MIT Sloan Finance Problems and Solutions Collection

Finance Theory I

Part 1

Andrew W. Lo and Jiang Wang

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Acknowledgements

Contents

1 Present Value

1. You can invest $10,000 in a certificate of deposit (CD) offered by your bank. The CD is for 5 years and the bank quotes you a rate of 4.5%. How much will you have in 5 years if the 4.5% is

   (a) an EAR?
   (b) a quarterly APR?
   (c) a monthly APR?

2. (W) **e-Money rates**. An internet company, e-Money, is offering a money market account with an A.P.R. of 4.75%. What is the effective annual interest rate offered by e-Money if the compounding interval is

   (a) annual
   (b) monthly
   (c) weekly
   (d) continuously?

3. You can invest $50,000 in a certificate of deposit (CD) offered by your bank. The CD is for 2 years and the bank quotes you a rate of 4%. How much will you have in 2 years if the 4% is

   (a) an EAR?
   (b) a quarterly APR?
   (c) a monthly APR?

4. You can invest $10,000 in a certificate of deposit (CD) offered by your bank. The CD is for 5 years and the bank quotes you a rate of 4.5%. How much will you have in 5 years if the 4.5% is

   (a) an EAR?
   (b) a quarterly APR?
   (c) a monthly APR?

5. **e-Money rates**. An internet company, e-Money, is offering a money market account with an A.P.R. of 5.25%. What is the effective annual interest rate offered by e-Money if the compounding interval is

   (a) annual
   (b) monthly
   (c) daily

(d) continuously?

6. True, false or "it depends" (give a brief explanation): PV is sometimes calculated by discounting free cash flow for several years, say from year 1 to T, and then discounting a forecasted terminal value at horizon date T. The choice of the horizon date can hae a significant effect on PV, particularly for rapidly growing firms.

7. Suppose you invest $10,000 per year for 10 years at an average return of 5.5%. The average future inflation rate is 2% per year.

   (a) The first investment is made immediately. What is your ending investment balance?

   (b) What is its purchasing power in todays dollars?

8. Overhaul of a production line generates the following incremental cash inflows over the line's 5-year remaining life.

	C_1	C_2	C_3	C_4	C_5
Cash inflow ($ million)	+1.5	+1.3	+1.05	+0.9	+0.75

   (a) What is the PV of the inflows? The cost of capital is 12%.

   (b) Part (a) used a nominal discount rate and the cash inflows incorporated inflation. Redo Part (a) with real cash flows and a real discount rate. The forecasted inflation rate is 3% per year.

9. You have just inherited an office building. You expect the annual rental income (net of maintenance and other cost) for the building to be $100,000 for the next year and to increase at 5% per year indefinitely. A expanding internet company offers to rent the building at a fixed annual rent for 5 years. After year 5, you could re-negotiate or rent the building to another tenant. What is the minimum acceptable fixed rental payments for this five-year agreement? Use a discount rate of 12%.

10. Two dealers compete to sell you a new Hummer with a list price of $45,000. Dealer C offers to sell it for $40,000 cash. Dealer F offers "0-percent financing:" 48 monthly payments of $937.50. (48x937.50=45,000)

    (a) You can finance purchase by withdrawals from a money market fund yielding 2% per year. Which deal is better?

    (b) You always carry unpaid credit card balances charging interest at 15% per year. Which deal is better?

11. Your sales are $10 million this and expected to grow at 5% in real terms for the next three years. The appropriate nominal discount rate is 10%. The inflation is expected to be 2% per year during the same period. What is the present value of your sales revenue for the next three years?

12. Company ABC's after-tax cash flow is $10 million (at the end of) this year and expected to grow at 5% per year forever. The appropriate discount rate is 9%. What is the value of company ABC?

13. You own three oil wells in Vidalia, Texas. They are expected to produce 7,000 barrels next year in total, but production is declining by 6 percent every year after that. Fortunately, you have a contract fixing the selling price at $15 per barrel for the next 12 years. What is the present value of the revenues from the well during the remaining life of the contract? Assume a discount rate of 8 percent.

14. A geothermal power station produces cash flow at a current rate of $14 million per year, after maintenance, all operating expenses and taxes. All the cash flow is paid out to the power stations owners. The cash flow is expected to grow at the inflation rate, which is forecasted at 2% per year. The opportunity cost of capital is 8%, about 3 percentage points above the long-term Treasury rate. (Assume this is an annually compounded rate.)

The power station will operate for a very long time. Assume for simplicity that it will last forever.

(a) What is the present value of the power station? Assume the first cash flow is received one year hence.

(b) Now assume that the power stations cash flow is generated in a continuous stream, starting immediately. What is the present value?

15. A foundation announces that it will be offering one MIT scholarship every year for an indefinite number of years. The first scholarship is to be offered exactly one year from now. When the scholarship is offered, the student will receive $20,000 annually for a period of four years, beginning from the date the scholarship is offered. This student is then expected to repay the principal amount received ($80,000) in 10 equal annual installments, interest-free, starting one year after the expiration of her scholarship. This implies that the foundation is really giving an interest-free loan under the guise of a scholarship. The current interest is 6% for all maturities and is expected to remain unchanged.

(a) What is the PV of the first scholarship?

(b) The foundation invests a lump sum to fund all future scholarships. Determine the size of the investment today.

16. You signed a rental lease for an office space in the Back Bay for five years with an annual rent of $1 million, paid at the beginning of each year of the lease. Just before you pay your first rent, the property owner wants to use the space for another purpose and proposes to buy back the lease from you. The rent for similar space is now $1.25 million per year. What would be the minimum compensation that you would ask from the property owner? Assume the interest rate to be 6%.

17. The annual membership fee at your health club is $750 a year and is expected to increase at 5% per year. A life membership is $7,500 and the discount rate is 12%. In order to justify taking out the life membership, what would be your minimum life expectancy?

18. You are considering buying a car worth $30,000. The dealer, who is anxious to sell the car, offers you an attractive financing package. You have to make a down-payment of $3,500, and pay the rest over 5 years with annual payments. The dealer will charge you interest at a constant annual interest rate of 2%, which may be different from the market interest rate.

    (a) What is the annual payment to the dealer?

    (b) The dealer offers you a second option: you pay cash, but get a $2,500 rebate. Should you go for the loan or should you pay cash? Assume that the market annual interest rate is constant at 5%.

    Note: the tradeoff between the two options is that in the first case, you can finance your purchase at a relatively low rate of interest. In the second case, you receive a lump-sum cash rebate.

19. Your brother-in-law asks you to lend him $100,000 as a second mortgage on his vacation home. He promises to make level monthly payments for 10 years, 120 payments in all. You decide that a fair interest rate is 8% compounded annually. What should the monthly payment be on the $100,000 loan?

20. Your cousin is entering medical school next fall and asks you for financial help. He needs $65,000 each year for the first two years. After that, he is in residency for two years and will be able to pay you back $10,000 each year. Then he graduates and becomes a fully qualified doctor, and will be able to pay you $40,000 each year. He promises to pay you $40,000 for 5 years after he graduates. Are you taking a financial loss or gain by helping him out? Assume that the interest rate is 5% and that there is no risk.

21. You are awarded $500,000 in a lawsuit, payable immediately. The defendant makes a counteroffer of $50,000 per year for the first three years, starting at the end of the first year, followed by $60,000 per year for the next 10 years. Should you accept the offer if the discount rate is 12%? How about if the discount rate is 8%?

22. You are considering buying a Back Bay two-bedroom apartment for $800,000. You plan to make a $200,000 down payment and take a $600,000 30-year mortgage for the rest. The interest rate on the mortgage is 6% monthly APR. Payments are due at the end of every month.

    (a) What is the effective annual rate?

    (b) What is the monthly payment?

(c) Suppose that exactly five years have passed, interest rates are now 5% and you decided to re-finance your mortgage. You have to pay the remaining portion of the principal on the mortgage to the bank. Exactly how much do you owe to the bank at that point?

Hint: There is a very quick and a very slow way to answer part (c).

23. True, false or "it depends" (give a brief explanation): U.S. Treasury securities have no risk because they give sure payoffs at fixed future dates.

24. A 10-year German government bond (bund) has face value of 10,000 and an annual coupon rate of 5%. Assume that interest rate (in euros) is equal to 6% per year.

    (a) What is the bond's PV?

    (b) Suppose instead that the bund paid interest semiannually like a U.S. bond. (The bond would pay .025 × 10,000 = 250 every 6 months.) What is the PV in this case?

25. You are considering buying a two bedroom apartment in Back Bay for $600,000. You plan to make a $100,000 down payment and take out a $500,000 30-year mortgage for the rest. The interest rate on the mortgage is 8.5% monthly APR.

    (a) What is the effective annual rate (EAR)?

    (b) What is the monthly payment?

    (c) How much do you owe the bank immediately after the 60th monthly payment?

26. John is 30 years old at the beginning of the new millennium and is thinking about getting an MBA. John is currently making $40,000 per year and expects the same for the remainder of his working years (until age 65). I f he goes to a business school, he gives up his income for two years and, in addition, pays $20,000 per year for tuition. In return, John expects an increase in his salary after his MBA is completed. Suppose that the post-graduation salary increases at a 5% per year and that the discount rate is 8%. What is miminum expected starting salary after graduation that makes going to a business school a positive-NPV investment for John? For simplicity, assume that all cash flows occur at the end of each year.

27. After doing well in your finance classes, you landed a job at the IMF. Your salary is $100,000, and your contract is for 5 years. Your salary will stay the same during the 5 years and, since you are at the IMF, you are not subject to taxes. If you do well (which we assume will happen with certainty), you will get a permanent contract. Under this contract, your salary will grow at the rate of 3% per year, until retirement. Retirement will occur in 30 years after your contract becomes permanent.

For simplicity, assume that your salary is paid at the end of each year. In other words,

(End of) Year	Salary
1	$100,000
2	$100,000
3	$100,000
4	$100,000
5	$100,000
6	$100,000(1+3%)
7	$100,000(1 + 3\%)^2
⋮	⋮
35	$100,000(1 + 3\%)^{30}

We assume that the interest rate is 4% (and will stay at 4% forever).

(a) What is the value of your human capital? That is, what is the PV (as of today) of all your future earnings?

(b) Assume that you spend 70% of your salary, and deposit the remainder in a savings account, which pays the rate 4%. How much money will you have in the savings account just after you received your fifth salary (end of year 5)? (You deposit only 30% of that salary in the savings account.)

28. Retirement planning: Mr. Jones is contemplating retirement. He is 55 and his net worth now is $2 million. He hopes that after retirement he can maintain a lifestyle that costs him $100,000 per year in today's dollars (i.e., real dollars, inflation adjusted).

If he retires, he will invest all his net worth in government bonds that yield a safe annual return of 5%. Inflation is expected to be 2% per year. Ignore taxes.

(a) Is Mr. Jones rich enough to retire today if he lives until (i) 80 (ii) 100 (iii) 115?

(b) Mr. Jones thinks he will live until about 100. What advice will you give him about retiring?

29. Suppose you invest $50,000 for ten years at a nominal rate of 7.5% per year. If the annual inflation rate is 3% for the next ten years, what is the real value of your investment at the end of ten years?

30. Fill in the blanks:

(a) ....% continuously compounded is equivalent to annual interest rate of 12%.

(b) 5% continuously compounded is equivalent to annual interest rate of ......%.

(c) ....% continuously compounded is equivalent to annual interest rate of 9%.

31. A 10-year U.S. Treasury bond with a face value of $10,000 pays a coupon of 5.5% (2.75% of face value every 6 months). The semi-annually compounded interest rate is 5.2 % (a 6-month discount rate of 5.2/2 = 2.6%).

(a) What is the present value of the bond?

(b) Generate a graph or table showing how the bond's present value changes for semi-annually compounded interest rate between 1% and 15%.

32. The Reborn VW Beetle.

You are considering the purchase of a new car, the reborn VW Beetle, and you have been offered two different deals from two different dealers. Dealer A offers to sell you the car for $20,000, but allows you to put down $2,000 and pay back $18,000 over 36 months (fixed payment each month) at a rate of 8% compounded monthly. Dealer B offers to sell you the car for $19,500 but requires a down payment of $4,000 with repayment of the remaining $15,500 over 36 months at 10% compounded monthly. Which deal would you choose? (Hint: Find ranges of market interest rates that make one deal more attractive than the other.)

