further details.

11 By definition, losers are stocks that have under-performed relative to some market average, implying a supply/demand imbalance in the direction of excess supply that has caused the prices of those securities to drop, and vice-versa for the winners. By buying losers and selling winners, the contrarians are adding to the demand for losers and increasing the supply of winners, thereby stabilizing supply/demand imbalances.

12 Note that positive profits for the contrarian strategy may arise from sources other than mean reversion. For example, positive lead-lag relations across stocks can yield contrarian profits (see Lo and MacKinlay, 1990 for details).
### 4.1 Factor Construction

We focus our analysis on five of the most studied and most highly cited quantitative equity valuation factors: three value measures, Price Momentum, and Earnings Momentum. The three value measures, Book-to-Market, Earnings-to-Price, and Cashflow-to-Market, are similar to the factors discussed in Lakonishok, Shleifer, and Vishny (1994). These factors are based on the most recent annual balance-sheet data from Compustat and constructed according to the procedure described below. The two remaining factors—Price Momentum and Earnings Momentum—have been studied extensively in connection with momentum strategies (see for example Chan, Jegadeesh and Lakonishok, 1996). The Earnings Momentum factor is based on quarterly earnings from Compustat, while the Price Momentum factor is based on the reported monthly returns from the CRSP database. At the end of each month, each of these five factors is computed for each stock in the S&P 1500 index using the following procedure:

1. The Book-to-Market factor is calculated as the ratio of the **Book Value Per Share** (item code BKVLPs in Compustat) reported in the most recent annual report (subject to the availability rules outlined in Section 3.1) divided by the closing price on the last day of the month. Share adjustment factor from CRSP and Compustat are used to correctly reflect changes in the number of outstanding common shares.

2. The Earnings-to-Price factor is calculate based on the **Basic Earnings Per Share Excluding Extraordinary Items** (item code EPSPX in Compustat) reported in the most recent annual report (subject to the availability rules outlined in Section 3.1) divided by the closing price on the last day of the month. Share adjustment factor available in CRSP and Compustat are used to correctly reflect stock splits and other changes in the number of outstanding common shares.

3. The Cashflow-to-Market factor is calculated based on the **Net Cashflow of Operating Activities** (item code OANCF in Compustat) reported in the most recent annual data (subject to the availability rules outlined in Section 3.1) divided by the total market cap of common equity on the last day of the month. Number of shares outstanding and the closing price reported in CRSP files are used to calculate the total market
value of common equity.

4. The Price Momentum factor is the stock’s cumulative total return (calculated using holding period return from CRSP files which includes dividends) over the period spanning the previous 2 to 12 months.\textsuperscript{13}

5. The Earnings-Momentum factor is constructed based on \textbf{Quarterly Basic Earnings Per Share Excluding Extraordinary Item} (item code EPSPXQ in Compustat) using the standardized unexpected earnings, SUE. The SUE factor is calculated as the ratio of the earnings growth in the most recent quarter (subject the availability rules outlined in Section 3.1) relative to the year earlier divided by the standard deviation of the same factor calculated over the prior 8 quarters (see Chan, Jegadeesh and Lakonishok, 1996, for a more detailed discussion of this factor).

At the end of each month during our sample period, we divide the S&P 1500 universe into 10 deciles according to each factor. Decile 1 will contain the group of companies with the lowest value of the factor; for example, companies whose stocks have performed poorly in the last 2 to 12 month will be in the first decile of the Price Momentum factor. Deciles 1 through 9 will have the same number of stocks and decile 10 may have a few more if the original number of stocks was not divisible by 10. We do not require a company to have data for all five factors or to be a U.S. common stock to be used in each ranking. However, we use only those stocks that are listed as U.S. common shares (CRSP Share Code “10” or “11”) to construct portfolios and analyze returns.\textsuperscript{14} For example, if a company does not have 8 quarters of earnings data, it cannot be ranked according to the Earnings Momentum factor, but it will still be ranked according to other measures if the information required for calculating those measures is available.

This process yields decile rankings for each of these factors for each month of our sample. In most months, we have the data to construct deciles for more than 1,400 companies. However, at the time we obtained the Compustat data for this analysis, the Compustat

\textsuperscript{13}The most recent month is not included, similar to the Price-Momentum factor available on Kenneth French’s data library (see footnote 8).

\textsuperscript{14}This procedure should not impact our analysis materially as there are only 50 to 60 stocks in the S&P indexes without these share codes, and these are typically securities with share code “12”, indicating companies incorporated outside the U.S.
database was still not fully populated with the 2007 quarterly data; in particular, the data for the quarter ending September 2007 (2007Q3) was very sparse. Given the 45-day lag we employ for quarterly data, the lack of data for 2007Q3 means that the deciles can be formed for only about 370 companies at the end of November 2007 (the comparable count was 1,381 in October 2007 and 1,405 at the end of September 2007). Since any analysis of factor models for December 2007 is impacted by this issue, we will limit all our study to the first 11 months of 2007.

Given the decile rankings of the five factors, we can simulate the returns of portfolios based on each of these factors. In particular, for each of the five factors, at the end of each month in our sample period, we construct a long/short portfolio by investing $1 long in the stocks in the 10th decile and investing $1 short in the stocks in the 1st decile of that month. Each $1 investment is distributed using equal weights among stocks in the respective decile and each portfolio is purchased at the closing price on the last trading day of the previous month. For the daily analysis, the cumulative return is calculated using daily returns based on the Holding Period Return available from CRSP daily returns files. For the intra-day return analysis, we compute the value of long/short portfolios using the most recent transactions price in each 5-minute interval based on TAQ Daily Trades File (see Section 3.2 for details). We use the Cumulative Factor to Adjust Price (CFACPR) from the CRSP daily files to adjust for stock splits, but do not adjust for dividend payments.\footnote{Our intra-day returns are unaffected by dividend payments, hence our analysis of marketmaking profits and price-impact coefficients should be largely unaffected by omitting this information. However, when we compute cumulative returns for certain strategies that involve holding overnight positions, small approximation errors may arise from the fact that we do not take dividends into account when using transactions data.}

Each portfolio is rebalanced on the last trading day of each month, and a new portfolio is constructed. For a few rare cases where a stock stops trading during the month, we assume that the final value of the initial investment in that stock is kept in cash for the remainder of the month.

4.2 Market Behavior in 2007

Figure 2 contains the daily cumulative returns for each of the five factors in 2007 through the end of November. The results are consistent with the patterns in Figure 1—the three value factors began their downward drift at the start of July 2007, consistent with the HML
factor-portfolio returns in Figure 1. On the other hand, the two momentum factors were the two best performers over the second half of 2007, again consistent with Figure 1. Also, the two momentum factors and the Cashflow-to-Market portfolio experienced very large drops and subsequent reversals during the second week of August 2007.

Of course, secular declines and advances of factor portfolios need not have anything to do with deleveraging or unwinding; they may simply reflect changing market valuations of value stocks, or trends and reversals that arise from typical market fluctuations. To establish a link between the movements of the five factor portfolios during July and August 2007 and the Unwind Hypothesis, we perform two cross-sectional regressions each day from January to November 2007 using daily stock returns and turnover as the dependent variables:

\[
R_{i,t} = \alpha_t + \sum_{f=1}^{5} \beta_{f,t} D_{i,f} + \epsilon_{i,t} \tag{1a}
\]

\[
TO_{i,t} = \gamma_t + \sum_{f=1}^{5} \delta_{f,t} |D_{i,f} - 5.5| + \eta_{i,t} \tag{1b}
\]

where \(R_{i,t}\) is the return for security \(i\) on day \(t\), \(D_{i,f}\) is the decile ranking of security \(i\) according to factor \(f\),\(^{16}\) and \(TO_{i,t}\), the turnover for security \(i\) on day \(t\), is defined as:\(^{17}\)

\[
TO_{i,t} \equiv \frac{\text{Number of Shares Traded for Security } i \text{ on Day } t}{\text{Number of Share Outstanding for Security } i \text{ on Day } t}.
\tag{2}
\]

If, as we hypothesize, there was a significant unwinding of factor-based portfolios in July and August 2007, the explanatory power of these two cross-sectional regressions should spike up during those months because of the overwhelming price-impact and concentrated volume of the unwind. If, on the other hand, it was business as usual, then the factors should not have any additional explanatory power during that period than any other period.

The lower part of Figure 2 displays the \(R^2\)’s for the regressions (1) each day during the __________

\(^{16}\)Note the decile rankings change each month, and they are time dependent, but we have suppressed the time subscript for notational simplicity.

\(^{17}\)Turnover is the appropriate measure for trading activity in each security because it normalizes the number of shares traded by the number of shares outstanding (see Lo and Wang, 2000, 2006). The values of the decile rankings are reflected around the “neutral” level for the turnover regressions because stocks that belong to either of the extreme deciles—deciles 1 and 10—are “equally attractive” according to each of the five factors (but in opposite directions), and should exhibit “abnormal” trading during those days on which portfolios based on such factors were unwound.
Figure 2: Cumulative performance of five long/short equity market-neutral portfolios constructed from commonly used equity-valuation factors, from January 3, 2007 to December 31, 2007. Also plotted are daily $R^2$’s and their 5-day moving averages of the following cross-sectional regressions of returns and turnover: $R_{i,t} = \alpha_t + \sum_{f=1}^{5} \beta_{f,t}D_{i,f} + \epsilon_{i,t}$, and $TO_{i,t} = \gamma_t + \sum_{f=1}^{5} \delta_{f,t}|D_{i,f} - 5.5| + \eta_{i,t}$, where $R_{i,t}$ is the return for security $i$ on day $t$, $D_{i,f}$ is the decile ranking of security $i$ according to factor $f$, and $TO_{i,t}$, the turnover for security $i$ on day $t$, is defined as the ratio of the number of shares traded to the shares outstanding.
sample period. To smooth the sampling variation of these $R^2$’s, we also display their 5-day moving average. These plots confirms that starting in late July, the turnover regression’s $R^2$ increased significantly, exceeding 10% in early August. Moreover, the turnover-regression $R^2$ continued to exceed 5% for the last three months of the our sample, a threshold that was not passed at any point prior to July 2007.

As expected, the daily return regressions typically have lower $R^2$’s, but at the same point in August 2007, the explanatory power of this regression also spiked above 10%, adding further support to the Unwind Hypothesis.

### 4.3 Evidence from Transactions Data

To develop a better sense of the market dynamics during August 2007, we construct the intra-day returns of long/short market-neutral portfolios based on the factors of Section 4.1 for the two weeks before and after August 6th. Figure 3 displays the cumulative returns of these portfolios from 9:30am on July 23rd to 4:00pm on August 17th. These patterns suggest that on August 2nd and 3rd, long/short portfolios based on Book-to-Market, Cashflow-to-Market, and Earnings-to-Price were being unwound, while portfolios based on Price Momentum and Earnings Momentum were unaffected until August 8th and 9th when they also experienced sharp losses. But on Friday, August 10th, sharp reversals in all five strategies erased nearly all of the losses of the previous four days, returning portfolio values back to their levels on the morning of August 6th.