33. Dear Financial Adviser,

My spouse and I are each 62 and hope to retire in 3 years. After retirement we will receive $5,000 per month after taxes from our employers pension plans and $1000 per month after taxes from Social Security. Unfortunately our monthly living expenses are $15,000. Our social obligations preclude further economies.

We have $1,200,000 invested in a high-grade corporate-bond mutual fund. Unfortunately the after-tax return on the fund has dropped to 3.5% per year. We plan to make annual withdrawals from the fund to cover the difference between our pension and social security income and our living expenses. How long will the money last?

Sincerely,

Luxury Challenged

Marblehead, MA

34. The <u>annually</u> compounded discount rate is 5.5%. You are asked to calculate the present value of a 12-year annuity with payments of $50,000 per year. Calculate PV for each of the following cases.

(a) The annuity payments arrive at one-year intervals. The first payment arrives one year from now.

(b) The first payment arrives in 6 months. Following payments arrive at one-year intervals, at 18 months, 30 months, etc.

35. IRA Accounts and Taxes.

An Individual Retirement Account (IRA) allows you to set aside a limited amount of money each year for retirement. These funds will have a special tax status that

depends on several factors. (These factors include your marital status, whether you have other sources of retirement savings, your income, etc.)

Suppose that you have $2,000 in pretax income to contribute to the IRA at the end of each year (starting with the end of the current year, i.e., year 1). You will retire in 30 years, and your marginal tax rate will be 28% for all years. Suppose that the account returns a fixed 6% each year until you retire. For simplicity, assume that you withdraw all money at your retirement, and any tax-deferred income is taxed at that time.

(a) How much money will you have in year 30 if neither the contribution nor the interest income is tax-deferred? (In this case, you can withdraw the money without paying any additional tax at year 30.)

(b) How much money will you have in 30 years if the contribution is not tax-deferred but the interest income is? (In this case, only the cumulative interest is taxed at year 30.)

(c) How much money will you have in 30 years if both the contribution and the interest income are tax-deferred?

(d) Would you expect the benefit of tax deferral to increase or decrease as the tax rate increases? Why?

2   Fixed Income Securities

1. True or False? Briefly explain (or qualify) your answers.

   (a) The duration of a coupon bond maturing at date T is always less than the duration of a zero-coupon bond maturing on the same date.

   (b) When investing in bonds, we should invest in bonds with higher yields to maturity (YTM) because they give higher expected returns.

   (c) The phrase "On the run" refers to junk bonds that have recently defaulted.

2. True or false? Briefly explain (or qualify) your answers.

   (a) Investors expect higher returns on long-term bonds than short-term bonds because they are riskier. Thus the term structure of interest rates is always upward sloping.

   (b) Bonds whose coupon rates fall when the general level of interest rates rise are called reverse floaters. Everything else the same, these bonds have a lower modified duration than their straight bond counterparts.

3. True, false or "it depends" (give a brief explanation):

   (a) Term structure of interest rates must be always upward sloping because longer maturity bonds are riskier.

   (b) Bonds with higher coupon rates have more interest rate risk.

4. True, false (give a brief explanation): The term structure of interest rates is always upward sloping because bonds with longer maturities are riskier and earn higher returns.

5. True or false (give a brief explanation): A flat term structure (identical spot rates for all maturities) indicates that investors do not expect interest rates to change in the future.

6. True or false (give a brief explanation): To reduce interest rate risk, an over-funded pension fund, i.e., a fund with more assets than liabilities, should invest in assets with longer duration than its liabilities.

7. Which security has a higher *effective* annual interest rate?

   (a) A three-month T-bill selling at $97,645 with face value of $100,000.

   (b) A coupon bond selling at par and paying a 10% coupon semi-annually.

8. The Wall Street Journal quotes 6.00% for the Treasury bill with a par value of $100,000 due two months from now. What is the effective annual yield on the bill?

9. Which security has a higher effective annual interest rate?

(a) A six-month T-bill selling at \$98,058 with face value of \$100,000.

(b) A coupon bond selling at par and paying a 4.2% coupon (2.1% every six months).

10. **Spot rates.**

You are given the following prices of US Treasury Strips (discount or zero coupon bonds):

Maturity	Price (per 100 FV)
1	96.2
2	91.6
3	86.1

(a) Compute the spot rates for years 1, 2 and 3.

(b) Now, suppose you are offered a project which returns the following cashflows:

\$300m at the end of year 1
\$210m at the end of year 2
\$400m at the end of year 3
The project costs \$600m today.

Calculate the NPV of the project using the spot rates computed above.

11. Assume that spot interest rates are as follows:

Maturity (year)	Spot Rate (%)
1	3.0
2	3.5
3	4.0
4	4.5

Compute the prices and YTMs of the following bonds:

(a) A zero-coupon bond with 3 years to maturity.

(b) A bond with coupon rate 5% and 2 years to maturity.

(c) A bond with coupon rate 6% and 4 years to maturity.

Assume that spot rates and YTMs are with annual compounding, coupon payments are annual, and par values are \$100.

12. Treasury bonds paying an 8% coupon rate with *semiannual* payments currently sell at par value. What coupon rate would they have to pay in order to sell at par if they paid their coupons *annually*?

13. Yields on three Treasury notes are given as follows:

Bonds and Notes

Coupon Rate	Maturity	Bid	Asked	Asked yield
7.250	Aug. 04	110:00	110:00	2.07%
10.750	Aug. 05	123:12	123:13	2.55%
5.750	Aug. 10	112:00	112:01	3.97%
5.00	Aug. 11	106:24	106:25	4.09%
8.750	Aug. 20	143:26	143:27	5.02%
6.125	Aug. 29	114:11	114:12	5.13%

Strips

Maturity	Bid	Asked	Asked yield
Aug. 04	96:03	96:04	2.00%
Aug. 05	92:19	92:21	2.58%
Aug. 10	71:13	71:17	4.24%
Aug. 11	67:06	67:08	4.47%

Table 1: Treasury Prices and Yields, August 20, 2002

Maturity (yrs)	Coupon rate (%)	Yield to maturity (%)
1	0	5.25
2	5	5.50
3	6	6.00

Coupons are paid annually.

(a) What are the prices of the 1-year, 2-year, and 3-year notes?

(b) What are the spot interest rates for years 1, 2 and 3?

(c) What is the implied forward rate for year 2 to year 3?

14. Using the following data given in Table 1, answer these questions:

(a) What were the 2-, 3- and 8-year spot interest rates?

(b) What is the forward interest rate from August 2004 to August 2005? From August 2010 to August 2011?

(c) What does the slope of the term structure imply about future interest rates? Explain briefly.

Express your answers to (a) and (b) as effective annual interest rates.

15. **Forward rates.**

You are given the following spot rates:

Maturity	Spot rate
1	2.9%
2	3.2%
3	3.6%
4	4.2%

(a) Compute the forward rate between years 1 and 2.

(b) Compute the forward rate between years 1 and 3.

(c) Suppose one of your 401 TAs, offers to *commit* to borrowing money from you between years 3 and 4 at a rate of 6.3%. Is there any way you can profit from this? What sort of risk are you exposed to? Is this strategy an arbitrage?

16. You are a bond trader and see on your screen the following information on three bonds with annual coupon payments and par value of $100:

Bond	Coupon rate (%)	Maturity (year)	YTM(%)
A	0	1	5.00
B	5	2	5.50
C	6	3	6.00

Coupon payments are annual.

(a) What are the prices of the above bonds?

(b) Construct the current term-structure of spot interest rates.

(c) Explain how you would synthetically replicate a zero-coupon bond with a maturity of 3 years and a par value of $100.

(d) What should be the price of the bond so that there is no arbitrage?

17. You are given the following information:

Bond	Coupon Rate	Maturity	Price
A	10%	1	106.80
B	5%	2	101.93
C	10%	3	111.31

All coupon payments are annual and par values are 100.

(a) Determine the 1-, 2- and 3-year spot interest rates from the given prices.

(b) Compute the annual forward rate from year one to year two, i.e., f_2.

18. The Wall Street Journal gives the following prices for STRIPS (with a principal of 100):

Bond	Maturity Year	Price
A	1	95.92
B	2	92.01
C	3	87.00

(a) Determine the 1-, 2- and 3-year spot interest rates from the given prices.

(b) Compute the annual forward rate from year two to year three, i.e., f_3 (or $f_{2,3}$).

(c) Compute the yield to maturity of a 2-year coupon bond with a principal of 100 and a coupon rate of 4.25%. Assume annual coupon payments.

19. The Wall Street Journal gives the following prices for the STRIPS:

Maturity (years)	1	2	3
Price (% of par value)	97.56	95.18	92.86

Suppose that you have a short term liability of $10 million every year for the next three years.

(a) Calculate the present value of the liability.

(b) Calculate the duration of your liability.

(c) Suppose that you want to set aside $20 million to pay part of the liability and the fund will be invested in STRIPS. In order to avoid interest rate risk, what maturity for the STRIPS should you pick?

(d) If the interest rates increase by 0.10%, how much will be the remaining short fall for your liability?

20. The Wall Street Journal gives the following prices for the STRIPS:

Maturity (years)	1	2	3
Price (% of par value)	96.1538	92.4556	88.8996

You are holding an asset which yields a sure income of $20 million every year for the next two years.

(a) Calculate the present value of the asset.

(b) Calculate the modified duration of the asset.

(c) If the interest rates increase by 0.10%, how much will the asset's value change in dollars?

(d) Suppose that you want to use an interest rate futures to hedge the interest rate risk. The futures has a contract value of $100,000 and a modified duration of 5. Assume a flat term structure of interest rates. What will be your hedging strategy?

(e) Show that with the hedging position in the futures, the value of your total position (the asset plus the futures position) is insensitive to the change in interest rate.

21. The term structure of spot interest rates is given in the table below:

Maturity (years)	1	2	3	4	5	¿5
Interest rate (%)	3.5	3.0	4.0	4.0	4.0	4.0

You have just signed a lease on an office building with a rental payment of $1 million per year forever. The first payment is due one year from now.

(a) What is the present value of the lease?

(b) New inflation figures imply that expected inflation will be 0.5% percent higher. As a result, interest rates for all maturities now increase by 0.5%. What is the PV of the lease under the new market conditions?

22. The following is a list of prices for zero-coupon bonds of various maturities. Calculate the yields to maturity of each bond and the implied sequence of forward rates.

Maturity (Years)	Price of Bond ($)
1	943.40
2	898.47
3	847.62
4	792.16

23. You have accounts receivable of $10 million due in one year. You plan to invest this amount in the Treasury market for one year after receiving it. You would like to lock into an interest rate today for this future investment. Current yields on Treasury STRIPS are as follows:

Maturity	Yield (%)
1	5.25
2	5.50
3	5.75

(a) Your bank quotes you a forward rate of 5.50%. Is this in line with the forward rate implied by market interest rates?

(b) Suppose that you can buy or sell short the STRIPS at the above yields without additional costs and the STRIPS have face value of $1,000. How can you use the STRIPS to structure the forward investment you wanted?

24. Refer to Table 1, use the quoted yields to calculate the present value for the cash payments on the

(a) August 2011 strip.

(b) August 2011 note.

Assume that the first note coupon comes exactly six months after August 20, 2002, and that principal is repaid after exactly 9 years. (This timing assumption is not exactly right. Also, the quoted yields are rounded. Your PV will not match the Asked Price exactly.)

25. **Spot Rates and Forward Transactions.**

Suppose you have the following bonds, which pay coupons at the end of each year:

Maturity (yrs)	YTM (%)	Coupon (%)
1	4%	4%
2	4.2%	5%
3	4.8%	5%

(a) Determine the price of each bond per $100 face value.

(b) What are the spot rates for years 1, 2 and 3?

26. Which of the following statements are correct?

With today's Yield Curve, you can compute exactly:

(a) The price at which a 5-year T-Strip with $1,000 face value trades today.

(b) The spot rates that will prevail in two years.

(c) The price at which a 5-year T-bond with 7% coupon and $1,000 face value will trade in one year.

(d) The forward rates that prevail today.

(e) The forward rates that will prevail in two years.

27. Suppose you are given the following prices for two U.S. Treasury strips.

Maturity date	Price	Yield to maturity
November 2012	41:25	6.83%
November 2013	38:27	6.87%

Assume for simplicity that the maturity dates are exactly 13 and 14 years from now (November). Calculate the forward rate of interest between November 2012 and November 2013.