Of course, this assumes that portfolio leverage did not change during this tumultuous week, which is an unlikely assumption given the enormous losses during the first few days. If, for example, a portfolio manager had employed a leverage ratio of 8:1 for the Book-to-Market portfolio on the morning of August 1st, he would have experienced a cumulative loss of 24% by the close of August 7th, which is likely to have triggered a reduction in leverage at that time if not before. With reduced leverage, the Book-to-Market portfolio would not have been able to recoup all of its losses, despite the fact that prices did revert back to their beginning-of-week levels by the close of August 10th.

To obtain a more precise view of the trading volume during this period, we turn to the cross-sectional regression (1) of individual turnover data of Section 4.2 on exposures to decile rankings of the five factors of Section 4.1. The estimated impact is measured in basis points
Figure 3: Cumulative returns for long/short portfolios based on five equity-valuation factors from 9:30am July 23, 2007 to 4:00pm August 17, 2007 computed from 5-minute returns using TAQ transactions data. Portfolios were rebalanced at the end of July 2007 to reflect the new factors rankings. Note that these returns are constructed under the assumption that only Reg-T leverage is used (see Khandani and Lo, 2007, for further details).
of turnover for a unit of difference in the decile ranking; for example, an estimated coefficient of 25 basis points for a given factor implies that ceteris paribus, stocks in the 10th decile of that factor had a 1% ($4 \times 25$ bps) higher turnover than stocks in the 6th decile.

![Graph showing estimated turnover impact and $R^2$ values for different valuation factors from July 23, 2007 to August 17, 2007.](image)

Figure 4: Estimated coefficients $\hat{\delta}_{f,t}$ and $R^2$ of the cross-sectional regression of daily individual-stock turnover on absolute excess decile rankings for five valuation factors from July 23, 2007 to August 17, 2007: $TO_{i,t} = \hat{\gamma}_t + \sum_{f=1}^{5} |D_{i,f} - 5.5|\hat{\delta}_{f,t} + \hat{\epsilon}_{i,t}$, where $TO_{i,t}$ is the turnover for stock $i$ on day $t$ and $D_{i,f}$ is the decile assignment for stock $i$ based on factor $f$, where the five factors are: Book-to-Market, Cashflow-to-Market, Earnings-to-Price, Price Momentum, and Earnings Momentum.

Figure 4 displays the estimated turnover impact $\hat{\delta}_{f,t}$ and $R^2$ of the daily cross-sectional regressions, which clearly shows the change in the trading activity and $R^2$ among stocks with extreme exposure to these five factors. The estimated coefficients are always positive, implying that the securities ranked as “attractive” or “unattractive” according to each of these measures, i.e., deciles 10 and 1, respectively, tend to have a higher turnover than the securities that are ranked “neutral” (deciles 5 or 6). Figure 4 shows that there was a substantial jump in the Price Momentum coefficient on August 8th, which coincides with the start of the steep losses shown in Figure 3.

The coefficients for the other factors also exhibit increases during this period, along with the $R^2$’s of the cross-sectional regressions, consistent with the Unwind Hypothesis. However, the explanatory power of these regressions and the estimated impact of the factors (other
than Price Momentum) on August 8th and 9th were not markedly different than earlier in the same week.

What changed on August 8th, 9th, and 10th that yielded the volatility spike in Figure 2? We argue in the next section that a sudden withdrawal of liquidity may be one explanation.

5 Measures of Market Liquidity

In Section 4, we have provided suggestive but indirect evidence supporting the Unwind Hypothesis for factor-based portfolios during the months of July and August 2007, but this still leaves unanswered the question of what happened during the second week of August. To address this issue head-on, in this section we focus on changes in market liquidity during 2007, and find evidence of a sharp temporary decline in liquidity during the second week of August 2007.

To measure equity-market liquidity, we begin in Section 5.1 by analyzing the contrarian trading strategy of Lehmann (1990) and Lo and MacKinlay (1990) from a marketmaking perspective, i.e., the provision of immediacy. Using analytical and empirical arguments, we conclude that marketmaking profits have declined substantially over the past decade, which is consistent with the common wisdom that increased competition—driven by a combination of technological and institutional innovations—has resulted in greater liquidity and a lower premium for liquidity provision services. We confirm this conjecture in Section 5.2 by estimating the price impact of equity trades using daily returns from 1995 to 2007, which shows a substantial increase in market depth, i.e., a reduction in the price impact of trades, in recent years. Markets were indeed much more liquid at the beginning of 2007 compared to just five years earlier. However, using transactions data for the months of July, August, and September 2007, in Section 5.3 we document a sudden and significant decrease in market liquidity in August 2007. And in Section 5.4, we use these tools to detect the exact date and time that the Meltdown started, and even the initial groups of securities that were involved.

5.1 Marketmaking and Contrarian Profits

The motivation behind the empirical analysis of this section can be understood in the context of Grossman and Miller’s (1988) model. In that framework, there are two types of
market participants—marketmakers and outside customers—and the provision of liquidity and immediacy by the marketmakers to randomly arriving outside customers generates mean-reverting prices. However, as observed by Campbell, Grossman and Wang (1993), when the price of a security changes, the change in price is partly due to new fundamental information about the security’s value, and partly due to temporary supply/demand imbalances. Although the latter yields mean-reverting prices, the former is typically modeled as a random walk where shocks are “permanent” in terms of the impulse-response function. To understand the role of liquidity in the Quant Meltdown of 2007, we need to separate these components.

Because the nature of liquidity provision is inherently based on mean reversion, i.e., buying losers and selling winners, the the contrarian strategy of Lehmann (1990) and Lo and MacKinlay (1990) is ideally suited for this purpose. As Khandani and Lo (2007) showed, a contrarian trading strategy applied to daily U.S. stock returns is able to trace out the market dislocation in August 2007, and in this section, we provide an explicit analytical explication of their results in the context of marketmaking and liquidity provision.

The contrarian strategy consists of an equal dollar amount of long and short positions across \( N \) stocks, where at each rebalancing interval, the long positions consist of “losers” (past underperforming stocks, relative to some market average) and the short positions consist of “winners” (past outperforming stocks, relative to the same market average). Specifically, if \( \omega_{i,t} \) is the portfolio weight of security \( i \) at date \( t \), then

\[
\omega_{i,t} = -\frac{1}{N} (R_{i,t-k} - R_{m,t-k}), \quad R_{m,t-k} \equiv \frac{1}{N} \sum_{i=1}^{N} R_{i,t-k}
\]

(3)

for some \( k > 0 \). Observe that the portfolio weights are the negative of the degree of outperformance \( k \) periods ago, so each value of \( k \) yields a somewhat different strategy. As in Khandani and Lo (2007), we set \( k = 1 \) day. By buying yesterday’s losers and selling yesterday’s winners at each date, such a strategy actively bets on one-day mean reversion across all \( N \) stocks, profiting from reversals that occur within the rebalancing interval. For this reason, (3) has been called a “contrarian” trading strategy that benefits from market overreaction, i.e., when underperformance is followed by positive returns and vice-versa for outperformance (see Khandani and Lo, 2007 for further details). A more ubiquitous source
of profitability for this strategy is the fact that liquidity is being provided to the marketplace, and investors are implicitly paying a fee for this service, both through the bid/offer spread and from price reversals as in the Grossman and Miller (1988) model. Historically, designated marketmakers such as the NYSE/AMEX specialists and NASDAQ dealers have played this role, but in recent years, hedge funds and proprietary trading desks have begun to compete with traditional marketmakers, adding enormous amounts of liquidity to U.S. stock markets and earning attractive returns for themselves and their investors in the process.

One additional subtlety in interpreting the profitability of the contrarian strategy (3) is the role of the time horizon over which the strategy’s profits are defined. Because the demand for immediacy arises at different horizons, disentangling liquidity shocks and informed trades can be challenging. All else equal, if marketmakers reduce the amount of capital they are willing to deploy, the price impact of a liquidity trade will be larger, and the time it takes for prices to revert back to their fundamental levels after such a trade will increase. To capture this phenomenon, we propose to study the expected profits of the contrarian trading strategy for various holding periods. In particular, consider a strategy based on the portfolio weights (3) but where the positions are held fixed for \( q \) periods. The profits for such a \( q \)-period contrarian strategy are given by:

\[
\pi_t(q) \equiv \sum_{i=1}^{N} \left( \omega_{i,t} \sum_{l=1}^{q} R_{i,t+l} \right). \tag{4}
\]

The properties of \( E[\pi_t(q)] \) under general covariance-stationary return-generating processes are summarized in the following proposition (see Appendix A.1 for the derivations):

**Proposition 1** Consider a collection of \( N \) securities and denote by \( \mathbf{R}_t \) the \((N \times 1)\)-vector of their period \( t \) returns, \([R_{1,t} \cdots R_{N,t}]'\). Assume that \( \mathbf{R}_t \) is a jointly covariance-stationary stochastic process with expectation \( E[\mathbf{R}_t] \equiv \mathbf{\mu} \equiv [\mu_1 \cdots \mu_N]' \) and autocovariance matrices \( E[(\mathbf{R}_{t-l} - \mathbf{\mu})(\mathbf{R}_t - \mathbf{\mu})'] \equiv \mathbf{\Gamma}_l \equiv [\gamma_{i,j}(l)] \). Consider a zero net-investment strategy that invests \( \omega_{i,t} \) dollars given by (3) in security \( i \). The expected profits over \( q \) periods, \( E[\pi_t(q)] \), is given

\[\text{This expression is an approximation since the return of security } i \text{ in period } t, R_{i,t}, \text{ is defined to be a simple return, and log of the sum is not equal to the sum of the logs. The approximation error is typically small, especially for short holding periods of just a few days.}\]
by:

$$E[\pi_t(q)] = M(\Gamma_1) + \cdots + M(\Gamma_q) - q \sigma^2(\mu) \tag{5}$$

where

$$M(A) \equiv \frac{1}{N^2} \iota' A \iota - \frac{1}{N} \text{tr}(A)$$

$$\sigma^2(\mu) \equiv \frac{1}{N} \sum_{i=1}^{N} (\mu_i - \mu_m)^2 \quad \text{and} \quad \mu_m \equiv \frac{1}{N} \sum_{i=1}^{N} \mu_i \tag{6}$$

and $\iota$ is an $(N \times 1)$-vector of ones.

If mean reversion implies that contrarian trading strategies will be profitable, then price momentum implies the reverse. In the presence of return persistence, i.e., positively autocorrelated returns, Lo and MacKinlay (1990) show that the contrarian trading strategy (3) will exhibit negative profits. As with other marketmaking strategies, the contrarian strategy loses when prices exhibit trends, either because of private information, which the market microstructure literature calls “adverse selection”, or a sustained liquidation in which the marketmaker bears the losses by taking the other side and losing value as prices move in response to the liquidation. We shall argue below that this can explain the anomalous pattern of losses during the second week of August 2007.

To develop this argument further, suppose that stock returns satisfy the following simple linear multivariate factor model:

$$R_{i,t} = \mu_i + \beta_i \nu_t + \lambda_{i,t} + \eta_{i,t} \tag{7a}$$

$$\lambda_{i,t} = \theta_i \lambda_{i,t-1} - \epsilon_{i,t} + \epsilon_{i,t-1}, \quad \theta_i \in (0, 1) \tag{7b}$$

$$\nu_t = \rho \nu_{t-1} + \zeta_t, \quad \rho \in (-1, 1) \tag{7c}$$

where $\epsilon_{i,t}$, $\zeta_t$, and $\eta_{i,t}$ are white-noise random variables that are uncorrelated at all leads and lags.

The impact of random idiosyncratic supply and demand shocks are represented by $\epsilon_{i,t}$, and $\lambda_{i,t}$ is the reduced-form expression of the interaction between public orders and marketmakers.
over the cumulative history of $\epsilon_{i,t}$'s. In particular, the reduced form (7) is an ARMA(1,1) process that exhibits negative autocorrelation to capture the mean reversion generated by marketmaking activity (e.g., bid/ask bounce, as in Roll, 1984). To develop a better sense of its time-series properties, we can express $\lambda_{i,t}$ as the following infinite-order moving-average process:

$$
\lambda_{i,t} = -\epsilon_{i,t} + \left(1 - \frac{\theta_i}{\theta_i}\right) \theta_i \epsilon_{i,t-1} + \left(1 - \frac{\theta_i}{\theta_i}\right) \theta_i^2 \epsilon_{i,t-2} + \cdots, \; \theta_i \in (0,1).
$$

(8)

Note that the coefficients in (8) sum to zero so that the impact of each $\epsilon_{i,t}$ is temporary, and the parameter $\theta_i$ controls the speed of mean reversion.

Market information is represented by the common factor $\nu_t$, which can capture mean reversion, noise, or momentum as $\rho$ is less than, equal to, or greater than zero, respectively. The error term $\eta_{i,t}$ represents idiosyncratic fluctuations in security $i$'s returns that are unrelated to liquidity or to common factors. The Appendix provides additional motivation and intuition for this specification.

By varying the parameters of the return-generating process (7), we can change the relative importance of fundamental shocks and marketmaking activity in determining security $i$'s returns. For example, if we assume parameters that cause $\nu_t$ to dominate $R_{i,t}$, this corresponds to a set of market conditions where liquidity traders are of minor importance and the common factor is the main driver of returns. If, on the other hand, the variability of $\nu_t$ is small in comparison to $\lambda_{i,t}$, this corresponds to a market where liquidity traders are the main drivers of returns. We shall see below that these two cases lead to very different patterns for the profitability of the contrarian strategy (3), which raises the possibility of inferring the relative importance of these two components by studying the empirical properties of contrarian trading profits. We perform this study below.

We now consider a few special cases of (7) to build the intuition and set the stage for the empirical analysis.

**Uncorrelated Returns**

Let $\beta_i \equiv 0$ and $\lambda_{i,t} \equiv 0$ in (7), implying that $R_{i,t}$ is driven only by firm-specific idiosyncratic shocks $\eta_{i,t}$ that are both serially and cross-sectionally uncorrelated. In this case, $\Gamma_t = 0$ for
all nonzero \( l \); hence,

\[
E[\pi_t(q)] = -q \sigma^2(\mu) < 0 .
\] (9)

Returns are both serially and cross-sectionally uncorrelated, and the expected profit is negative as long as there is some cross-sectional variation in expected returns. In this special case, the contrarian strategy involves shorting stocks with higher expected return and buying stocks with lower expected return, which yields a negative expected return that is linear in the holding period \( q \) and the cross-sectional variance of the individual securities’ expected returns.

**Idiosyncratic Liquidity Shocks**

Let \( \rho = 0 \) in (7) so that the common factor is serially uncorrelated. The expected \( q \)-period profit is then given by (please see Appendix A.2 for the derivation):

\[
E[\pi_t(q)] = \frac{N - 1}{N^2} \sum_{i=1}^{N} \frac{1 - \theta_i^q}{2} \sigma_{\lambda_i}^2 - q\sigma^2(\mu) .
\] (10)

In this case, the strategy benefits from correctly betting on mean reversion in \( \lambda_{i,t} \) but again loses a small amount due to the cross-sectional dispersion of expected returns. For a fixed holding period \( q \), the expected profit is an increasing function of the volatility \( \sigma_{\lambda_i}^2 \) of the liquidity component, and a decreasing function of the speed of mean reversion, \( \theta_i \), which is consistent with our intuition for the returns to marketmaking. Moreover, as long as the cross-sectional dispersion of expected returns, \( \sigma^2(\mu) \), is not too large, the expected profit will be an increasing and concave function of the length of the holding period, \( q \).

**Common-Factor and Idiosyncratic Liquidity Shocks**

Now suppose that \( \rho \neq 0 \) in (7), so that the common factor is autocorrelated. In this case, the expected \( q \)-period profit is given by (see Appendix A.2 for the derivation):

\[
E[\pi_t(q)] = \frac{N - 1}{N^2} \sum_{i=1}^{N} \frac{1 - \theta_i^q}{2} \sigma_{\lambda_i}^2 - \rho \frac{1 - \rho^q}{1 - \rho} \sigma^2(\beta) - q\sigma^2(\mu) .
\] (11)
In contrast to (10), when $\rho \neq 0$ the contrarian strategy can profit or lose from the common factor (depending on the sign of $\rho$) if there is any cross-sectional dispersion in the common-factor betas $\beta_i$. If the common factor is negatively serially correlated, i.e., $\rho < 0$, then the first two terms of (11) are unambiguously positive. In this case, mean reversion in both the common and liquidity components both contribute positively to expected profits.

If, however, the common-factor component exhibits momentum, i.e., $\rho > 0$, then the first two terms in (11) are of opposite sign. By buying losers and selling winners, the contrarian strategy (3) profits from mean-reverting liquidity shocks $\lambda_{i,t}$ but suffers losses from momentum in the common factor. For sufficiently large $\rho$ and large cross-sectional variability in the $\beta_i$’s, the second term of (11) can dominate the first, yielding negative expected profits for the contrarian strategy over all holding periods $q$.

However, if $\rho$ is small and positive, (11) yields an interesting relation between expected profits and the holding period $q$—the contrarian strategy can exhibit negative expected profits over short holding periods due to momentum in the common factor, and yield positive expected profits over longer holding periods as the influence of the common factor decays and as the liquidity term grows with $q$.\footnote{Of course, for (11) to be positive, the liquidity term must dominate both the common-factor term as well as the cross-sectional variability in expected returns. However, since this latter component is independent of the parameters of the liquidity and common-factor components, our comparisons center on the first two terms of (11).} To see this possibility more directly, consider the following simplifications: let $\theta_i = \theta$ and $\sigma^2_{\lambda_i} = \sigma^2_{\lambda}$ for all $i$, $\sigma^2(\mu) \approx 0$, and assume that $N$ is large. Then the expected profit under (7), normalized by the variance of the liquidity component $\sigma^2_{\lambda}$, reduces to:

$$
E[\pi_t(q)] \approx 1 - \theta^q \quad \frac{1}{2} - \rho \frac{1 - \rho^q}{1 - \rho} \frac{\sigma^2_\nu}{\sigma^2_\lambda} \sigma^2(\beta) .
$$

(12)

Figure 5 plots the normalized expected profit (12) as a function of the common factor’s autocorrelation coefficient $\rho$ and the holding period $q$, assuming the following values for the $\rho$.
remaining parameters:\textsuperscript{20}

\[ \theta = \frac{1}{2}, \quad \frac{\sigma_\nu^2}{\sigma_\lambda^2} = 50, \quad \sigma^2(\beta) = \left(\frac{1}{4}\right)^2. \]

Figure 5 shows that the expected profit is always increasing and concave in the holding period, \( q \). While mean reversion in the common factor increases the expected profits of the strategy, for sufficiently large momentum, i.e. positive \( \rho \), the expected profits becomes negative for all holding periods. The more interesting intermediate region is one in which expected profits are negative for short holding periods, but becomes positive if positions are held for a sufficiently long period of time. This pattern will be particularly relevant in light of the events of August 2007, as we will see below.

\textsuperscript{20}These calibrations are arbitrary, but the motivation for the value of \( \frac{\sigma_\nu^2}{\sigma_\lambda^2} \) is the importance of the common factor during the Quant Meltdown, and the motivation for the value of \( \sigma^2(\beta) \) is that 95\% of the betas will fall between plus and minus 0.5 of its mean if they are normally distributed with a variance of 1/16.
Empirical Results

We now apply the contrarian strategy (3) to historical U.S. stock returns from January 3, 1995 to December 31, 2007. As in Lehmann (1990), Lo and MacKinlay (1990), and Khandani and Lo (2007), we expect to find positive expected profits for short-term holding periods (small values of $q$), and the expected profits should be increasing in $q$ but at a decreasing rate, given the timed decay implicit in liquidity provision and its implied mean reversion.\textsuperscript{21} By construction, the weights for the contrarian strategy (3) sum to zero, hence the return of the strategy is ill-defined. We follow Lo and MacKinlay (1990) and the practice of most equity market-neutral managers in computing the return of this strategy $R_{p,t}$ each period $t$ by dividing each period’s dollar profit or loss by the total capital $I_t$ required to generate that profit or loss, hence

$$R_t(q) \equiv \frac{\pi_t(q)}{I_t}, \quad I_t \equiv \frac{1}{2} N \sum_{i=1}^{N} |\omega_{i,t}|$$

(13)

where $\pi_t(q)$ is given by (4).\textsuperscript{22}

Figure 6 displays the average return of the contrarian strategy when applied to components of S&P 1500 index between January 1995 to December 2007.\textsuperscript{23} When averaged over the entire sample, the results confirm that the average return increases in the holding period $q$ and, as predicted, increases at a decreasing rate. Even though the average return remains generally an increasing function of the holding period, its absolute level has decreased over time. For example, the average return for years prior to 2002 are all above the full-sample average in Figure 6 and in the years 2002 and after, they are below the full-sample average. This pattern is consistent with the common intuition that increased competition and technological innovations in the equity markets have reduced the profitability of such mar-

\textsuperscript{21} Note that as a matter of convention, we date the multi-holding-period return $R_t(q)$ as of the date $t$ on which the positions are established, not the date on which the positions are closed out and the return is realized, which is date $t+q$.

\textsuperscript{22} This expression for $R_t(q)$ implicitly assumes that the portfolio satisfies Regulation-T leverage, which is $1$ long and $1$ short for every $1$ of capital. However, most equity market-neutral managers used considerably greater leverage just prior to August 2007, and returns should be multiplied by the appropriate leverage factor when comparing properties of this strategy between 2007 and earlier years. See Khandani and Lo (2007) for further discussion.