28. Here are closing quotes for 4 Treasury securities on October 11, 2002.

	Coupon	Maturity	Asked Price	Asked Yield
(Note)	6.5	Feb. 2010	119:08	3.50%
(Note)	5.0	Feb. 2011	109:20	3.65%
(Strip)	0	Feb. 2010	76:22	3.65%
(Strip)	0	Feb. 2011	73:08	3.77%

(a) Suppose you buy the Feb. 2011 note and hold it to maturity. How much would you have to pay (approximately)? What cash flows would you receive, on what dates?

(b) What are the spot interest rates for February 2010 and February 2011?

(c) What is the forward rate of interest between February 2010 and February 2011?

(d) Which of these securities has the shortest duration? Explain.

29. Yankee Inc. has sold the Super Coupon Absolute Marvel (SCAM) security to raise new funds. Unlike ordinary bonds, it pays no par value/face value at the end of its life. It only pays coupons every year as follows: $100(1 + 0.05)$ at the end of year one, $100(1 + 0.05)^2$ at the end of year two, and so on. This security lasts for 4 years (i.e., makes 4 payments). The current interest rate is 5% for all maturities.

   (a) What is the price today of SCAM?

   (b) What is the duration today of SCAM?

   (c) Yankee Inc. sold $10 million worth of SCAM. It plans to invest the proceeds in two assets, A1 and A2, for the short run. A1 is a 12-month T-Bill, whereas A2 is a 4-year STRIPS. How much should Yankee Inc. invest in A1 and A2 to avoid interest rate risks?

30. You manage a pension fund, and your liabilities consist of two payments as follows:

Time	Payment
10 years	$20 million
30 years	$30 million

   Your assets are $18 million. The term structure is currently flat at 5%.

   (a) Compute the present value of your liabilities.

   (b) Compute modified duration of your liabilities.

   (c) Compute an approximate change in the present value of your liabilities, using duration, when interest rates fall by 0.25%.

   (d) Suppose that you invest the $18 million in 1-year Treasury bills (i.e., 1-year zero-coupon bond) and in a Treasury bond with modified duration of 20. How would you allocate your assets to avoid interest rate risk of your portfolio, which includes both assets and liabilities?

31. As a mid-size company, you have a pension plan which pays out $10 million a year forever. The first payment is exactly one year from now. The term structure is currently flat at 5%.

    (a) Compute the present value of your pension liabilities.

    (b) Suppose that the interest rate goes down by 0.1%. How does the value of your liability change?

    (c) Given your answer to (b), what is the modified duration of your pension liability?

    (d) Suppose that the pension plan is fully funded (i.e., the value of your assets equal the value of pension liabilities). You want to invest all your assets in bonds to avoid any interest rate risk. What should the duration of your bond portfolio be?

    (e) Suppose that this portfolio is a single zero-coupon bond. What should its maturity and total par value be?

32. On a job interview, you were handed the following quotes on U.S. Treasuries:

Bond	Maturity (years)	Coupon Rate	Yield to Maturity
1	1	5%	4.5%
2	2	5%	5.0%
3	3	0%	5.5%

Assume that the par value is $100 and coupons are paid annually, with the first coupon payment coming in exactly one year from now. The yield to maturity is also quoted as an annual rate. You are then asked the following questions:

    (a) What should be the price of a bond with a maturity of 3 years and coupon rate of 5%, given the above information?

    (b) What should be the 1-year forward rate between years 2 and 3?

    (c) What is the modified duration of a bond portfolio with 30% invested in bond 1 and 70% invested in bond 3?

    (d) How much would the value of the portfolio in (c) change if the yields of all bonds increased by 0.15%?

33. You have the following data on Treasury bonds. Assume that there are no taxes, only annual coupon payments are made and the first coupon payment occurs a year from now.

Bond	Year of Maturity	Coupon	Face Value at T	Price Today
A	1	25	100	100.00
B	2	50	500	422.61
C	3	20	300	232.28
D	10	0	1000	192.31

(a) Calculate the following four annualized forward rates: f from 0 to 1, f from 1 to 2, f from 2 to 3, and f from 3 to 10.

(b) Is it a good investment if it costs $21 million now and yields the following risk-free cash inflows?

Year	1	2	3
Cash Flow (in million dollars)	9	10	11

34. Which of the following investments is most affected by changes in the level of interest rates? Suppose interest rates go up or down by 50 basis points ($\pm$ 0.5%). Rank the investments from most affected (largest change in value) to least affected (smallest change in value).

(a) $1 million invested in short-term Treasury bills.

(b) $1 million invested in Treasury strips (zero coupons) maturing in December 2016.

(c) $1 million invested in a Treasury note maturing in December 2016. The note pays a 5.5% coupon.

(d) $1 million invested in a Treasury bond maturing in January 2017. The bond pays a 9.25% coupon.

Explain your ranking briefly.

35. Valerie Smith is attempting to construct a bond portfolio with a duration of 9 years. She has $500,000 to invest and is considering allocating it between two zero coupon bonds. The first zero coupon bond matures in exactly 6 years, and the second zero coupon bond matures in exactly 16 years. Both of these bonds are currently selling for a market price of $100. Suppose that the yield curve is flat at 7.5%. Is it possible for Valerie to construct a bond portfolio having a duration of 9 years using these two types of zero coupon bonds? If so, how? (Describe the actual portfolio.) If not, why not?

36. Given the bond prices in the question above, you plan to borrow $15 million one year from now (end of year 1). It will be a two-year loan (from year 1 to year 3) with interest paid at the ends of year 2 and 3. The cash flow is as follows:

Year 1	Year 2	Year 3
Borrow $15M	Pay interest	Pay interest plus principal of $15M$

Explain how you could arrange this loan today and "lock in" the interest rate on the loan. What transactions today would be required? What would the interest rate be? You can buy or sell any of the bonds listed above (in the previous question).

37. You purchased a 3 year coupon bond one year ago. Its par value is $1,000 and coupon rate is 6%, paid annually. At the time you purchased the bond, its yield to maturity was 6.5%. Suppose you sell the bond after receiving the first interest payment.

(a) What is the total rate of return from holding the bond for the year if the yield to maturity remains at 6.5% when you sell it?

(b) What if the yield to maturity becomes 6.0% when you sell it?

38. You manage a pension fund, which provides retired workers with lifetime annuities. The fund must pay out $1 million per year to cover these annuities. Assume for simplicity that these payments continue for 20 years and then cease. The interest rate is 4% (flat term structure). You plan to cover this obligation by investing in 5- and 20-year maturity Treasury strips.

(a) What is the duration of the funds 20-year payout obligation?

(b) You decide to minimize the funds exposure to changes in interest rates. How much should you invest in the 5- and 20- year strips? What will be the par value of your holdings of each strip?

(c) After three months, you reexamine the pension funds investment strategy. Interest rates have increased. You still want to minimize exposure to interest rate risk. Will you invest more in 20-year strips and less in 5-year strips? Explain briefly.

39. **Duration and Convexity.**

Consider a 10 year bond with a face value of $100 that pays an annual coupon of 8%. Assume spot rates are flat at 5%.

(a) Find the bond's price and duration.

(b) Suppose that 10yr yields increase by 10bps. Calculate the change in the bond's price using your bond pricing formula and then using the duration approximation. How big is the difference?

(c) Suppose now that 10yr yields increase by 200bps. Repeat your calculations for part (b).

(d) Given that the bond has a convexity of 33.8, use the convexity adjustment and repeat parts (b) and (c). Has anything changed?

40. The yield to maturity of a 10-year zero-coupon bond is 4%.

(a) Suppose that you buy the bond today and hold it for 10 years. What is your return? (Express this return as an annual rate.)

(b) Given only the information provided, can you compute the return on the bond if you hold the bond only for 5 years? If you answered yes, compute the return. If you answered no, explain why.

41. Refer to Table 2.

(a) What was the quoted ask price (in dollars) for the 8.75s of 2020? Assume par value = $10,000. You can ignore accrued interest.

(b) What cash flows would you receive if you bought this bond on August 13, 2006 and held it to maturity? Specify amounts and timing (by month).

(c) Suppose you buy $10 million (par value) of the 4.125s of 2008 and sell short $10 million (par value) of the 3.25s of 2007. You hold each trade until the bond matures. What cash flows would you pay or receive? Specify amounts and timing. You can ignore any fees or margin requirements for the short sale.

Table 2: Treasury Prices and Yields, August 3, 2006

Coupon Rate	Maturity	Bid	Asked	Asked yield
Bonds and Notes:				
3.25	Aug. 07	98:04	98:05	5.09%
4.125	Aug. 08	98:16	98:17	4.89
6.0	Aug. 09	103:00	103:00	4.92
5.75	Aug. 10	103:06	103:07	4.86
4.375	Aug. 12	97:11	97:12	4.88
12.5	Aug. 14	121:08	121:09	4.86
8.75	Aug. 20	136:02	136:03	5.11
6.125	Aug. 29	113:18	113:19	5.11
Strips:				
	Aug. 07	94:30	94:31	5.08%
	Aug. 08	90:18	90:19	4.93
	Aug. 09	86:10	86:10	4.91
	Aug. 10	82:18	82:19	4.80
	Aug. 12	74:25	74:26	4.87
	Aug. 14	67:21	67:22	4.92
	Aug. 16	60:20	60:21	5.05

42. Refer again to Table 2.

(a) What were the 1, 2, 3, 4, 6 and 10-year spot interest rates?

(b) What was the forward interest rate from August 2007 to August 2008? From August 2009 to August 2010?

(c) The 8.75s of August 2020 will pay a coupon in August 2010. What was the PV of this payment in August 2006?

(d) What did the slope of the term structure imply about future interest rates? Explain briefly.

Express your answers to (a) and (b) as effective annual interest rates.

43. Refer again to Table 2. Use the quoted yield on the August 2012 note to calculate the present value for the cash payments on the August 2012 note.

Assume that the first note coupon comes exactly six months after August 13, 2006. Note: The quoted yields are rounded. Your PV may not match the Asked Price exactly.

44. In August 2006 you learn that you will receive a $10 million inheritance in August 2007. You have committed to invest it in Treasury securities at that time, but worry that interest rates may fall over the next year. Assume that you can buy or sell short any of the Treasuries in Table 2 at the prices listed in the table.

(a) How would you lock in a one-year interest rate from August 2007 to August 2008? What transactions would you make in August 2006? Show how the transactions that lock in the rate.

(b) Suppose you wanted to lock in a 5-year interest rate from August 2007 to August 2012. How does your answer to part a change?

45. Assume the yield curve is flat at 4%. There are a 3-year zero coupon bond and a 3-year coupon bond that pays a 5% coupon annually.

(a) What are the YTMs of these two bonds?

(b) Suppose the yield curve does not change in the future. You invest $100 in each of the two bonds. You re-invest all coupons in zero coupon bonds that mature in year 3. How much would you have at the end of year 3?

46. The attached chart shows the fixed obligations of the Edison Mills pension plan, which is also managed by the Renssalear Advisors.

Year	Benefits ($MM)
2000	$10.60
2001	$11.24
2002	$11.91
2003	$12.62
Total	$46.37

Using an assumed interest rate of 6%, the present value of this stream of fixed cash outflows is $40 million. You are given $40 million to invest in U.S. Treasury bonds for the pension plan. Your boss insists that you only invest in 1 year and 10 year STRIPS. Your task is to minimize the exposure of the Edison Mills pension fund to unexpected changes in the level of interest rates. Your performance will be evaluated after one year.

Answer the following questions. Use the backs of this page and the next page if needed to complete your answer.

(a) What is duration of your obligation?

(b) Describe – step by step – how you would choose and manage the portfolio of 1-year and 10-year STRIPS to minimize the exposure to interest rate risk.

47. Bond underwriting.

Bond underwriters agree to purchase a corporate client's new bonds at a specific price, usually near 100% of face value, and then attempt to resell the bonds to the public. The act of reselling takes some time. Underwriting fees increase with the maturity of the bonds. Provide an explanation for this pattern of fees.

48. You have just been given the following bond portfolio:

Bond	Maturity (yrs)	Coupon rate (%)	Holdings ($ million)
A	2	7.00	10
B	5	7.25	20
C	10	7.50	20
D	20	8.00	10

Coupons are paid semi-annually. The current yield curve is flat at 6%.