\textsuperscript{23} The S&P indexes are based on the last day of the previous month and not changed through out each month. Plots based on strategy profits, $\pi_t(q)$, are qualitatively the same but the values are harder to interpret since they are not scaled by the amount of capital used.
Figure 6: Average return of the contrarian strategy when portfolios are constructed based on 1-day lagged returns and positions are held for 1 to 10 days for January 3, 1995 to December 31, 2007. A return is assigned to the year in which the position was established and averages are then calculated for different holding periods. Components of the S&P 1500 are based on memberships as of the last day of the previous month.

The decline in profitability also suggests that U.S. equity markets may be more liquid now than a decade ago, and we will confirm this conjecture in the next section by studying the link between trading volume and price changes.

5.2 Market Liquidity: 1995 to 2007

Many methods have been suggested for measuring liquidity in financial markets. For example, volume is used in Brennan, Chordia and Subrahmanyam (1998), quoted bid-ask spreads and depths are used in Chordia, Roll, and Subrahmanyam (2000), and the ratio of absolute stock returns to dollar volume is proposed by Amihud (2002). Because the days leading up to August 2007 exhibited unusually high volume (see Figure 1), any volume-based measure is not likely to capture the full extent of illiquidity during that time. Instead, we take an approach motivated by Kyle’s (1985) model in which liquidity is measured by a linear-regression

---

24 See Amihud, Mendelson, and Pedersen (2005) and Hasbrouck (2007) for a comprehensive discussion of different theoretical and empirical aspects of liquidity with an overview of most relevant studies in this area.
estimate of the volume required to move the price by one dollar.\textsuperscript{25} Sometimes referred to as “Kyle’s lambda”, this measure is an inverse proxy of liquidity, with higher values of lambda implying lower liquidity and lower market depth. From an empirical perspective, this is a better measure of liquidity than quoted depth since it captures undisclosed liquidity not reflected in the best available quotes, and correctly reflects lower available depth if narrower spreads come at the expense of smaller quantities available at the best bid and offer prices.

We estimate this measure using daily returns for individual stocks each month from January 1995 to December 2007. To be included in our sample, the stock must be in the corresponding S&P index as of the last day of the previous month and have at least 15 days of returns in that month. Given the sequence of daily returns, \( \{R_{i,1}, R_{i,2}, \ldots, R_{i,T}\} \), closing prices, \( \{p_{i,1}, p_{i,2}, \ldots, p_{i,T}\} \), and volumes, \( \{v_{i,1}, v_{i,2}, \ldots, v_{i,T}\} \) for security \( i \) during a specific month, we estimate the following regression:

\[
R_{i,t} = \hat{c}_i + \hat{\lambda}_i \cdot \text{Sgn}(t) \log(v_{i,t}p_{i,t}) + \epsilon_{i,t}
\]  

(14)

where \( \text{Sgn}(t) \equiv \{+1 \text{ or } -1\} \) depending on the direction of the trade, i.e., “buy” or “sell”, as determined according to the following rule: if \( R_{i,t} \) is positive we assign a +1 to that entire day (to indicate net buying), and if \( R_{i,t} \) is negative we assign a -1 to the entire day (to indicate net selling).\textsuperscript{26} Any day with zero return receives the same sign as that of the most recent prior day with a non-zero return (using returns from the prior month, if necessary). Because of evidence that the impact of trade size on price adjustment is concave (see, for example, Hasbrouck, 1991, Dufour and Engle, 2000, and Bouchaud, Farmer, and Lillo, 2008), we use the natural logarithm of trade size in our analysis. Days with zero volume were dropped from the sample.\textsuperscript{27} The monthly cross-sectional average of the estimated price impact coefficients, \( \sum_i \hat{\lambda}_i / N \), then yields an aggregate measure of market liquidity. This approach is similar to the aggregate liquidity measure of Pastor and Stambaugh (2003).

\textsuperscript{25}For examples of prior empirical work based on a similar measure, see Brennan and Subrahmanyam (1996) and Anand and Weaver (2006), or Pastor and Stambaugh (2003) for a closely related measure.

\textsuperscript{26}This approach is similar to the so-called “tick test” used in many studies of transactions data for signing trades (see, for example, Cohen et al., 1986). We adopt this method since it can be applied easily to daily returns. Some studies such as Finucane (2000) suggest that this method is the most reliable method for determining whether a trade is buyer- or seller-initiated.

\textsuperscript{27}Zero-volume days are rare in general, and extremely rare among S&P 500 stocks.
Figure 7: Monthly cross-sectional averages of price impact coefficients $\hat{\lambda}_i$ of stocks in the S&P 1500 and sub-components between January 1995 and December 2007, based on daily time-series regressions for each stock $i$ within each month: $R_{i,t} = \hat{c}_i + \hat{\lambda}_i \cdot \text{Sgn}(t) \log(v_{i,t}p_{i,t}) + \epsilon_{i,t}$ where $\text{Sgn}(t) \log(v_{i,t}p_{i,t})$ is the signed volume of security $i$ on date $t$ and the sign is determined by “tick test” applied to daily returns. Components of these indexes are based on the memberships on the last day of the previous month.
Figure 7 graphs the time series of our aggregate price-impact measure, and shows that equity markets are more liquid today than a decade ago, and have become progressively more liquid over the past five years. The spikes in price impact correspond well with known periods of uncertainty and illiquidity: the first large spike starts in August 1998 and reaches its highest level in October 1998 (the LTCM crisis), another spike occurs in March 2000 (the end of the Technology Bubble), a third occurs in September 2001 (the September 11th terrorist attacks), and the most recent spike occurs in August 2007 (the Quant Meltdown).

Kyle’s (1985) framework yields a market-depth function that is decreasing in the level of informed trading, hence it is no surprise that spikes in this measure coincide with periods of elevated uncertainty about economic fundamentals, where trading activity is more likely to be attributed to informed trading. However, the importance of the patterns in Figure 7 for our purposes involves the systematic nature of liquidity. Studies such as Huberman and Halka (2001), Hasbrouck and Seppi (2001), and Chordia, Roll and Subrahmanyam (2000,2001) have documented commonality in liquidity using different measures and techniques. Chordia, Sarkar and Subrahmanyam (2005) observe common liquidity shocks between equity and bond markets. Perhaps motivated by some of these studies, Pastor and Stambaugh (2003) propose the sensitivity to a common liquidity factor as a risk measure in an asset-pricing framework. But how does such commonality in liquidity arise?

Chordia, Roll and Subrahmanyam (2000) argue that a common liquidity factor can emerge from co-movements in optimal inventory levels of marketmakers, inventory carrying costs, commonality in private information, and common investing styles shared by large institutional investors. Carrying costs are considered explicitly in Chordia, Sarkar and Subrahmanyam (2005) and are modeled by Brunnermeier and Pedersen (2008). These studies suggest that the availability of credit and low carrying costs may have contributed to the very low price-impact levels observed between 2003 and August 2007 (see, also, Brunnermeier 2008). Institutional changes such as decimalization and technological advances are also likely contributors to the overall trend of increasing liquidity over the past decade.

5.3 Market Liquidity in 2007

Having documented the historical behavior of marketmaking profits and liquidity in Sections 5.1 and 5.2, we now turn our attention to the events of August 2007 by applying similar
measures to transactions data from July to September 2007 for the stocks in the S&P 1500 universe.

For computational simplicity, we use a simpler mean-reversion strategy than the contrarian strategy (3) to proxy for marketmaking profits. This high-frequency mean-reversion strategy is based on buying losers and selling winners over lagged \( m \)-minute returns, where we vary \( m \) from 5 to 60 minutes. Specifically, each trading day is broken into non-overlapping \( m \)-minute intervals, and during each \( m \)-minute interval we construct a long/short dollar-neutral portfolio that is long those stocks in the lowest return-decile over the previous \( m \)-minute interval, and short those stocks in the highest return-decile over the previous \( m \)-minute interval.\(^{28}\) The value of the portfolio is then calculated for the next \( m \)-minute holding period, and this procedure is repeated for each of the non-overlapping \( m \)-minute intervals during the day.\(^{29}\)

Figure 8 plots the cumulative returns of this mean-reversion strategy from July 2 to September 28, 2007 for various values of \( m \), and a clear pattern emerges. For \( m = 60 \) minutes, the cumulative return is fairly flat over the three-month period, but as the horizon shortens, the slope increases, implying increasingly larger expected returns. This reflects the fact that shorter-horizon mean reversion strategies are closer approximations to marketmaking, with correspondingly more consistent profits. However, for all values of \( m \), we observe the same dip in profits during the second week of August. Consider, in particular, the case where \( m = 5 \) minutes—on August 6th, the cumulative profit levels off, and then declines from August 7th through August 13th, after which it resumes its growth path at nearly the same rate. This inflection period suggests that marketmakers may have reduced their risk capital from August 7–13, returning to the market once the Quant Quake had passed. The logic is straightforward: the existence of marketmakers typically provides a counterbalancing force to attenuate such correlated liquidation-driven momentum among a large group of stocks. Therefore, a sudden withdrawal of marketmaking capital in the face of mounting price pressure would yield exactly the kind of price patterns observed over the week of

\(^{28}\) Stocks are equal-weighted. In case there is a tie between returns for several securities that cross the decile threshold, we ignore all securities with equal returns to focus on the supply-demand imbalance, and also to enhance the reproducibility of our numerical results. No overnight positions are allowed.

\(^{29}\) We always use the last traded price in each \( m \)-minute interval to calculate returns; hence, the first set of prices for each day are the prices based on trades just before 9:30am plus \( m \) minutes, and the first set of positions are established at 9:30am plus 2\( m \) minutes.
August 6, 2007.

Although NYSE/AMEX specialists and NASDAQ dealers have an affirmative obligation to maintain orderly markets and stand ready to deal with the public, in recent years, a number of hedge funds and proprietary trading desks have become de facto marketmakers by engaging in high-frequency program-trading strategies that exploit mean reversion in intra-daily stock prices.\textsuperscript{30} But in contrast to exchange-designated marketmakers, such traders are under no obligation to make markets, and can cease trading without notice. We conjecture that these traders may have left the market during the second week of August, either because of losses sustained during the start of the week, or because they were forced to reduce their exposures due to unrelated losses in credit portfolios and other investments within their organizations.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{cumulative_return.png}
\caption{Cumulative return of \textit{m}-minute mean-reversion strategy applied to stocks in the S&P 1500 universe from July 2, 2007 to September 28, 2007 for \textit{m} = 5, 10, 15, 30, and 60 minutes. No overnight positions are allowed, initial positions are established at 9:30am plus \textit{m} minutes each day and all positions are closed at 4:00pm. Components of the S&P 1500 are based on memberships as of the last day of the previous month.}
\end{figure}

\textsuperscript{30}The advent of decimalization in 2001 was a significant factor in the growth of marketmaking strategies by hedge funds because of the ability of such funds to “step in front of” designated marketmakers (achieve price priority in posted bids and offers) at much lower cost after decimalization, i.e., a penny versus 12.5 cents.
To explore the impact of varying holding periods on marketmaking profits, we use lagged 5-minute returns to establish the positions of the mean-reversion strategy, and hold those positions for \( m \) minutes where \( m \) varies from 5 to 60 minutes, after which new positions are established based on the most recent lagged 5-minute returns. This procedure is applied for each day and average returns are computed for each week and each holding period, and displayed in Figure 9. With the notable exception of the week of August 6th, the average return is generally increasing in the length of holding period, consistent with the patterns in the daily strategy in Section 5.1 and Figure 6.

![Figure 9: The average return for the contrarian strategy applied to 5-minute returns from July 2, 2007 to September 28, 2007.](image)

To interpret the observed patterns in Figure 9, recall that this mean-reversion strategy provides immediacy by buying losers and selling winners every 5 minutes. As quantitative factor portfolios were being deleveraged and unwound during the last two weeks of July 2007,
the price for immediacy presumably increased, implying higher profits for marketmaking strategies such as ours. This is confirmed in Figure 9. However, during the week of August 6, 2007, the average return to our simplified mean-reversion strategy turned sharply negative, with larger losses for longer holding periods that week. This pattern is consistent with the third special case of Proposition 1 in Section 5.1—a trending common factor and mean reversion in the idiosyncratic liquidity factor. In particular, the pattern of losses in Figure 9 supports the Unwind Hypothesis in which sustained liquidation pressure for a sufficiently large subset of securities created enough price pressure to overcome the profitability of our short-term mean-reversion strategy, resulting in negative returns for holding periods from 5 minutes to 60 minutes.

To distinguish between the unwinding of factor portfolios and the withdrawal of marketmaking capital, in Table 1 we compute the performance of the short-term marketmaking strategy based on 5-minute returns for holding periods from 5 minutes to 1 hour (Panel A), and based on daily returns for holding periods from 1 to 5 days (Panel B). Recall that this strategy can be viewed as providing immediacy to the market at regular intervals (every 5 minutes or every day). Accordingly, the strategy will suffer losses if information flows generate price trends over intervals that match or exceed the typical holding period of the marketmaker.

Panel A of Table 1 indicates that even as early as Monday, August 6th, the deleveraging that began earlier (see Figure 2) seems to have overwhelmed the amount of marketmaking capital available, and prices began exhibiting momentum during the day. This situation became more severe on August 8th, implying that marketmakers may have reduced their risk exposure after having observed the unusual trading and price patterns earlier in the week. By reducing their exposure, marketmakers reduced the stabilizing influence of their trading activity in the face of mounting price pressure from unwinding portfolios, which greatly increased the price impact of the unwind. The entries in Panel B of 1 also show that the premiums collected by those marketmakers on the 8th and 9th who were intrepid (and well-capitalized) enough to hold their positions for five days would have earned unleveraged returns of 8.20% and 8.96% from positions established on those two days, respectively. These extraordinary returns are yet another indication of just how much dislocation occurred during that fateful week.
Panel A: High-Frequency Contrarian Strategy

<table>
<thead>
<tr>
<th>Date</th>
<th>5 Minutes</th>
<th>10 Minutes</th>
<th>15 Minutes</th>
<th>30 Minutes</th>
<th>60 Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>8/1/07</td>
<td>5.06</td>
<td>6.81</td>
<td>5.15</td>
<td>6.02</td>
<td>7.37</td>
</tr>
<tr>
<td>8/2/07</td>
<td>6.74</td>
<td>6.97</td>
<td>8.47</td>
<td>9.48</td>
<td>9.02</td>
</tr>
<tr>
<td>8/3/07</td>
<td>4.28</td>
<td>2.93</td>
<td>1.77</td>
<td>1.47</td>
<td>-0.62</td>
</tr>
<tr>
<td>8/6/07</td>
<td>-1.30</td>
<td>-2.57</td>
<td>-3.57</td>
<td>-8.75</td>
<td>-5.30</td>
</tr>
<tr>
<td>8/7/07</td>
<td>-1.12</td>
<td>-6.32</td>
<td>-10.14</td>
<td>-14.55</td>
<td>-15.43</td>
</tr>
<tr>
<td>8/8/07</td>
<td>-18.69</td>
<td>-31.60</td>
<td>-40.99</td>
<td>-56.82</td>
<td>-62.49</td>
</tr>
<tr>
<td>8/9/07</td>
<td>-9.82</td>
<td>-16.86</td>
<td>-20.87</td>
<td>-27.65</td>
<td>-26.06</td>
</tr>
<tr>
<td>8/10/07</td>
<td>-4.38</td>
<td>-11.41</td>
<td>-16.25</td>
<td>-25.15</td>
<td>-42.97</td>
</tr>
<tr>
<td>8/13/07</td>
<td>-4.90</td>
<td>-10.29</td>
<td>-15.17</td>
<td>-23.18</td>
<td>-28.69</td>
</tr>
<tr>
<td>8/14/07</td>
<td>5.39</td>
<td>7.72</td>
<td>8.30</td>
<td>10.12</td>
<td>10.28</td>
</tr>
<tr>
<td>8/15/07</td>
<td>6.79</td>
<td>8.96</td>
<td>9.63</td>
<td>8.46</td>
<td>8.35</td>
</tr>
</tbody>
</table>

| July Sigma | 1.58 | 1.96 | 2.15 | 2.58 | 3.53 |

Panel B: Daily Contrarian Strategy

<table>
<thead>
<tr>
<th>Construction Date</th>
<th>1 Day</th>
<th>2 Days</th>
<th>3 Days</th>
<th>4 Days</th>
<th>5 Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>8/1/07</td>
<td>0.14</td>
<td>-1.03</td>
<td>-2.69</td>
<td>-2.57</td>
<td>-0.34</td>
</tr>
<tr>
<td>8/2/07</td>
<td>-0.76</td>
<td>-1.62</td>
<td>-2.57</td>
<td>-2.63</td>
<td>-2.79</td>
</tr>
<tr>
<td>8/3/07</td>
<td>-0.30</td>
<td>-0.57</td>
<td>0.65</td>
<td>0.29</td>
<td>2.04</td>
</tr>
<tr>
<td>8/6/07</td>
<td>-1.47</td>
<td>-1.79</td>
<td>-1.75</td>
<td>1.24</td>
<td>3.44</td>
</tr>
<tr>
<td>8/7/07</td>
<td>-2.68</td>
<td>-4.49</td>
<td>1.38</td>
<td>4.00</td>
<td>4.52</td>
</tr>
<tr>
<td>8/8/07</td>
<td>-3.99</td>
<td>3.79</td>
<td>7.81</td>
<td>8.31</td>
<td>8.20</td>
</tr>
<tr>
<td>8/10/07</td>
<td>-1.46</td>
<td>-1.71</td>
<td>-1.48</td>
<td>-1.84</td>
<td>-1.49</td>
</tr>
<tr>
<td>8/13/07</td>
<td>0.19</td>
<td>0.82</td>
<td>3.79</td>
<td>4.61</td>
<td>3.77</td>
</tr>
<tr>
<td>8/14/07</td>
<td>-0.95</td>
<td>-0.83</td>
<td>0.22</td>
<td>0.34</td>
<td>0.56</td>
</tr>
<tr>
<td>8/15/07</td>
<td>-1.34</td>
<td>-0.58</td>
<td>0.31</td>
<td>0.76</td>
<td>1.69</td>
</tr>
</tbody>
</table>

| Jan. to Jul. 2007 Sigma | 0.36 | 0.49 | 0.59 | 0.66 | 0.69 |

Table 1: Performance of the contrarian strategy applied to 5-minute and daily returns from August 1 to 15, 2007. Each entry in Panel A shows the average return over the specified day for the contrarian strategy applied to 5-minute returns when positions are held for 5, 10, 15, 30, or 60 minutes. Panel B shows the performance of the contrarian strategy based on daily returns, with positions established based on stock returns on the “Construction Date”, and positions are held for 1 to 5 days afterward.
By August 10th, it seems that supply/demand imbalances returned to more normal levels as the daily contrarian strategies started to recover on that day (see Table 1, Panel B), presumably as new capital flowed into the market to take advantage of opportunities created by the previous days’ dislocation. For example, a daily contrarian portfolio based on stock returns on August 6th suffered a 1.47% loss by the close of the market on the 7th, and these losses continued for the next two days, reaching the high of 1.75% by the close of the 9th. But there was a reversal on the 10th which continued through the 13th, resulting in a cumulative profit of 3.44% over the 5-day period. Daily contrarian portfolios constructed on the 7th and 8th both suffered for two days and one day, respectively, but both recovered on the 10th. In short, while prices trended intra-day (Panel A) and even over multiple days (Panel B) during this week, they eventually reverted back to their beginning-of-week levels by the 10th. While the managers that had the ability to stay fully invested for the duration of this dislocation had recovered most, if not all, of their losses by Monday, August 13th (see Figure 3), those marketmakers with sufficient capital and fortitude to hold their positions throughout this period would have profited handsomely. Not surprisingly, during periods of extreme dislocation, liquidity becomes scarce and highly valued, hence those able to provide liquidity stand to earn outsized returns.

It should be emphasized that the returns reported in these tables and figures are all unleveraged (2:1 or Regulation-T leverage) returns. The volatility and drawdowns would have been substantially higher for leveraged portfolios—for example, a portfolio with 8:1 leverage and constructed based on the Price Momentum factor would have lost about 31% of its value over the two-day period from August 8th to the 9th! And as argued by Khandani and Lo (2007), the use of leverage ratios ranging from 4:1 to 10:1 was quite common among quantitative equity market-neutral strategies, where the higher leverage ratios were used by those managers engaged in high-frequency mean-reversion strategies because those strategies exhibited the lowest volatilities and highest Sharpe ratios.

To obtain a more concrete measure of changes in liquidity during this period, we estimate a price-impact model according to (14) but using transactions volume and prices.\textsuperscript{31} To focus on the change in market liquidity during this period, in Figure 10 we display the relative

\textsuperscript{31}We use only those transactions that occur during normal trading hours, and discard all trades that are reported late, out of sequence, or have a non-zero correction field (see Section 3.2 for further details).
increase in our price-impact measure as compared to its value on July 2, 2007 (the first day of our sample of transactions data). The empirical evidence suggests that relative increases in price impact was common to all market-cap groups and not limited to the smaller stocks. Note that the specification given in (14) uses the natural logarithm of transactions dollar volume as a regressor, hence a relative increase of 1.5 (which first occurred on July 26, 2007) indicates a very large increase in trading costs, implying, for example, that trading 100 shares of a stock at price of $50/share on July 26th—a total dollar volume of $5000—would have had the same price impact as trading approximately $353,000 of stock on July 2nd!