(a) What is duration for each of the bonds in your portfolio?

(b) What is the duration of your total portfolio?

(c) What is the percentage change in the value of your portfolio if the yield moves up by 20 basis points?

49. A U.S. Treasury bond makes semi-annual payments of $300 for 10 years. (The investor receives 20 $300 payments at 6-month intervals.) At the end of 10 years, the bonds principal amount of $10,000 is paid to the investor.

(a) What is the present value of the bond if the annual interest rate is 5%?

(b) Suppose the bond is observed trading at $11,240. What discount rate are investors using to value the bonds cash flows? (This discount rate is called the bonds "yield to maturity.")

50. A savings bank has the following balance sheet ($ millions, market values).

Assets	Liabilities
Treasuries:$200	Deposits:$900
Floating rate mortgage loans:$300	Equity:$100
Fixed rate mortgage loans:$500	
Total: $1,000	Total: $1,000

Durations are as follows:

Treasuries	6 months
Floating rate mortgage loans	1 year
Fixed rate mortgage loans	5 years
Deposits	1 year

(a) What is the duration of the banks equity? Briefly explain what this duration means for the banks stockholders.

(b) Suppose interest rates move from 3% to 4% (flat term structure). Use duration to calculate the change in the value of the banks equity. Will the actual change be more or less than your calculated value? Explain briefly.

51. Fixed Income Management:

A pension fund has the following liability:

A 20-yr annuity, that will pay coupons of 7% at the end of each year.(t=1...t=20). The pension fund's liability has a face value of 100. The yield curve is flat at 5%.

(a) Calculate the PV and duration of this liability.

(b) The same pension fund has the following assets:

a 1-yr discount bond with face value 100, and a 20-yr discount bond which also has a face value of 100.

Calculate the PV and duration of the portfolio of assets.

(c) How would you change the portfolio composition of assets (keeping the PV of assets the same), so that the NPV of the firm, defined as $PV_A - PV_L$, that is Present Value of assets minus the Present Value of liabilities, is unaffected by interest rate changes?

(d) After making the change above in (c), what is the change in the NPV of the firm if interest rates increase by 10 basis points.

52. Three bonds trade in London and pay annual coupons

Bond	Coupon	Maturity	Price
A	5%	1	100.96%
B	6.5%	3	106.29%
C	2%	3	93.84%

Prices are in decimals, not 32nds.

(a) What is each bond's yield to maturity?

(b) What are the 1, 2 and 3-year spot rates? What are the forward rates?

53. Assume the spot rates for year 1, year 2 and year 3 are 3.5%, 4% and 4.5%, respectively. There are a 3-year zero coupon bond and a 3-year coupon bond that pays a 5% coupon annually.

   (a) What are the YTMs of the bonds?

   (b) Calculate all 1-year forward rates.

   (c) Calculate the realized returns of the two bonds over the next year if the yield curve does not change. (In year 1 the 1-year spot rate is 3.5%, the 2-year spot rate is 4% and the 3-year spot rate is 4.5%.)

54. A pension plan is obligated to make disbursements of $1 million, $2 million and $1 million at the end of each of the next three years, respectively. Find the durations of the plan's obligations if the interest rate is 10% annually.

55. A local bank has the following balance sheet:

Asset	Liability
Loans $100 million	Deposits $90 million
	Equity $10 million

The duration of the loans is 4 years and the duration of the deposits is 2 years.

   (a) What is the duration of the bank's equity? How would you interpret the duration of the equity?

   (b) Suppose that the yield curve moves from 6% to 6.5%. What is the change in the bank's equity value?

   The term structure is flat at 6%. A bond has 10 years to maturity, face value $100, and annual coupon rate 5%. Interest rates are expressed as EARs.

56. (a) Compute the bond price.

   (b) Compute the bond's duration and modified duration.

   (c) Suppose the term structure moves up to 7% (still staying flat). What is the bond's new price?

   (d) Compute the approximate price change using duration, and compare it to the actual price change.

57. Suppose Microsoft, which has billions invested in short-term debt securities, undertakes the following two-step transaction on Dec. 30, 2009. (1) Sell $1 billion market value of 6-month U.S. Treasury bills yielding 4% (6 month spot rate). (2) Buy $1 billion of 10-year Treasury notes. The notes have a 5.5% coupon and are trading at par. Microsoft does not need the $1 billion for its operations and will hold the notes to maturity.

(a) What is the impact of this two-step transaction on Microsoft's earnings for the first 6 months of 2010?

(b) What is the transaction's NPV?

Briefly explain your answers.

58. The Treasury bond maturing on August 15, 2017 traded at a closing ask price of 133:16 (i.e., \$133 16/32) on August 31, 2007. The coupon rate is 8.875%, paid semi-annually. The yield to maturity was 4.63% (with semi-annual compounding).

(a) Explain in detail how this yield to maturity was calculated.

(b) Discount the bond's cash flows, using the yield to maturity. Can you replicate the ask price? (The replication should be close but won't be exact.) Show your calculations.

59. The following questions appeared in past CFA Examinations. Give a brief explanation for each of your answers.

(a) Which set of conditions will result in a bond with the greatest volatility?

  i. A high coupon and a short maturity.
  ii. A high coupon and a long maturity.
  iii. A low coupon and a short maturity.
  iv. A low coupon and a long maturity.

(b) An investor who expects declining interest rates would be likely to purchase a bond that has a . . . coupon and a . . . term to maturity.

  i. Low, long.
  ii. High, short.
  iii. High, long.
  iv. Zero, long.

(c) With a zero-coupon bond:

  i. Duration equals the weighted average term to maturity.
  ii. Term to maturity equals duration.
  iii. Weighted average term to maturity equals the term to maturity.
  iv. All of the above.

60. Please circle your answer to the following questions and provide a one-line explanation.

(a) Holding the one-year real interest rate constant, if the nominal one-year interest rate where to increase by 1%, it would imply that the inflation rate over the same period

  i. Increased.

ii. Declined.

iii. Stayed the same.

iv. It can go either way, impossible to tell from the provided data.

(b) Consider two treasury bonds, A and B. Both have 5 years to maturity, A pays a 5% coupon rate, B pays a 7% coupon rate. Which of bonds A and B has higher modified duration,

   i. A.

   ii. B.

   iii. The same for A and B.

   iv. It can go either way, impossible to tell from the provided data.

(c) A ten-year bond with a coupon rate of 6% and a face value of $100 is priced at $98. Let the yield to maturity be denoted by y. Which of the following statements is true:

   i. $y > 6\%$.

   ii. $y < 6\%$.

   iii. $y = 6\%$.

   iv. It can go either way, impossible to tell from the provided data.

(d) (This may require a calculation.) Suppose the one-year spot rate $r_1 = 5\%$ and the two-year rate $r_2 = 6\%$. At time 0 you enter into a forward contract to buy, in exactly one year from now, a one-year zero-coupon bond. Suppose that in one year from now the term structure of interest rates changes, so that a one year rate becomes 6%. Will you experience a profit or a loss on your forward contract?

   i. Profit.

   ii. Loss.

   iii. No effect.

   iv. It can go either way, impossible to tell from the provided data.

61. Consider two bonds (i) 3 year bond with zero coupons, and (ii) 3 year bond with 5% annual coupon. Assume the yield curve is flat at 5.5%.

(a) Calculate the price and modified duration of each bond.

(b) Suppose the yield curve shift up by 0.1%. What are the new prices of each bond? Check that the % change is close to MD times the yield change.

(c) Suppose the yield curve shifts up by 2%. Show that the approximation is not very close.

(d) You have $10 million and invested 40% of them in the zero coupon bond and 60% in the 5-year coupon bond. What is the modified duration of your portfolio?

62. You manage a pension fund that will provide retired workers with lifetime annuities. You determine that the payouts of the fund are (approximately) level perpetuities of $1 million per year. The interest rate is 10%. You plan to fully fund the obligation using 5-year maturity and 20-year maturity zero-coupon bonds.

(a) How much *market value* of each of the zeros will be necessary to fund the plan if you desire an immunized position?

(b) What must be the *face value* of the two zeros to fund the plan?

63. Do agree with the following statements? Explain your reason.

(a) Higher YTM means higher bond return.

(b) If the forward rates are lower than the current short term spot rate, then we should not enter into the forward rate agreement to lend money because the rates we get are too low.

(c) The market expects a rate cut in next month's Fed meeting, therefore I should load up on bonds to take advantage of the opportunity.

64. You will be paying $10,000 a year in tuition expenses at the end of the next two years. Bonds currently yield 8%.

(a) What is the present value and duration of your obligation?

(b) What maturity zero-coupon bond would immunize your obligation?

(c) Suppose you buy a zero-coupon with value and duration equal to your obligation. Now suppose that rates immediately increase to 9%. What happens to your net position, that is, to the difference between the value of the bond and that of your tuition obligation? What if rates fall to 7%?

65. You bought a 5-year treasury with 5% annual coupon. You decide to hold it until maturity. The yield curve is flat at 5%. The current projected inflation is 2% per year.

(a) Suppose inflation is 2% per year as forecasted. What will be your real return?

(b) Suppose inflation jumps to 3% right after you buy the bond and stay at 3% for the next 5 years. What would your real return be?

(c) TIPS are government risk-free bond that provides protection for inflation. Let's look at a 5-year TIPS with 2% real coupon (and $100 principal). The way TIPS work is that the $100 principal is expressed in real terms. Therefore in nominal (dollar), the principal increases each year with inflation. Therefore if inflation is 2% each year, then the principal becomes $102 (100*(1+2%))in year 1, and $102*1.02 = $104.04 in year 2, etc. The coupon payment each year is based on the new principal adjusted for inflation. For this 5-year TIPS, calculate the cash flow from the bond each year if inflation is (i)2%, and (ii) 3%, respectively.

3 Common Stock

1. True or False? Briefly explain (or qualify) your answers.

   (a) The present value of a share of common stock is an increasing function of the future growth rate of earnings per share.

2. True or false? Briefly explain (or qualify) your answers.

   (a) In principle, the market price of a share of stock equals the discounted value of the stream of future earnings per share.

3. True, false or "it depends" (give a brief explanation):

   (a) Present value is good to value only traded assets since the discount rate comes from returns on traded assets.

   (b) Growth stocks should either have growing dividends or earnings.

4. True, false or "it depends" (give a brief explanation): Managers should maximize the firm's current market value, but only when maximization does not create unacceptable risks for shareholders.

5. True, false or "it depends" (give a brief explanation): The DDM (Dividend Discount Model or the DCF Valuation model) works only for firms with a dividend history.

6. True or false. Briefly explain your answer in each case.

   (a) Growth stocks usually have growing dividends.

7. Question 1 (40 points) True, false or "it depends"? Briefly explain or qualify your answer.

   (a) A company that has not made a profit since its IPO (initial public offering) cannot possibly be a growth stock.

8. True, false (give a brief explanation):

   (a) Firms with higher than average plow back ratios are growth companies.

   (b) Small stocks earn higher average returns because their returns are more volatile.

9. True or false (give a brief explanation): The price of a stock equals the present value of expected future earnings per share.

10. True or false (give a brief explanation): Within an industry, holding business risk and financial leverage constant, differences in firms' P/Es depend only on differences in rates of asset growth.

11. MetaTrend Corp. earns a book rate of return (ROE) of 12%. It reinvests one-half its earnings and pays out the other half as cash dividends. The nominal cost of capital is 12%.

   (a) Given this ROE and dividend payout ratio, what is the growth rate of Meta-Trends earnings and dividends?

   (b) Assume this growth rate is expected to continue in perpetuity. What is the present value of MetaTrend shares? Assume that book value per share is $10.

   (c) What does your answer to (b) assume about the timing of dividend payments? Explain briefly.

   (d) Your calculation in (b) assumes a nominal cost of capital and a nominal growth rate. Restate the cost of capital and growth rate in real (inflation-adjusted) terms and recompute the present value of MetaTrend shares. Show that the present value does not change.

   (e) Suppose MetaTrend decides to pay out all its earnings as cash dividends. Therefore it does not grow. What is the change, if any, in MetaTrends stock price? Why?

12. Consider the following three stocks.

   (a) Stock Q is expected to pay a dividend of $2.20 per share forever – no growth or decline.

   (b) Stock R will pay a dividend of $1.40 next year. Dividends are expected to grow at 3% per year forever.