Figure 10: The relative increase in the estimated price impact $\hat{\lambda}_i$ based on transactions data regression estimates for each day from July 2, 2007 to September 28, 2007. Using all transactions data for each security on each day, the following price-impact equation is estimated: $R_{i,t} = \hat{c}_i + \hat{\lambda}_i \cdot \text{Sgn}(t) \cdot \log(v_{i,t}P_{i,t}) + \epsilon_{i,t}$ where $\text{Sgn}(t) \cdot \log(v_{i,t}P_{i,t})$ is the signed volume of trade $t$ in security $i$, where sign is determined by “tick test”. The relative increase in the average of all $\hat{\lambda}_i$ for each day, using July 2, 2007 (the first day of our sample) as the base, is then calculated as a measure of the relative decrease in market depth. Components of various S&P indexes as of the last day of the previous month are used.

Although Figure 9 shows that short-term marketmaking profits did not turn negative until the second week of August, the pattern of price impact displayed in Figure 10 documents a substantial drop in liquidity in the days leading up to August 6, 2007. Given the increased
trading activity in factor-based portfolios as documented in Figure 2, sustained pressure on market makers' inventory levels due to the unwinding of correlated investment portfolios seems to be the most likely explanation for such a large increase in the price impact of trades during this time.\textsuperscript{32}

\subsection*{5.4 Determining the Epicenter of the Quake}

When applied to transactions data, the contrarian strategy can be used to pinpoint the origins of the Quant Meltdown even more precisely. In particular, we apply the contrarian strategy to the following subsets of stocks: three market-cap based subsets (Small-, Mid-, and Large-Cap subsets representing the bottom 30%, middle 40% and top 30% of stocks by market capitalization), five factor-based subsets (each subset consists of stocks in either decile 1 or decile 10 of each of the five quantitative factors of Section 4), and six industry based subsets, based on the twelve-industry classification codes available from Kenneth French’s website.\textsuperscript{33} To each of these subsets, we apply the simpler version of the contrarian strategy described in Section 5.3.

Table 2 and Figure 11 contain the returns of these portfolios from July 23rd to August 17th, the two weeks before and after August 6th. Each entry in Table 2 is the average return of the 5-minute contrarian strategy applied to a particular subset of securities over the specified day. As discussed in Section 5.1, days with negative average returns in Table 2 correspond to those days when price pressure due to a trending common factor overwhelmed mean-reverting idiosyncratic liquidity shocks. This interpretation, coupled with the cumulative returns of various factor-based portfolios in Figure 11 allows us to visually detect the intra-day emergence of price pressure and determine when the liquidation began and in which factor-based portfolios it was concentrated. Based on these results, we have developed the following set of hypotheses regarding the epicenter of the Quant Quake of August 2007:

1. The first wave of deleveraging began as early as August 1 around 10:45am, with the ac-

\textsuperscript{32}Our analysis also shows a sudden change in liquidity on September 18th, which was the day the U.S. Federal Open Market Committee lowered the target for the federal funds rate by 50 bps and the CBOE Volatility Index dropped by more than 6 point to close at about 20. Based on cumulative intra-day returns (which we have omitted to conserve space), the market values of our factor-based portfolios dropped and subsequently recovered over the following two days, similar to the pattern observed in Figure 3.

\textsuperscript{33}See footnote 8. Note that industries with fewer than 100 stocks are included in the \textit{Other Industries} subset.
tivity apparently concentrated among factor-based subsets of stocks. One can visually detect the sudden abnormal behavior of the long/short portfolios at the exact same time in Figure 3. Portfolios based on Book-to-Market and Earnings-to-Price dropped in value while portfolios based on the other three factors, Cashflow-to-Market, Earnings Momentum and Price Momentum, gained a little, suggesting that portfolios being deleveraged or unwound during this time were probably long Book-to-Market and Earnings-to-Price factors and short the other three. This wave of activity was short-lived and by approximately 11:30am that day, markets returned to normal. By the end of the day, the contrarian strategy applied to all subsets except for Earnings Momentum and Book-to-Market yielded positive average returns for the day (see Table 2), implying that the liquidation may have been more heavily concentrated on portfolios formed according to these two factors.

2. The second wave started on August 6th at the market open, and lasted until approximately 1:00pm. Once again, the action was concentrated among factor-based subsets. This time, the price pressure due to the hypothesized forced liquidation was strong enough to overcome the idiosyncratic liquidity shocks and, as such, the contrarian strategy applied to all factor-based subsets of stocks yielded negative returns for the entire day. Earnings-Momentum and Book-to-Market portfolios within the financial sector suffered the largest losses, implying that the deleveraging was strongest among these groups of stocks. The patterns in Figure 3 suggest that the portfolios being deleveraged were probably long Book-to-Market, Price Momentum, and Cashflow-to-Market, and short Earnings Momentum and Earnings-to-Price. Appendix A.3 contains a more detailed analysis of the specific stocks that were affected. August 6th was remarkable in another respect—for the first time in our sample, the contrarian strategy applied to all stocks also registered a loss for the day (see Table 2), implying widespread and strong price pressure due to a forced liquidation on this day.

3. On August 7th, portfolios based on Price Momentum and Cashflow-to-Market continued to drift downward as Figure 3 shows, suggesting continued deleveraging among portfolios based on these two factors. Furthermore, the contrarian strategy applied to all stocks yielded a second day of negative returns, suggesting that the forced liquida-
tion carried over to this day.

4. August 8th was the start of the so-called “Meltdown”. On this day, the contrarian strategy suffered losses when applied to any subset of stocks (factor-based, industry, and market-cap). The sudden drop and subsequent reversal is clearly visible in Figure 3.

5. Starting on Friday August 10th, the long/short factor-based portfolios sharply reversed their losing trend, and by the closing bell on Monday August 13th, all five long/short portfolios were within 2% of their values on the morning of August 8th. We conjecture that this reversal was due to two possible causes: new capital that came into the market to take advantage of buying and selling opportunities created by the price impact of the previous days’ deleveraging, and the absence of further deleveraging pressure because the unwind that caused the initial losses was completed.

6. In addition to the hypothesized forced liquidation, we conjecture that part of the losses from August 8th and 9th also stemmed from a reduction in liquidity, most likely from certain hedge funds engaged in high-frequency marketmaking activities. Unlike NYSE specialists and other designated marketmakers that are required to provide liquidity, even in the face of strong price trends, hedge funds have no such obligation. However, in recent years, such funds have injected considerable liquidity into U.S. equity markets by their high-frequency program-trading activities. We have two reasons to believe that reduced marketmaking activity is partly responsible for the August 2007 Meltdown. First, as seen in Figure 10, the price impact of trades suddenly increased as of Friday, August 10th—above and beyond the already elevated levels of the prior weeks—and stayed high until the end of the following week, implying reduced market depth and lower liquidity during that period. Second, Figure 4 shows that the $R^2$’s of the turnover regressions based on (1b) were also elevated during the week of August 6th, with values of 10% or higher which were the highest $R^2$’s for this regression during all of 2007. Also, the $R^2$ of the return-based regression (1a) was 13.9% on August 8th, once again a record-setting level for all of 2007. But these $R^2$’s and impact estimates were—with the exception of the Price-Momentum factor—elevated throughout the week of August
6th, hence they cannot explain the widespread losses that occurred on August 8th and 9th. We suspect that at least part of the meltdown that began on August 8th was due to a specific reduction in marketmakers’ capital, most likely by hedge funds engaged in high-frequency mean-reversion trading strategies.

7. We conjecture that the motivation for the reduction in marketmaking capital is the negative average returns for the all-stocks contrarian strategy during August 6th and 7th (see Table 2), which revealed a much larger pending unwind than marketmakers could handle, and those marketmakers who had the option of reducing their exposure, e.g., hedge funds, did so on August 8th.

In Appendix A.3, we show how the contrarian strategy can be used to identify with even greater precision the specific stocks and sectors that were involved at the start of the Quant Meltdown of August 2007.
Table 2: The average returns for the contrarian strategy applied to various subsets of the S&P 1500 index using 5-minute returns for July 23, 2007 to August 17, 2007. Each day is divided into non-overlapping 5-minute intervals, and positions are established based on lagged 5-minute returns and held for the subsequent 5-minute interval. The average return for each subset of stocks over each day of this period is then calculated. No overnight positions are allowed, initial positions are established at 9:40am each day and all positions are closed at 4:00pm. All entries are in units of basis points.
Figure 11: The cumulative returns for the contrarian strategy applied to various subset of the S&P 1500 index using 5-minute returns for July 23, 2007 to August 17, 2007. Each day is divided into non-overlapping 5-minute intervals, and positions are established based on lagged 5-minute returns and held for the subsequent 5-minute interval. The average return for each subset of stocks over each day of this period is then calculated. No overnight positions are allowed, initial positions are established at 9:40am each day and all positions are closed at 4:00pm.
6 Conclusions

The events of August 2007 in U.S. equity markets provide a living laboratory for developing insights into the dynamics of portfolio liquidity and marketmaking activity. By simulating the performance of simple trading strategies that proxy for factor bets like Book-to-Market and Cashflow-to-Market, we find indirect evidence of the unwinding of factor-based portfolios starting in July 2007, and continuing through August and September 2007. By simulating the performance of a high-frequency (5-minute-return) mean-reversion strategy that proxies for marketmaking activity, we find indirect evidence that liquidity declined sharply during the second week of August 2007, raising the possibility that marketmakers reduced their risk capital in the face of mounting losses from the onslaught of portfolio liquidations by long/short equity managers.

If these conjectures are well-founded, they point to a new financial order in which the “crowded trade” phenomenon now applies to entire classes of hedge-fund strategies, not just to a collection of overly popular securities. In much the same way that a passing speedboat can generate a wake with significant consequences for other ships in a crowded harbor, the scaling up and down of portfolios can have significant consequences for all other portfolios and investors. Managers and investors involved in long/short equity strategies must now incorporate this characteristic in designing their portfolios and implementing their risk management protocols. The Quant Meltdown of 2007 is another piece of evidence supporting the claim by Chan et al. (2007) that systemic risk in the hedge-fund industry has risen.

The hypothesized interplay between long/short equity managers and marketmakers is consistent with the ecological view of financial markets in Farmer and Lo (1999) and Farmer (2002), and the Adaptive Markets Hypothesis of Lo (2004, 2005). In that framework, market participants are not infinitely rational, but they do learn over time and adapt to changing market conditions. As the size (as measured by assets under management or risk capital) of one “species” grows, the population dynamics change to reflect the impact of its dominance, and in the case of the hedge-fund industry, we can observe these changes in real time given how quickly managers and investors adapt and evolve. Although the fallout from August 2007 was severe for many market participants, nevertheless this narrow slice of time and
industry has been a boon to academics interested in market dynamics.

Yet another interpretation of the Quant Meltdown of August 2007 is a case study in the how betas are born. The fact that the entire class of long/short equity strategies moved together so tightly during August 2007 implies the existence of certain common factors within that class. The analysis in this paper confirms the identities of several such factors, but more refined simulations may uncover others. In any case, there should be little doubt now about the existence of “alternative betas”, which is the next step in the natural progression from long-only investing to indexation to long/short investing to the nascent hedge-fund beta replication industry. To the extent that the demand for alternative investments continues to grow, the increasing amounts of assets devoted to such endeavors will create its own common factors that can be measured, benchmarked, managed, and, ultimately, passively replicated as proposed in Hasanhodzic and Lo (2007).

However, our conclusions must be circumscribed by the warning that we began with in Section 1: all of our inferences are indirect, tentative, and speculative. We have no inside information about the workings of the hedge funds that were affected in August 2007, nor do we have any access to proprietary prime brokerage records, trading histories, or other confidential industry data. Therefore, our academic perspective of the events during the week of August 6–10 should be interpreted with some caution and a healthy dose of skepticism. These qualifications were highlighted in Khandani and Lo (2007) and we repeat them here for completeness.

Our empirical findings are based on very simple strategies applied to U.S. stocks, which may be representative of certain short-term market-neutral mean-reversion strategies, but is not likely to be as good a proxy for the broader set of quantitative long/short equity products that involve both U.S. and international equities, and other securities. A more refined analysis using more sophisticated strategies and a broader set of assets will no doubt yield a more complex and accurate picture of the very same events.

More importantly, even if our hypothesis is correct that an unwind initiated the losses during the second week of August 2007, we cannot say much about the ultimate causes of such an unwind. It is tempting to conclude that a multi-strategy proprietary trading desk’s exposure to sub-prime mortgage portfolios caused it to reduce leverage by liquidating a portion of its most liquid positions, e.g., a statistical arbitrage portfolio. However, another
possible scenario is that several quantitative equity market-neutral managers decided at the beginning of August that it would be prudent to reduce leverage in the wake of so many problems facing credit-related portfolios. They could have deleveraged accordingly, not realizing that this strategy was so crowded and that the price impact of their liquidation would be so severe. Once this price impact had been realized, other funds employing similar strategies may have decided to cut their risks in response to their losses, which then led to the kind of “death spiral” that we witnessed in August 1998 as managers attempted to unwind their fixed-income arbitrage positions to meet margin calls.

Finally, we conjecture that liquidations of a number of strategies and asset classes may have started earlier. For example, other liquid investment categories such as global macro, managed futures, and currency strategies seem to have experienced similar unwinds earlier in 2007 as problems in the sub-prime mortgage markets became more prominent in the minds of managers and investors. The so-called “carry trade” among currencies was supposedly unwound to some extent in July and August 2007, generating losses for a number of global macro and currency-trading funds. Obviously, our long/short equity strategies are incapable of detecting dislocation among currency strategies, but a simple carry-trade simulation—similar to our simulation of the contrarian trading strategy—could shed considerable light on the dynamics of the foreign exchange markets in recent months. Indeed, a collection of simulated strategies across all of the hedge-fund categories can serve as a kind of multi-resolution microscope, one with many lenses and magnifications, with which to examine the full range of financial-market activity and vulnerabilities. It is only by deconstructing every market dislocation that we will eventually learn how to minimize their disruptive impact on market participants, and we hope that the insights drawn from our simulations will encourage others to take up this important challenge.
Appendix

In this Appendix, we provide the details of the computation of expected profits for the contrarian trading strategy of Lehmann (1990) and Lo and MacKinlay (1990), and its application to transactions data on August 6, 2007. In Section A.1, we derive a general expression for the expected profit under the assumption of jointly covariance-stationary returns for individual securities. In Section A.2, we derive the expected profit for the special case of the linear factor model (7) which incorporates an idiosyncratic marketmaking component and a common factor that may exhibit either mean reversion or momentum. And in Section A.3, we show how the simulated returns of the contrarian strategy can be used to identify the specific stocks that were at the center of the Quant Meltdown of August 2007.

A.1 Expected Profits for Stationary Returns

Consider the collection of $N$ securities and denote by $\mathbf{R}_t$ the $N \times 1$ vector of their period $t$ returns, $[R_{1,t}, \ldots, R_{N,t}]'$. Assume that $\mathbf{R}_t$ is a jointly covariance-stationary stochastic process with expectation $E[\mathbf{R}_t] = \mathbf{\mu} = [\mu_1, \ldots, \mu_N]'$ and auto covariance matrices

$E[(\mathbf{R}_{t-1} - \mathbf{\mu})(\mathbf{R}_t - \mathbf{\mu})'] = \Gamma_t = [\gamma_{i,j}(l)].$

Define $R_{i,t}(q)$ as the $q$-period return of security $i$ starting at time $t$:

$$R_{i,t}(q) \equiv \prod_{k=0}^{q-1} (1 + R_{i,t+k}) - 1 \approx \sum_{k=0}^{q-1} R_{i,t+k}$$

where the approximation is needed because period returns, $R_{i,t}$, are simple returns and logarithm of a sum is not equal to the sum of the logarithms.

We will consider the return of a market-neutral strategy which invests an amount $\omega_{i,t}$ in security $i$ at time $t$, where:

$$\omega_{i,t} \equiv -\frac{1}{N}(R_{i,t-1} - R_{m,t-1}) , \quad R_{m,t-1} \equiv \frac{1}{N} \sum_{i=1}^{N} R_{i,t-1}.$$

Define $\pi_t(q)$ as the profit for this strategy for a portfolio constructed at date $t$ and held fixed for $q$ periods. Then we have:

$$\pi_t(q) = \sum_{i=1}^{N} \omega_{i,t} R_{i,t}(q) \approx \sum_{i=1}^{N} \left( \omega_{i,t} \sum_{l=0}^{q-1} R_{i,t+l} \right)$$

$$\approx \sum_{i=1}^{N} \left( -\frac{1}{N}(R_{i,t-1} - R_{m,t-1}) \sum_{l=0}^{q-1} R_{i,t+l} \right)$$

$$\approx -\frac{1}{N} \sum_{i=1}^{N} (R_{i,t-1}(R_{i,t} + \cdots + R_{i,t+q-1})) + R_{m,t-1}(R_{m,t} + \cdots + R_{m,t+q-1})$$
Now, taking expectations yields:

\[
E[\pi_t(q)] \approx -\frac{1}{N} \sum_{i=1}^{N} \left( \gamma_{i,i}(1) + \cdots + \gamma_{i,i}(q) + q \mu_i^2 \right) + \\
\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} (\gamma_{i,j}(1) + \cdots + \gamma_{i,j}(q) + q \mu_i \mu_j)
\] (A.1)

For lag \( l \), the group of terms has the following structure:

\[
-\frac{1}{N} \sum_{i=1}^{N} \left( \gamma_{i,i}(l) + \mu_i^2 \right) + \\
\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} (\gamma_{i,j}(l) + \mu_i \mu_j) = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{i,j}(l) - \frac{1}{N} \sum_{i=1}^{N} \gamma_{i,i}(l) + \\
\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \mu_i \mu_j - \frac{1}{N} \sum_{i=1}^{N} \mu_i^2
\]

\[
= \frac{1}{N^2} \Gamma_i \Gamma_i - \frac{1}{N} \text{tr}(\Gamma_i) + \\
\frac{1}{N^2} \Gamma_i ' (\mu \mu') \Gamma_i - \frac{1}{N} \text{tr}(\mu \mu')
\] (A.2)

To simplify this expression, define the following operator

\[
\mathcal{M} (A) \equiv \frac{1}{N^2} \Gamma_i ' A \Gamma_i - \frac{1}{N} \text{tr}(A)
\] (A.3)

Note that \( \mathcal{M}(\cdot) \) is linear, i.e.,

\[
\mathcal{M}(A + B) = \mathcal{M}(A) + \mathcal{M}(B) \quad (A.4)
\]

\[
\mathcal{M}(\alpha A) = \alpha \mathcal{M}(A) \quad (A.5)
\]

Now for any \((N \times 1)\) column-vector \( c \), define \( \sigma^2(c) \) as:

\[
\sigma^2(c) \equiv \frac{1}{N} \sum_{i=1}^{N} (c_i - c_m)^2, \quad c_m \equiv \frac{1}{N} \sum_{i=1}^{N} c_i
\] (A.6)

Note that if \( A = cc' \) where \( c \) is an \((N \times 1)\) column-vector, it is easy to show that

\[
\mathcal{M}(A) = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} c_i c_j - \frac{1}{N} \sum_{i=1}^{N} c_i^2
\]

\[
= -\sigma^2(c)
\] (A.7)

Combining these relations and substituting (A.2) into (A.1) yields:

\[
E[\pi_t(q)] \approx \mathcal{M}(\Gamma_1) + \cdots + \mathcal{M}(\Gamma_q) + q \mathcal{M}(\mu \mu')
\]
\[ \approx \mathcal{M}(\Gamma_1) + \cdots + \mathcal{M}(\Gamma_q) - q\sigma^2(\mu) \quad (A.8) \]

### A.2 Expected Profits for a Linear Factor Model

Consider the following return-generating process:

\[ R_{i,t} = \mu_i + \beta_i \nu_t + \lambda_{i,t} + \eta_{i,t} \quad (A.9a) \]

\[ \lambda_{i,t} = -\epsilon_{i,t} + \left( \frac{1 - \theta_i}{\theta_i} \right) \theta_i \epsilon_{i,t-1} + \left( \frac{1 - \theta_i}{\theta_i} \right) \theta_i^2 \epsilon_{i,t-2} + \cdots , \quad \theta_i \in (0, 1) \quad (A.9b) \]

\[ \nu_t = \rho \nu_{t-1} + \zeta_t , \quad \rho \in (-1, 1) \quad (A.9c) \]

where \( \epsilon_{i,t}, \zeta_t \), and \( \eta_{i,t} \) are white-noise random variables that are uncorrelated at all leads and lags.

In this specification, \( \beta_i \nu_t \) represents a market-wide or common factor, \( \eta_{i,t} \) represents firm-specific fundamental shocks, and \( \lambda_{i,t} \) represents firm-specific liquidity shocks. The specification of this liquidity component in (A.9b) may seem odd at first, but has a natural interpretation. It is an infinite-order moving average where each term \( \epsilon_{i,t} \) is meant to capture idiosyncratic liquidity shocks to security \( i \) at time \( t \), hence a positive realizations represents buying pressure and vice versa for negative realizations. The coefficients for these idiosyncratic shocks are meant to decay at a rate of \( \theta_i \) to reflect the gradual decline in buying or selling pressure, and they sum to one so as to eliminate any long-term impact of these shocks on prices.

The expressions for expected profits derived in Section A.1 are functions of the variance-covariance matrix of returns \( R_{i,t} \). Since \( \epsilon_{i,t} \) and \( \zeta_t \) are uncorrelated at all leads and lags, the covariance matrix of the \( R_{i,t} \)'s can be decomposed into two parts: a liquidity component and a common-factor component. We will refer to these two parts as \( \Gamma_{i,\lambda} \) and \( \Gamma_{i,\nu} \), respectively, which are related to the covariance matrix \( \Gamma_i \) by:

\[ \Gamma_i = \Gamma_{i,\lambda} + \Gamma_{i,\nu} \quad (A.10) \]

Because \( \mathcal{M}(\cdot) \) is linear, the above decomposition will simplify our derivation of expected profits. We now turn to computing each component of \( \Gamma_i \).