   (c) Stock S is recovering from several years of losses, during which its dividend was cut to $.50 per year. The now-profitable company is expected to increase dividends by $.50 per year for four years (DIV1 = $1.00, DIV2 = $1.50, DIV3 = $2.00, DIV4 = $2.50). Thereafter dividends are not expected to grow or decline.
   The cost of capital is 9.5%. How much is each stock worth today (ex-dividend)?

13. Northern Co. expects to pay a dividend of $1 per share next year, which will grow at 10% for the following 5 years. Afterwards, the growth becomes 2%. The cost of capital for Northern Co. is 8%. What is today's share price of Northern Co.?

14. Company iTV's earnings per share are expected to be $5 next year and to grow at 8% per year indefinitely. The firm's cost of capital is 10%. The firm's payout ratio will stay at 30%.

   (a) What is iTV's share price?

   (b) Is iTV a growth company? Explain.

15. Company ABC's earnings per share this year are $5. ABC's earnings are expected to grow at rate g every year. The return that investors expect on ABC is 10%. ABC's current stock price is $80. ABC's payout ratio is 0.4.

(a) Determine g.

(b) Determine the present value of ABC's growth opportunities.

16. Company ABC has a book value of $100 per share. Investors demand an expected return of 10%.

(a) For the next three years, the return on equity (ROE) will be 15% and the payout ratio will be 0.3. After year 3, the ROE will drop to 10% and the payout ratio increase to 0.6. What is the stock value per share today? What are ABC's price to earnings ratios (P/E) for the next 4 years?

(b) Re-do part (a) if the payout ratio in the next three years increases to 0.6 and all the other assumptions remain the same as in part (a)). Is the price lower or higher than the price in part (a) and by how much?

(c) Re-do part (a) if the payout ratio after three years increases to 0.9 and all the other assumptions remain the same as in part (a). Is the price lower or higher than the price in part (a) and by how much?

(d) Explain the changes in share price in part (b) and (c)

17. You remain skeptical about some of the assumptions in 15.401. For example, if the DCF (discounted cash-flow formula) is correct, how could the stock market fall by 23% on October 19, 1987?

Try to convince yourself (and us) that such an event is possible even in a world of efficient markets; all you need is only a slight change of expectations.

[HINT: Recall the dividend discount model: P0=D1/(r-g). Explain what each variable of the formula means. Now put some reasonable numbers for the right-hand side variables for the case of S&P 500: say D1 = 20, r=12% and g=10%. Taking each variable in turn in the right-hand side of the equation, and holding constant the other variables in the right-hand side of the equation, estimate how much it should change to cause a 23% drop in P0. Is this just a slight change? Briefly explain your answer.]

18. Digital Organics has been growing at a rate of 6 percent per year and is expected to continue to do so indefinitely. The next dividend is expected to be $5 per share.

(a) If the market expects a 10 percent rate of return on Digital Organics, at what price must it be selling?

(b) Next year Digital Organics earnings per share will be $8. What part of Digital Organicss value is due to assets in place, and what part to growth opportunities?

19. The Sloan Management Group expects to pay a dividend of $4.50 per share one year from now. After this payment, the annual dividend is expected to grow (in perpetuity) in real terms at 4% per year. The appropriate nominal discount rate for valuing the dividends is 9.5% per year. Inflation is expected to be 2% per year. Given these assumptions, what is the present value of the stream of future dividends paid by Sloan Management Group stock?

20. Company Ts current return on equity (ROE) is 16%. It pays out one-quarter of earnings as cash dividends (payout ratio = .25). Current book value per share is $35. The company has 5 million shares outstanding.

   Assume that ROE and payout ratio stay constant for the next four years. After that, competition forces ROE down to 10% and the company increases the payout ratio to 60%. The company does not plan to issue or retire shares. The cost of capital is 9.5%.

   (a) What is stock T worth?

   (b) How much of stock Ts value is attributable to growth opportunities (PVGO)?

21. Now suppose the management of company T decides to increase the dividend payout ratio to 60% starting next year, at the same time maintaining the investment and growth rates that you calculated in answering pervious question. The company will cover any cash shortfall by issuing additional shares.

   (a) How many additional shares will Company T have to issue next year?

   (b) What is the effect on the (ex-dividend) value of stock T today?

   You can assume that the stock issue is made immediately after the dividend at year 1. The company can take out a short-term loan if necessary to cover the additional dividend at year 1.

22. Company XYZ is expected to pay a dividend of $5 a year from now. This dividend is expected to grow at 10% for the next two years and at 5% forever after. The return that investors expect on XYZ is 12%. XYZ's payout ratio is 0.3.

   (a) Determine XYZ's stock price.

   (b) Determine XYZ's PVGO.

   (c) Determine XYZ's P/E ratio.

   (d) What will be the P/E ratio of XYZ in one year from now?

   (e) To see how sensitive your conclusions are to the assumption about discount rates, re-compute your answers to previous questions assuming that the required rate of return on XYZ is either 10% or 14%.

23. Firm XYZ has two lines of business, organized as two divisions, A and B. Division A generates a risk-free cash flow. It will produce $2 million in free cash flow next year and it will grow at 2% each year thereafter forever. The second line of business, run by Division B, is risky. It expects to generate a cash flow of $2 million next year and will grow at a rate of 4%. Currently, the total market value of XYZ is $87 million. The term structure of interest rate is flat at 5%.

   (a) What is the cost of capital for the second line of business?

(b) Assume the company comes across a new technology that can improve the Division B's profitability. It requires an initial investment of $5 million and will increase next year's cash flow by $0.8 million as well as future cash flows at so that their growth rate stays at 4%. If the management decides to take on this new technology, what will the market value of XYZ now be?

24. Unigene Labs has existing assets that generate an EPS of $5 per year, which is expected to remain constant if the firm does not invest except to maintain exiting assets. Unigene is all-equity financed and its stock has a beta of 1.2. You estimate the market risk premium to be 8% and the risk free rate to be 4%.

Next year (year 1), the firm has the opportunity to invest $3 per share to launch a new product, which will increase its EPS earnings by $0.80 per year permanently, starting the year after (year 2). The earnings from this product are highly volatile, with an annual standard deviation to be 50% and a correlation to the market to be 0.1. The market's standard deviation is 20%.

   (a) What is the cost of capital for the company's existing assets?

   (b) What would be stock price and the price-to-earnings ratio at time zero if the firm did not plan to launch the new product?

   (c) What is the appropriate discount rate for the new product? Explain your answer.

   (d) What would be stock price and the price-to-earnings ratio at time zero if the firm did plan to launch the new product?

25. PDQ Corp. is earning EPS of $3.00 next year, 15% of book value per share (BVPS) of $20.00. The companys sales revenues are expanding at 10% per year, and assets and BVPS will grow proportionally. But growth of revenues and assets will drop to 5% per year after year 5.

   (a) Assume earnings will continue to be 15% of BVPS. What is PDQ stock worth? The cost of capital is 10.

   (b) Your answer epended in part on a horizon value for the companys shares in year 5. What does that horizon value assume about the NPV of PDQs growth opportunities from year 6 on?

26. Company Us earnings and dividends have been growing at a steady 15% per year. You are confident that the growth will continue for at least one more year, but the growth is not sustainable for the long run. Eventually the companys growth rate will drop below 10%. The current price is $62, and next years dividend is forecasted at $.50 per share.

A security analyst forecasts the expected rate of return over the next year as:

$$r = \frac{DIV_1}{P_0} + g = \frac{0.50}{62} + 0.15 = 15.8\%$$

Explain why 15.8% is an upward-biased forecast of next years rate of return.

27. Consider a company XYZ. As of today, it has 1,000,000 shares of stock outstanding, trading at $50 per share and no debt. Assume that tomorrow the company decides, unexpectedly, to invest in a new project, which requires an initial investment of $10,000,000 today and is expected to produce a growing stream of cash flows, starting from $1,200,000 in one year from now and growing at 3% per year. The cost of capital for the new project is estimated at 12%. These new investment plans are not yet known. The company decides to finance the new project by issuing equity. It would like to issue just enough shares to raise the necessary $10,000,000 for the initial investment.

   (a) How many shares should the company issue?

   (b) What will be the share price after the new project is adopted?

   (c) How would your answer change if the investment in the new project was not unexpected, but rather was well known to the market a month in advance?

28. Your rich aunt has died. Her will gives you ownership of 5,000 shares of Plum Creek Timber Company, currently trading at $22 per share. But various legal complications will delay distribution of your aunts shares to you until October 2008.

   Plum Creek now pays an annual dividend of $1.40 per share. Assume for simplicity that the next dividend will be paid in September 2008, just before you will receive the shares. (Therefore you will not receive the next dividend.) In the past Plum Creeks dividend has increased by 3% per year. This is a reasonable long-term trend, but security analysts are pessimistic for the immediate future. They forecast no growth in earnings and dividends for the next 3 years.

   Plum Creek is a relatively safe security. Investors are content with an 8% expected rate of return.

   A bank offers to buy the shares from you for $18.50 per share paid immediately. (The bank would pay $18.50 per share now and receive the shares next October.) Is this a fair offer?

29. Chucky Cheese has a cost of capital of 9% per year. Its expected EPS next year is $5.00. The firm plans to plow back 40% of its earnings for new investments in the following years. The ROE on the new investments is 12%.

   (a) Calculate the share price and the P/E ratio of this firm.

   (b) If the plowback ratio increases, will the P/E ratio increase, decrease or remain unaffected? State any assumptions you make and give a brief justification.

   (c) Given your answer to b), what will be your advice to this firm on its dividend policy?

30. iDoc, a health service company, is expected to generate $1 in earnings per share next year and in the years to follow, from its existing assets. It plans to announce a new program to expand its business. This new program will increase its plow back ratio

from zero to 50% next year. The return on equity for the new investments will be 12%. The expansion is expected to continue forever at the same rate. The cost of capital is 10%.

(a) What is the expected dividend next year and its future growth under the new program?

(b) What is the change in iDoc's stock price in response the announcement?

(c) Is iDoc a growth company? What is its P/E ratio before and after the announcement?

31. MW Co. expects earnings of $1.25 per share next year, out of which $0.50 will be paid out as dividends. Earning and dividends are expected to grow at a constant rate g each year afterwards. MW shares are now traded at $20. The cost of capital for MW Co. is 10%.

(a) What is the expected growth rate of earnings g?

(b) What is the ROE for MW?

(c) Is MW a growth company? Justify your answer.

32. Fast Track, a local bus company providing direct service between New York and Boston, had after-tax earnings of $2.5 million in the past year (year 0) and expected the same earnings forever. An investment bank had valued Fast Track at $20 million. Now by allowing passengers to book online, Fast Track expects its after-tax earnings to grow at 2.5% per year, starting this year (year 1). How much would the value of Fast Track increase by this change? (Assume the cost of capital stays the same.)

33. Dragon and Tiger Island (DTI), an online game company, expects next year's after tax earnings to be $20 per share. Its business is still expanding. It plows back 80% of its earnings. The ROE on its new investments is 15%. Its cost of capital is 12.5%.

(a) What is the share price of DTI? What is its PVGO?

(b) Suppose that a new competitor comes in and cuts DTI's ROE to 12%. How would this impact DTI's investment decisions and its share price? Explain why.

34. The dividend yield for shares of the Union Pacific Railroad is 1.9%. Security analysts are forecasting rapid growth in Union Pacific's earnings per share (EPS), about 12.7% per year for the next three years. Does that imply an expected rate of return of $1.9 + 12.7 = 14.6\%$? Explain.

35. The Northern Company is a utility company with existing assets that generates an EPS (earnings per share) of $5. If the firm only maintains existing assets, EPS is expected to remain constant at $5 a year. However, next year, the Northern Company has the opportunity to invest $3 per share to develop a new electricity generator using solar energy. The development of the new generator will be completed next year. This investment is expected to generate a return (ROE) of 20% per year forever. The cost

of capital is 10%. What will be the Northern Companys share price if it decides to develop the new generator? Use the back of this page if needed to complete your answer.

36. BetaTrend is an exact match for MetaTrend except for one thing: it generates a continuous stream of earnings and dividends. Thus it generates earnings in a continuous stream at 12% per year, starting immediately, and pays out half of earnings as dividends. What is the present value of BetaTrend stock? The annually compounded cost of capital is 12%.

1 Present Value Solutions

1. (a) $\$10,000 \times (1.045)^5 = \$12,461.82$.

   (b) $\$10,000 \times (1 + \frac{0.045}{4})^{(4 \times 5)} = \$12,507.51$.

   (c) $\$10,000 \times (1 + \frac{0.045}{12})^{(12 \times 5)} = \$12,517.96$.

2. Using the following formula $r_{EAR} = (1 + \frac{r_{APR}}{k})^k - 1$

   (a) Annual: $\approx 4.75\%$

   (b) Monthly: $\approx 4.85\%$

   (c) Weekly: $\approx 4.86\%$

   (d) Contious using the following formula $r_{EAR} = e^{r_{APR}} - 1 \approx 4.86\%$

3. (a) $\$50,000 \times (1.04)^2 = \$54,080$

   (b) $\$50,000 \times (1.01)^8 = \$54,142.84$

   (c) $\$50,000 \times (1 + \frac{0.04}{12})^{(12 \times 2)} = \$54,157.15$

4. (a) $\$10,000 \times (1.045)^5 = \$12,461.82$.

   (b) $\$10,000 \times (1 + \frac{0.045}{4})^{(4 \times 5)} = \$12,507.51$.

   (c) $\$10,000 \times (1 + \frac{0.045}{12})^{(12 \times 5)} = \$12,517.96$.

5. **e-Money**. The effective annual interest rate for an A.P.R. of 5.25% is

   (a) 5.2500% with annual compounding

   (b) 5.3782% with monthly compounding

   (c) 5.3899% with daily compounding

   (d) 5.3903% with continuous compounding.

   We used the following two formulas for this question: $r_{EAR} = (1 + \frac{r_{APR}}{k})^k - 1$ and $r_{EAR} = e^{r_{APR}} - 1$.

6. False. The choice of the horizon date should not affect the present value.

7. (a) $1.055 \times (\frac{\$10,000}{0.055})(1 - \frac{1}{1.055^{10}}) \times (1.055)^{10} = \$135,835$ (end of year 10)

   (b) $\frac{\$135,835}{(1+0.02)^{10}} = \$111,432$ (NPV=79,521)

8. (a) $PV = \frac{1.5}{1.12} + \frac{1.3}{1.12^2} + \frac{1.05}{1.12^3} + \frac{0.9}{1.12^4} + \frac{0.75}{1.12^5} = \$4.12$ million.

   (b) Inflation affects the cash flow and the discount rate equally, and the two effects cancel out in the PV calculation. Therefore, PV = $4.12 million.

9. Assuming the rent payments are paid at the beginning of the year, the PV of the floating rent for the next 5 years is

$$\$100,000 + \frac{\$100,000 * 1.05^1}{1.12^1} + \cdots + \frac{\$100,000 * 1.05^4}{1.12^4} = \$441,285.71$$

This must be equal to

$$\$441,285.71 = F + \frac{F}{1.12} + \frac{F}{1.12^2} + \frac{F}{1.12^3} + \frac{F}{1.12^4}$$

Solve for F, to get $F = \$109,300.67$

10. (a) Cash option PV = \$40,000

Financing option PV 2% $= 937.5 \times \frac{1}{1.02^{1/12}-1} \times \left(1 - \frac{1}{\left(1.02^{(1/12)}\right)^{48}}\right) = \$43,228$

Cash option costs less

(b) Cash option PV = \$40,000

Financing option 15% $= 937.5 \times \frac{1}{1.15^{1/12}-1} \times \left(1 - \frac{1}{\left(1.15^{(1/12)}\right)^{48}}\right) = \$34,271$

Cash option costs more

11. Year 1: $10 \times \left(\frac{1.05 \times 1.02}{1.10}\right)^1 = 9.7364$
Year 2: $10 \times \left(\frac{1.05 \times 1.02}{1.10}\right)^2 = 9.4797$
Year 3: $10 \times \left(\frac{1.05 \times 1.02}{1.10}\right)^3 = 9.2298$
PV: $9.7364 + 9.4797 + 9.2298 = \$28.4458$ million.

12. The value of company ABC is the PV of the growing annuity of cash flow:
PV $= \frac{10}{1.09 - 1.05} = \$250$ million.

13. Assume all the oil is sold at the end of each year; for example the PV of the production for the first year is $\frac{\$7000 \times \$15}{1.08} = \$97,222$. The PV of the second year production is $\frac{\$6580 \times \$15}{1.08^2} = \$84,619$. The PV of the total production in the first 12 years is \$608,254 and by then the annual production has declined to 3,544 barrels.

14. (a) Assuming first cash flow one year from now in real terms is \$14m:
$r_{real} = (1.08/1.02) - 1 = 5.88\%$
$PV = 14/0.0588 = \$238m$ (no growth)
Using nominal values: $14 * 1.02/(0.08 - 0.02) = \$238m$
Alternatively, if assuming first cash flow one year from now in real terms is
$14/1.02 = \$13.73m$, $PV = (14/1.02)/0.0588 = \$233.33m$
Using nominal values: $14/(0.08 - 0.02) = \$233.33m$
Note: in order to get full credit, the assumptions made should have been clearly stated.

(b) To calculate PV of cash flows generated in a continuous stream, starting immediately, we need to use the same perpetuity formula (PV = C / r), but substitute the continuously compounded rate

New $r = log(1.08/1.02) = 5.72\%$ and $PV = 14/0.0572 = \$244.93m$

Common mistake: not using this compounding formula / forgetting that the inflation rate also must be adjusted. Using nominal values instead, $14/(log1.08 - log1.02) = \$244.93m$

Note: in order to get full credit, the assumptions made should have been clearly stated.

15. The PV of the first scholarship from the foundations point of view is

$$-(20,000/0.06)[1 - (1/(1.06)^4)] + (1/(1.06)^5)(8,000/0.06)[1 - (1/(1.06)^{10})] = -25,303$$

So it loses \$25,303 every year beginning from t=0. The PV of this perpetuity is -25,303/0.06-25,303 = -447,020

This implies that the investment needed to fund this is \$447,020

16. The market value of the lease is \$1.25 million for 5 years, with the first payment due right now. So the value is
$1.25 + \frac{1.25}{1+6\%} + \frac{1.25}{(1+6\%)^2} + \frac{1.25}{(1+6\%)^3} + \frac{1.25}{(1+6\%)^4} = \$5.5814$ million.
The value of your current lease is \$1 million for 5 years, and the total PV is \$4.4651 million. Therefore the difference is \$1.1163 million, and this should be the compensation you asked from the owner.

17. Use growing annuity formula assuming that the payments are made at the begining of each year and you pay in full for the year that you die (unfortunately).

The value if you have an expected life of T years is:

$$
\begin{aligned}
PV &= \$750 + \$750 * \left(\frac{1.05}{1.12}\right) + \cdots + \$750 * \left(\frac{1.05}{1.12}\right)^T \\
&= \$750 + \$750 \left[\frac{1}{0.12 - 0.05} - \frac{1}{0.12 - 0.05} \times \left(\frac{1.05}{1.12}\right)^T\right]
\end{aligned}
\tag{1}
$$

Solve for T. The breakeven point is $T >= 16$

18. (a) Let the annual payment $= C$. The PV of all my payments, discounted at the dealer's rate, must equal to the price, i.e.,
$3500 + \frac{C}{0.02}(1 - \frac{1}{1.02^5}) = 30000$.
$3500 + 4.71346C = 30000$.
$C = \$5,622.20$.

(b) Since I can save at a higher rate, the cost of the financing plan in (a) is only
$3500 + \frac{C}{0.05}(1 - \frac{1}{1.05^5}) = \$27,841.17$.
The cost of the second option is $30,000 - 2,500 = \$27,500$. I should pay cash.

19. The fair monthly rate is $(1 + 0.08)^{1/12} - 1 = 0.006434$.
Let C be the appropriate monthly payment. Then, $\frac{C}{0.006434}\left[1 - \frac{1}{1.006434^{120}}\right] = 100,000$.
$C = \$1,198.58$.

20. The PV of all cash inflows and outflows is
$\text{PV} = \frac{-65}{1.05} + \frac{-65}{1.05^2} + \frac{10}{1.05^3} + \frac{10}{1.05^4} + \frac{40}{1.05^5} + \ldots + \frac{40}{1.05^9} = \$38,478,57$.
I am taking a financial gain by helping him out.

21. The PV of the offer at discount rate of 12% is $359,921. With the discount rate of 8% the PV increases to $446,821. In both cases accepting the award of $500,000 is preferable. This calculation ignores other consideration such as difference tax treatment.

22. (a) $\text{EAR} = (1 + \frac{.06}{12})^{12} - 1 = 0.06168$.

(b) Let C be the monthly payment. The effective interest rate per month is $\frac{0.06}{12} = 0.005$. The PV of the mortgage annuity must equal the amount borrowed:
$\frac{C}{0.005}\left[1 - \left(\frac{1}{1.005}\right)^{(12 \times 30)}\right] = 600,000$.
$C = \$3,597.30$.

(c) $\frac{3597.30}{0.005}\left[1 - \left(\frac{1}{1.005}\right)^{(12 \times 25)}\right] = \$558,326.10$.

23. False. US treasury bond carries inflation risk.

24. (a) $\text{PV} = \frac{500}{0.06}\left[1 - \frac{1}{1.06^{10}}\right] + \frac{10000}{1.06^{10}} = \$9,263.99$.

(b) The effective interest rate per 6 months is $1.06^{1/2} - 1 = 0.02956$.
$\text{PV} = \frac{250}{0.02956}\left[1 - \frac{1}{1.02956^{20}}\right] + \frac{10000}{1.06^{10}} = \$9,318.39$.

25. (a) $\text{EAR} = (1 + \frac{.085}{12})^{12} - 1 = 0.088391$.

(b) The effective interest rate per month is $\frac{.085}{12} = 0.0070833$.
Let C be the monthly payment. The PV of the mortgage annuity must equal the amount borrowed:
$\frac{C}{0.0070833}\left[1 - \frac{1}{1.0070833}^{(12 \times 30)}\right] = 500,000$.
$C = \$3,844.57$.

(c) $\frac{3844.57}{0.0070833}\left[1 - \frac{1}{1.0070833}^{(12 \times 30 - 60)}\right] = \$477,451.33$.

26. The PV of the current salary at the discount rate of 8% is $468,687. It is assumed that salaries are paid at the end of each year (only an approximation to the more accurate monthly payment) and he works up to the end of his 65th year.

To calculate the break-even starting salary after MBA, one needs to include the negative NPV of the cost of MBA as well as the lost income for the 2-years spent in school and compare that against the positive NPV of the increased earning power. The negative NPV of the cost of MBA is $35,665 and the loss in salary is $71,330 for a total of $-106,995.

Even if the starting salary after the MBA is as low as $32,687 it still pays off to get the MBA. Of course, getting an MBA makes the skill set more flexible (such as the

general management skills or the network of people you know) which reduced the risk of the income as well. The aspect if not considered here but definitely would make the MBA even more valuable.

27. (a) PV at $t = 5$ of the growing annuity from $t = 6$ to 35:
$100,000\frac{1.03}{0.04-0.03}\left[1 - \left(\frac{1.03}{1.04}\right)^{30}\right] = \$2,591,785.58$.
PV at $t = 0$ of the growing annuity:
$2591785.58 \times (1.04)^{-5} = \$2,130,258.82$
PV at $t = 0$ of the annuity from $t = 1$ to 5:
$\frac{100,000}{0.04}\left[1 - \frac{1}{1.04^5}\right] = \$445,182.23$.
Total human capital:
$2,130,258.82 + 445,182.23 = \$2,575,441.06$.

(b) FV at $t = 5$ of savings:
$\frac{30,000}{0.04}\left[1.04^5 - 1\right] = \$162,489.68$.

28. (a) Assume that every year, Mr. Jones withdraws the year's living expense at the beginning of the year. If he retires today and lives for another T years, he would need the following amount:
$S = 100,000\frac{1.05}{1.05-1.02}\left[1 - \frac{1.02}{1.05}^T\right]$.
(i) $T = 25 : S(25) = \$1,804,335.76$ (enough).
(ii) $T = 45 : S(45) = \$2,550,363.68$ (not enough).
(iii) $T = 60 : S(60) = \$2,885,219.04$ (not enough).