#### Derivation of \( \Gamma_{i,\nu} \)

Observe that \( \nu_t \) is a simple AR(1) process so its variance and covariances are given by:

\[ \sigma_{\nu}^2 = \frac{1}{1 - \rho^2} \sigma_{\zeta}^2 \]

\[ \gamma_{\nu_{m},\nu_{m+i}} = \frac{\rho^i}{1 - \rho^2} \sigma_{\zeta}^2 = \rho^i \sigma_{\nu}^2. \]
Note that $\nu_t$ is multiplied by $\beta_i$ in the $R_{i,t}$, which will cause the covariance terms due to the common factor between different securities to scaled accordingly. Therefore, we have:

$$\Gamma_{l,\nu} = \beta \beta^T \rho' \sigma^2_{\nu}$$  \hspace{1cm} (A.11)

where $\beta$ is a column vector of the $\beta_i$'s. Appealing to the linearity and other properties of $\mathcal{M}(\cdot)$ discussed in Section A.1, we have:

$$\mathcal{M}(\Gamma_{l,\nu}) = \mathcal{M}(\beta \beta^T \rho' \sigma^2_{\nu}) = -\rho' \sigma^2_{\nu} \sigma^2(\beta) .$$  \hspace{1cm} (A.12)

**Derivation of $\Gamma_{l,\lambda}$**

The variance of $\lambda_{i,t}$ is given by:

$$\sigma^2_{\lambda_i} = \left( 1 + \left( \frac{1 - \theta_i}{\theta_i} \right)^2 \theta_i^2 + \left( \frac{1 - \theta_i}{\theta_i} \right)^2 \theta_i^4 + \cdots \right) \sigma^2_{\epsilon_i}$$

$$= \left( 1 + \left( \frac{1 - \theta_i}{\theta_i} \right)^2 \left( \frac{\theta_i^2}{1 - \theta_i^2} \right) \right) \sigma^2_{\epsilon_i} = \left( 1 + \frac{1 - \theta_i}{1 + \theta_i} \right) \sigma^2_{\epsilon_i}$$

$$= \frac{2}{1 + \theta_i^2} \sigma^2_{\epsilon_i} .$$  \hspace{1cm} (A.13)

To compute the covariance between $\lambda_{i,t}$ and $\lambda_{i,t+l}$, we need to focus on the common $\epsilon_{i,t}$'s in the cross product of the MA coefficients of $\lambda_{i,t}$ and $\lambda_{i,t+l}$. This yields:

$$\gamma_{\lambda_{i,t},\lambda_{i,t+l}} = \left( -\left( \frac{1 - \theta_i}{\theta_i} \right) \theta_i^l + \left( \frac{1 - \theta_i}{\theta_i} \right)^2 \theta_i^{l+2} + \left( \frac{1 - \theta_i}{\theta_i} \right)^2 \theta_i^{l+4} + \cdots \right) \sigma^2_{\epsilon_i}$$

$$= \left( -\left( \frac{1 - \theta_i}{\theta_i} \right) \theta_i^l + \left( \frac{1 - \theta_i}{\theta_i} \right)^2 \theta_i^l \frac{\theta_i^2}{1 - \theta_i^2} \right) \sigma^2_{\epsilon_i}$$

$$= \left( \frac{1 - \theta_i}{\theta_i} \right) \theta_i^l \left( -1 + \frac{1 - \theta_i}{\theta_i} \frac{\theta_i^2}{1 - \theta_i^2} \right) \sigma^2_{\epsilon_i}$$

$$= \left( \frac{1 - \theta_i}{\theta_i} \right) \theta_i^l \left( -1 + \frac{\theta_i}{1 + \theta_i} \right) \sigma^2_{\epsilon_i}$$

$$= -\left( \frac{1 - \theta_i}{\theta_i} \right) \theta_i^l \frac{1}{1 + \theta_i} \sigma^2_{\epsilon_i}$$

$$= -\left( \frac{1 - \theta_i}{\theta_i} \right) \theta_i^l \sigma^2_{\lambda_i}$$  \hspace{1cm} (A.14)

where we use our expression for $\sigma^2_{\lambda_i}$ to simplify. Combining these yields the following expression for $\Gamma_{l,\lambda}$

$$\Gamma_{l,\lambda} = \text{diag} \left( -\frac{1 - \theta_1}{2\theta_1} \theta_1^l \sigma^2_{\lambda_1} , \ldots , -\frac{1 - \theta_N}{2\theta_N} \theta_N^l \sigma^2_{\lambda_N} \right)$$
\[ \mathcal{M}(\Gamma_{l,\lambda}) = \frac{N - 1}{N^2} \sum_{i=1}^{N} \frac{1 - \theta_i^l \theta_i^l \sigma_{\lambda_i}^2}{2 \theta_i} \]  \tag{A.15} \\

**Derivation of Expected Profit**

We can now combine \( \Gamma_{l,\nu} \) and \( \Gamma_{l,\lambda} \) to yield an expression for the expected profit of the contrarian strategy. Recall that the expected profit is given by:

\[ E[\pi_t(q)] \approx \mathcal{M}(\Gamma_1) + \cdots + \mathcal{M}(\Gamma_q) - q \sigma^2(\mu) . \tag{A.16} \]

Due to the linearity of \( \mathcal{M}(\cdot) \) and using (A.10), we can rewrite this expression as:

\[ E[\pi_t(q)] \approx \mathcal{M}(\Gamma_{1,\lambda}) + \cdots + \mathcal{M}(\Gamma_{q,\lambda}) + \mathcal{M}(\Gamma_{1,\nu}) + \cdots + \mathcal{M}(\Gamma_{q,\nu}) - q \sigma^2(\mu) . \tag{A.17} \]

The two parts of this expression can be simplified as follows:

\[ \mathcal{M}(\Gamma_{1,\lambda}) + \cdots + \mathcal{M}(\Gamma_{q,\lambda}) = \sum_{l=1}^{q} \left[ \frac{N - 1}{N^2} \sum_{i=1}^{N} \frac{1 - \theta_i^l \theta_i^l \sigma_{\lambda_i}^2}{2 \theta_i} \right] \]

\[ = \frac{N - 1}{N^2} \sum_{l=1}^{q} \sum_{i=1}^{N} \frac{1 - \theta_i^l \theta_i^l \sigma_{\lambda_i}^2}{2 \theta_i} \]

\[ = \frac{N - 1}{N^2} \sum_{i=1}^{N} \frac{1 - \theta_i^q \theta_i^q \sigma_{\lambda_i}^2}{2 \theta_i} \]

\[ = \frac{N - 1}{N^2} \sum_{i=1}^{N} \frac{1 - \theta_i^q \theta_i^q \sigma_{\lambda_i}^2}{2} \]  \tag{A.18} \\

\[ \mathcal{M}(\Gamma_{1,\nu}) + \cdots + \mathcal{M}(\Gamma_{q,\nu}) = -\sum_{l=1}^{q} \rho' \sigma_{\nu}^2 \sigma^2(\beta) \]

\[ = -\rho \frac{1 - \rho\theta}{1 - \rho} \sigma_{\nu}^2 \sigma^2(\beta) . \tag{A.19} \]

Substituting (A.18) and (A.19) into (A.17) yields the final expression for expected profits:

\[ E[\pi_t(q)] \approx \frac{N - 1}{N^2} \sum_{i=1}^{N} \frac{1 - \theta_i^q \theta_i^q \sigma_{\lambda_i}^2}{2} - \rho \frac{1 - \rho\theta}{1 - \rho} \sigma_{\nu}^2 \sigma^2(\beta) - q \sigma^2(\mu) . \tag{A.20} \]

**A.3 Extreme Movers on August 6, 2007**

Simulations of simple strategies such as the contrarian strategy can be used to pinpoint the beginning of market dislocations when applied to transactions data. Recall that the intra-day contrarian strategy of Section 5 invests $1 long in the worst performing decile and $1 short in the best performing decile of lagged 5-minute returns. Given the position \( \omega_{i,t} \) of security \( i \) at time \( t \), the security’s contribution to the portfolio’s profit or loss over the next period is simply \( \omega_{i,t} R_{i,t} \). If this value is negative, it suggests that the security experienced either a
negative return following a period of under-performance (recall that we invest $1 long in the worst performing decile), or a positive return following a period of out-performance. While such an outcome may be purely random, a sufficiently high number of such occurrences over a given day indicates a price trend for that security and systematic losses for the contrarian strategy. Therefore, the number of periods in which a security exhibited negative contributions to the portfolio:

$$\sum_t I_{\{\omega_{i,t} R_{i,t} < 0\}}$$  \hspace{1cm} (A.21)

can be used as a metric to detect the start of an unwind of mean-reversion strategies, as well as a possible decline in market liquidity due to losses accumulated by marketmaking strategies.

Under the scenario of pure randomness, i.e., independently and identically distributed mean-zero returns, each security has a 1/5 chance of being included in the contrarian portfolio in each time period (recall that we long and short the bottom- and top-performing deciles). Once the portfolio is established, each position has a 1/2 chance of contributing a loss (negative returns following a period of under performance or position return following a period of outperformance).\(^{34}\) Therefore, each security has a 1/10 chance of contributing a negative value to the return of the contrarian strategy over each interval so the expected value of (A.21) for each security on any given day is 7.6 (recall that the contrarian strategy takes position 76 times each day, starting at 9:40am and closing final positions at 4:00pm).

We have ranked securities according to this metric for August 6, 2007 and list the securities with the top 50 values in Table A.1. We have also reported the decile ranking of each security according to each of the five valuation factors as well as their market-capitalization decile. The Open, High, Low and the Closing price as well as the High-Low spread, as a measure of the intraday volatility, and the overall return for the day are also reported.

The stocks’ factor rankings in Table A.1 do not look random, but clearly show that the extreme losers were concentrated in the financial sector, and had extreme factor rankings in at least three of our valuation factors and in size—high Book-to-Market, high Earnings-to-Price, low Earnings Momentum, and low market cap.

\(^{34}\)Recall that we are using 5-minute returns, which is close to zero mean, hence the loss probability of 1/2 is a reasonable approximation.
Table A.1: Top 50 securities with highest loss rankings from the contrarian strategy applied to 5-minute returns of stocks in the S&P 1500 on August 6, 2007. Securities are ranked based on $\sum wt_i, R_{t,c<0}$ where $\omega_t$ is the weight assigned to security $i$ based on the returns over the preceding 5-minute interval and $R_{t,c}$ is the return over the subsequent 5-minute interval. The realized value for this metric is contained in the column "Periods with Loss".
References


Rothman, M., 2007c, “Rebalance of Large Cap Quant Portfolio,” U.S. Equity Quantitative Strategies, Lehman Brothers Research