(b) Mr. Jones can (i) work longer and retire later, (ii) downgrade his lifestyle and reduce living expense, or (iii) find better investment opportunities that give higher returns (though he must also consider the risk involved).

29. The nomilar value is $\$50,000 \times (1.075^{10})$ Inflation reduced this by a factor of $(1.03)^{10}$. So the real value is only $\$76,680$.

30. (a) $r_{APR} = ln(r_{EAR} + 1) = ln(1.12) = 0.1133$

(b) $e^{0.05} - 1 = 0.0513$

(c) $ln(1.09) = 0.08617$

31. (a) PV $= \frac{275}{0.026}\left[1 - \frac{1}{1.026^{20}}\right] + \frac{10000}{1.026^{20}} = \$10,231.64$.

(b) (A convex, downward sloping curve)

32. **The Reborn VW Beetle**.

First use the annuity formula to determine the monthly payments C_a and C_b for dealerships A and B, respectively, ignoring the initial down payments:

(a) Dealership A: PV$_a$ = \$18,000, $r_a = 0.08/12$, and $t = 36$ months $\Rightarrow C_a = \$564.05$.

(b) Dealership B: PV$_b$ = \$15,500, $r_b = 0.10/12$, and $t = 36$ months $\Rightarrow C_b = \$500.14$.

If the monthly discount rate is currently r, then the net present values of the two packages are

$$\text{NPV}_a = 2{,}000 + C_a \left[\frac{1}{r} - \frac{1}{r(1+r)^{36}} \right]$$

$$\text{NPV}_b = 4{,}000 + C_b \left[\frac{1}{r} - \frac{1}{r(1+r)^{36}} \right].$$

It is clearly more advantageous to accept dealership A's offer if and only if $\text{NPV}_a <$ NPV_b. Substituting the expressions from above and simplifying, we have that $\text{NPV}_a <$ NPV_b if and only if

$$\left[\frac{1}{r} - \frac{1}{r(1+r)^{36}} \right] < 31.29.$$

By trial and error, the cross-over point is at $r = 0.00778$. The conclusion is that if the current annual interest rate for a 36-month period (compounded monthly) is above 9.34%, you should choose dealership A. If the current annual interest rate for a 36-month period (compounded monthly) is below 9.34%, you should choose dealership B.

33. Monthly amount to fund from the mutual fund = 15,000-6,000 = \$9,000

PV of the fund in three years $= \$1{,}200{,}000 \times 1.035^3 = \$1{,}330{,}461$

Number of years that it will last:

$\$1330461 = 1.035 \times 9000 \times 12 \times \frac{1}{0.035} \times \left(1 - \frac{1}{(1+0.035)^T}\right)$

(assume you take the money out at the beginning of the year)

$T = 16.4$ years

34. (a) $\text{PV} = \frac{50000}{0.055} \left[1 - \frac{1}{1.055^{12}}\right] = \$430{,}925.89$.

(b) $\text{PV}_{-0.5} = \frac{50000}{0.055} \left[1 - \frac{1}{1.055^{12}}\right] = \$430{,}925.89$.
$\text{PV}_0 = 430{,}925.89 \times (1.055)^{0.5} = \$442{,}617.74$.

35. IRA Accounts and Taxes

(a) Yearly contribution: $2000 \times (1 - 28\%) = \$1440$:

$$\sum_{t=1}^{30} \left\{ 2000 \times (1 - 0.28) \times [1 + 0.06 \times (1 - 0.28)]^{30-t} \right\} = \$85{,}218.$$

(b) To get the total amount at end of 30 years:

$$\sum_{t=1}^{30} \left\{ 2000 \times (1 - 0.28) \times [1 + 0.06 \times (1 - 0.00)]^{30-t} \right\} = \$113{,}843.$$

of which the principal is $30 \times 2000 \times (1 - 0.28) = \$43{,}200$, implying that the interest is $113{,}843 - 43200 = \$70{,}643.79$. Tax on this is 70643.79×0.28 implying that the take-home amount is \$94,063.

(c) The total amount is

$$\sum_{t=1}^{30} \left[2000 \times (1 + 0.06)^{30-t} \right] = \$158,116.37.$$

After taxes, this is only $158116.37 \times (1 - 0.28) = 113,843.789$. Note: The answers to (b) and (c) are identical. Given the after–tax interest rate, it doesn't matter whether your money is taxed on the way in or the way out.

(d) Increase, because more deferred tax can be used to accrue interest.

2 Fixed Income Securities Solutions

1. (a) TRUE

   Coupon payments accelerate cash flows received from the investment reducing the duration.

   (b) FALSE

   Higher yield to maturity (YTM) does not mean that the bond is a good investment; NPV=0 in any case.

   Two good answers to this question were:

   1. Bonds with longer duration have usually higher yield to maturity (YTM) as a result of increasing yield curve.

   2. Bonds with higher probability of default have higher YTM to compensate for credit risk.

   Full answer must mention at least one of these two points. Many students concentrated on reinvestment risk only.

   (c) FALSE

   "On the run" refers to a newly issued bond. These bonds are usually more liquid and trade at a premium.

2. (a) False: yield of longer term bond also incorporates the expectation of future short term interest rates and depending on the this factor, the term structure may be up- or down-ward sloping.

   (b) False: If interest rates rise, future coupons of such bonds fall, and so their price (which is the present value of future coupons) falls more than the price of its straight bond counterpart. If interest rates fall, future coupons of such bonds rise, and so their price (which is the present value of future coupons) rises more than the price of its straight bond counterpart. So these bonds are more interest rate sensitive.

3. (a) False. The term structure of interest rates depends on expected future interest rates, if short term interest rates are expected to decrease substantially, the term structure can be inverted, flat, or downward sloping even when there is a liquidity premium for long term interest rates.

   (b) False. Bonds with higher duration are more sensitive to changes in interest rates. If we examine coupon bonds, all things equal a bond with a higher coupon rate will have a lower duration and thus be less sensitive to changes in interest rates.

4. False. The term structure depends on the expected path of interest rates (among other factors): if interest rates are expected to fall, the term structure will slope downward.

5. False. If there is liquidity premium, then a flat term structure means that investors expect interest rates to fall.

6. No. To minimize interest rate risks, we want $MD(A) \times V(A) - MD(L) \times V(L) = 0$. If $V(A) > V(L)$, we want $MD(A) < MD(L)$. That means we should invest in assets with shorter duration.

7. (a) Effective annual rate on 3-month T-bill:

$$\left(\frac{100,000}{97,645}\right)^4 - 1 = (1.02412)^4 - 1 = .10 \text{ or } 10\%.$$

(b) Effective annual interest rate on coupon bond paying 5% semiannually:

$$(1.05)^2 - 1 = .1025 \text{ or } 10.25\%.$$

Therefore, the coupon bond has the higher effective annual rate.

8. The qoutes yield is calculated as

$$y = \frac{1}{t}\frac{F - P}{P}$$

where t is in fraction of year, i.e. 1/6 in this example. Using $F = \$100,000$ the current price is $99,009$. So the effective annual rate is $\left(\frac{\$100,000}{\$99,009}\right)^6 - 1 = 6.15\%$

9. (a) $1 + EAR = (100/98.058)^2$ so EAR=4%

(b) Bond at par, so yield=coupon. So $1 + EAR = (1 + 0.042/2)^2$. So EAR=4.24%
The coupon bond has the higher EAR.

10. (a) $r_1 = \frac{\$100}{\$96.2} - 1 = 3.95\%$, $r_2 = \left(\frac{\$100}{\$91.6}\right)^{1/2} - 1 = 4.45\%$ $r_3 = \left(\frac{\$100}{\$86.6}\right)^{1/3} - 1 = 5.11\%$

(b) $PV = \$300M \times \frac{96.2}{100} + \$210M \times \frac{91.6}{100} + \$400M \times \frac{86.1}{100} - \$600M = \$225.36$

11. (a) Price $= \frac{100}{1.04^3} = 88.90$; YTM $= 0.040$.

(b) Price $= \frac{5}{1.03^1} + \frac{105}{1.035^2} = \$102.87$; YTM solves $\frac{5}{(1+y)^1} + \frac{105}{(1+y)^2} = \$102.87$: $y = 0.03488$.

(c) Price $= \frac{6}{1.03^1} + \frac{6}{1.035^2} + \frac{6}{1.04^3} + \frac{106}{1.045^4} = 105.65$;
YTM solves $\frac{6}{(1+y)^1} + \frac{6}{(1+y)^2} + \frac{6}{(1+y)^3} + \frac{106}{(1+y)^4} = 105.65$: $y = 0.04428$.

12. The effective annual yield on the semi-annual coupon bonds is 8.16%. If the annual coupon bonds are to sell at par, they must offer the same yield, which will require an annual coupon of 8.16%.

13. (a) $P_{1-year} = \frac{\$100}{1+5.25\%} = \$95.01$
$P_{2-year} = \frac{\$5}{(1+5.50\%)} + \frac{\$100}{(1+5.50\%)^2} = \$99.08$
$P_{3-year} = \frac{\$6}{(1+6\%)} + \frac{\$6}{(1+6\%)^2} + \frac{\$106}{(1+6\%)^2} = \$100$

(b) The first year, is easy. It is simply the yield of the 1-year note so, $r_1 = 5.25\%$. Then solve for r_2 and r_3 recursively. $r_2 = 5.505\%$ and $r_3 = 5.7\%$.

(c) Use the fact that $\left(\frac{1}{1+r_3}\right)^3 = \left(\frac{1}{1+r_2}\right)^2 \times (1 + f_{2,3})$ so $f_{2,3} = 6.1\%$

14. (a) Spot rate at 2 years is 2.0%. At 3 years is 2.58%. At 8 years is 4.24%.

(b) Use this relation: $(1 + r_{t-1})^{t-1}(1 + f_{t-1,t}) = (1 + r_t)^t$
The one year forward rate from Aug 2004 to Aug 2005 is 3.74%.
Similarly the one year forward rate from Aug 20010 to Aug 20011 is 6.33%.

(c) Under the Expectation Hypothesis, the upward slope implies that interest rates are expected to go up. In practice, the long-term average term structure is upward sloping. However, the August 20, 2002 slope is very steep. So interest rates are expected to rise.

15. (a) $(1 + r_2)^2 = (1 + r_1)^1(1 + f_{1,2}) \rightarrow f_{1,2} = 3.5\%$

(b) $(1 + r_3)^3 = (1 + r_1)^1(1 + f_{1,3})^2 \rightarrow f_{1,3} = 3.95\%$

(c) Let's calculate the forward rate first. $(1+r_4)^4 = (1+r_3)^3(1+f_{3,4}) \rightarrow f_{1,3} = 6.02\%$
So the TA is offering too high of a rate. What you need to do is as follows:

  i. Enter into a contract to lend him money at 6.3% between years 3 and 4.

  ii. Short a 4 year strip

  iii. Use the proceeds form the shorting to buy 3 year strip. The amount should be based on how much you enter into a contract with the TA for.

If you follow these steps you will have more than enough to cover the short position in year 4 (since the rate you lend at is higher than the fair forward rate). Hence, this is an arbitrage. The risks you are exposed are contract and counter-party risk, for example the TA may go bankrupt before year 3 if he continues to enter into contracts of this form and may not be able to honor his/her end. In that case, you will end up with hanging position and expose to various interest rate risks.

16. (a) A: $\frac{100}{1.05} = 95.24$.
B: $\frac{5}{1.055} + \frac{105}{1.055^2} = 99.08$.
C: 100 (coupon rate = YTM).

(b) $r_{0,1} = 0.05$.
$r_{0,2} = \left[\frac{105}{99.08-5/1.05}\right]^{0.5} - 1 = 0.05513$.
$r_{0,3} = \left[\frac{106}{100-6/1.05-6/1.05513^2}\right]^{1/3} - 1 = 0.06041$.

(c) Long $100\frac{100}{106} = 0.9434$ C
Short $0.9434\frac{6}{105} = 0.05391$ B
Short $\frac{1}{100}[6 \times 0.05513 - 5 \times 0.06041] = 0.05391$ A

(d) No arbitrage means that the bond in (c) should cost $0.9434 \times 100 - 0.05391 \times 99.08 - 0.05391 * 95.24 = 83.86$.
Note that $\left[\frac{100}{83.86}\right]^{1/3} - 1 = 0.06041$.

17. (a)

$$r_1 = \frac{110}{106.8} - 1 = 3.00\%$$

$$101.93 = \frac{5}{1 + r_1} + \frac{105}{(1 + r_2)^2} \Rightarrow r_2 = 4.00\%$$

$$111.31 = \frac{10}{1 + r_1} + \frac{10}{(1 + r_2)^2} + \frac{110}{(1 + r_3)^3} \Rightarrow r_3 = 6.00\%$$

(b)

$$f_2 = \frac{(1 + r_2)^2}{1 + r_1} - 1 = 5.0\%$$

18. (a)

$$95.92 = \frac{100}{1 + r_1}$$

$$92.01 = \frac{100}{(1 + r_2)^2}$$

$$87.00 = \frac{100}{(1 + r_3)^3}$$

$$r_1 = \frac{100}{95.92} - 1 = 4.25\%$$

$$r_2 = \left(\frac{100}{92.01}\right)^{1/2} - 1 = 4.25\%$$

$$r_3 = \left(\frac{100}{87.00}\right)^{1/3} - 1 = 4.75\%$$

(b)

$$f_{2,3} = \frac{(1 + r_3)^3}{(1 + r_2)^2} - 1$$
$$= 5.76\%$$

(c) The yield to maturity is simply 4.25% since the one year and two year spot rates are roughly equivalent, more specifically when we calculate the present value of a coupon bond with a coupon rate of 4.25% this bond has a current price at par.

$$PV = \frac{4.25}{1 + r_1} + \frac{104.25}{(1 + r_2)^2}$$
$$= \frac{4.25}{1.0425} + \frac{104.25}{(1.0425)^2} = 100$$

19. (a) $PV = 10 \times (0.9756 + 0.9518 + 0.9286) = \$28.56MM$

(b) Duration of liability

$D = 10 \times (0.9756 + 2 \times 0.9518 + 3 \times 0.9286)/28.56 = 1.98$ years

(c) To hedge interest rate risk $\Delta P/P = -D\Delta y/(1+y)$, so if we want $\Delta P(liability) = \Delta P(asset)$, i.e. perfect hedge, then $D(asset)P(asset) = D(liability)P(liability)$ Leading to $20 \times D(asset) = 1.98 \times 28.56 \rightarrow D(asset) = 2.83year$ To achieve such a duration, invest $x\%$ in 2 year strips and (1-x)% in 3 year strips such as $2x + 3(1-x) = 2.83 \rightarrow x = 17\%$

(d) Impact on net liability

Since by investing in portfolio calculated in c, you are perfectly hedge, the change in interest rate has no impact on your net liability. There will be some effect due to convexity but there is not first order effect.

20. (a) P $= 20(0.961538+0.924556) = \$37.72$.

(b) Duration is given by:

$$\frac{20 \times 0.961538 + 20 \times 0.924556 \times 2}{37.72188} = 1.490.$$

The yield is 4% (we have a flat term structure) so the modified duration is:

$$\frac{1.490196141}{1.04} = 1.433.$$

(c) Asset value falls by 0.1433%, i.e. by $54,050.96.

(d) Let F be the dollar value of the futures contracts that we wish to short.

$$5F = 37.72188 \times 1.433.$$

Hence $F = \$10.810$ million so you will short 108 futures contracts. (You can only long or short whole numbers of futures contracts.)

(e) Let the yield change by y percentage points. Portfolio value will change by

$$-5 \times 10.8 \times \Delta y + 37.72188 \times 1.433 \times \Delta y \approx 0.$$

21. (a) $PV_1 = \frac{1}{1+3.5\%} = 0.9662, PV_2 = \frac{1}{(1+3\%)^2} = 0.9426$
From PV_3 on, sum PV $= 1/(1 + 4\%)^3 + 1/(1 + 4\%)^4 + \ldots$
$= \frac{1}{(1+4\%)^2}(\frac{1}{1.04} + \frac{1}{1.04^2} + \frac{1}{1.04^3} \ldots)$
$= \frac{1}{(1+4\%)^2} \times \frac{1}{0.04} = 23.1139$
Therefore the total $= 25.02268$

(b) If you increase all rates by 0.5%, then the new values become 0.9615, 0.9335 and 20.3500. The total is 22.2446.

22.

Maturity	Price	YTM	Forward Rates
1	$943.40	6.00%	
2	$898.47	5.50%	$(1.055^2/1.06 - 1) = 5\%$
3	$847.62	5.67%	$(1.0567^3/1.055^2 - 1) = 6\%$
4	$792.16	6.00%	$(1.06^4/1.0567^3 - 1) = 7\%$

23. (a) No, the rate of 5.50% is too low. To see why, use the identity that

$$(1 + r_t)^t \times f_{t,t+1} = (1 + r_{t+1})^{t+1}$$

So the forward rate for year 1 to 2 should be:

$$f_{1,2} = \frac{1.055^2}{1.0525} - 1 = 5.75\%$$

(b) Borrow PV of $10 million due in one year now for one year. Invest the amount for two years. The amount you have to borrow is $10M/(1 + 5.25\%) = \$9,501,187$. In one year, you willl have enough to pay back your loan. To borrow for one year, you can short sell the 1-year strip and to invest for 2-years, you can buy the 2-year strip.

24. (a) $P = \frac{100}{(1+y/2)^{18}}$
So P=$67.17

(b) $P = \frac{100}{(1+y/2)^{18}} + \frac{c/2}{y/2}(1 - \frac{1}{(1+y/2)^{18}})$
So P=$106.79

25. Not available but identical (different numbers) to question 000202

26. (a), (d)

27. Not available but almost identical to question 02MM03

28. (a) Assuming a par value of $100. The price is the quoted price plus accrued interests: $109 + 20/32 + 5*41/365 = \$110.2$ (Any reasonable approximation to the accrued interest was considered correct).

You will receive $2.5 in Feb and August every year from 2003 to 2010. In Feb 2011, you will receive $102.5 Common mistake: Most people didnt take into account the difference between clean and dirty price

(b) You were expected to give EAR. Using the quoted yield (semi-annually compounded rate): $r_{2010} = (1 + 0.0365/2)^2 - 1 = 3.68\%$
$r_{2011} = (1 + 0.0377/2)^2 - 1 = 3.81\%$
Common mistake: give the APR (-2/4), or compute it using prices without taking correct time to maturity (-2/4).

(c) $f_{2010,2011} = \frac{1.0381^{8.4}}{1.0368^{7.4}} - 1 = \frac{B_{2010}}{B_{2011}} = \frac{76.6875}{73.25} = 4.6\%$

(d) The Feb 2010 Note with 6.5 coupon rate has the shortest duration, because its coupons (given out early) are bigger than the Feb 2011 bond and the principal is also given earlier.

29. (a) Price = PV of cash flows = $100(1+r)/(1+r) + 100(1+r)^2/(1+r)^2 + \cdots = 400.00$

(b) $Duration = (1 + 2 + 3 + 4)/4 = 2.5$ years

(c) Put 50% in each. Then the duration of your liability = 2.5 = duration of your assets = 0.5(1)+0.5(4)

30. (a)
$$PV = \frac{20}{(1+5\%)^{10}} + \frac{20}{(1+5\%)^{30}} = 12.28 + 6.94 = 19.22 \text{million}$$

(b)
$$D = \frac{12.28 * 10 + 6.94 * 30}{19.22} = 17.22$$

$$MD = \frac{D}{1+y} = 16.40$$

y = 5% because the yield curve is flat

(c) When rates drop by 0.25%, the PV of liabilities will go up by $0.25\% * MD * PV = 0.7881$ million

(d) First, you calculate the desired MD of your assets. We want:
$MD_{asset} * PV_{asset} = MD_{liabilites} * PV_{liabilities}$
Therefore, $MD_{asset} = \frac{16.4*19.22}{18} = 17.51$
Now we can determine the allocation of our portfolio. Suppose we invest a fraction of x of our portfolio into 1-year bond and the rest into treasury bond, then the MD of our portfolio will be:
$MD_{portfolio} = x * MD_{1yrbond} + (1-x) * MD_{tbond} = x * \frac{1}{1+5\%} + (1-x) * 20$
Equating $MD_{portfolio} = 17.51$, we get x = 13.05%

31. (a) Using the perpetuity formula
$$PV_{Liability} = \frac{10M}{r} = \frac{10M}{0.05} = 200M$$

(b)
$$PV_{Liability} = \frac{10M}{r} = \frac{10M}{0.049} = 204.0816M$$

The value of the liabilities would increase by 4.0816M.

(c)
$$P_{new} = P_{old} - P_{old} \times MD \times \Delta y$$
$$\rightarrow MD = \frac{P_{old} - P_{new}}{P_{old} \times \Delta y}$$
$$= \frac{200 - 204.0816}{200 \times -0.001} = 20.4082$$

(d) You should match the modified duration to neutralize first order interest rate risk. MD=20.4082

(e) We must match both the modified duration and the present value of the zero coupon bond. Let PAR be the par value of the zero coupon bond and t be the maturity. Since the duration of a zero coupon bond is its maturity $t = 21.4286$. We can then solve for the PAR value

$$200M = \frac{PAR}{(1.05)^t}$$
$$\rightarrow PAR = 200(1.05)^{20.4082} = 541.331M$$

32. (a) $r_1 = 4.5\%, r_3 = 5.5\%$
$B_2 = 100$ (coupon $=$ YTM), so $100 = 5/(1+r_1) + 105/(1+r_2)^2$, so $r_2 = 5.0126\%$
Therefore the price of the 5% 3-year bond is
$5/(1+4.5\%) + 5/(1+5.0126\%)2 + 105/(1+5.5\%)3 = 98.7382$

(b) $f_3 = (1+r_3)^3/(1+r_2)^2 - 1 = 6.4817\%$

(c) MD(bond 1) $= 1/(1+4.5\%) = 0.9569$, MD(bond 3) $= 3/(1+5.5\%) = 2.8436$, so
MD(portfolio) $= 30\%*0.9569+70\%*2.8436 = 2.2776$

(d) That means the portfolio will decrease in value by $0.15\%*2.2776 = 0.3416\%$

33. (a) We have the following system of equations:
$100 = (100 + 25)/(1 + f_{0,1})$
$422.61 = 50/(1 + f_{0,1}) + (500 + 50)/((1 + f_{0,1})(1 + f_{1,2}))$
$232.28 = 20/(1+f_{0,1})+20/((1+f_{0,1})(1+f_{1,2}))+(300+20)/((1+f_{0,1})(1+f_{1,2})(1+f_{2,3}))$
$192.31 = 1000/((1 + f_{0,1})(1 + f_{1,2})(1 + f_{2,3})(1 + f_{3,10})^7$
The solutions are
$f_{0,1} = 0.25$,
$f_{1,2} = 0.15$,
$f_{2,3} = 0.10$,
$f_{3,10} = 0.18.54$.

(b) $NPV = -21 + 9/1.25 + 10/((1.25)(1.15)) + 11/((1.25)(1.15)(1.10)) = 12$

34. (b) the entire value of the treasury strip is in the principal repayment in the distant future; it has the highest duration and is most sensitive to a change in the interest rate.
(c) despite having the same maturity as (b), (c) has 5.5% coupon payments thatdampen its sensitivity to interest rate changes.
(d) higher coupon payment $\rightarrow$ lower duration
(a) T-bills are only for 1 to 6 months; they have the smallest duration.

35. Since we want portfolio $D = 9$, we can invest x in the 6-year bond ($D = 6$) and $(1-x)$ in 16-year bond ($D = 16$).
Then $D_p = x * 6 + (1 - x) * 16, x = 70\%$ (*)
Therefore invest 70% (350,000 or 3500 shares) in 6-year bond and 30% (150,000 or

1500 shares) in 16-year bond.

*: This equation holds for D here because the bonds have the same YTM.

36. Let the annual interest payment for the two-year forward loan be C. You can arrange the loan and lock in the forward interest rate loan as follows

- Buy \$15M in par value of 1-year zero, costing $(15M)(943.40/1000) = \$14,151,000$.
- Sell \$$C$ in par value of 2-year zero, receiving $\$C \times (898.47/1000)$
- Sell \$$C + 15M$ in par value of 3-year zero, receiving $\$(C + 15M)(847.62/1000)$
- Choose C such that proceeds from the selling of 2-year and 3-year zeros just cover the cost of buying the 1-year zero.

Thus,

$$(15M)(943.40/1000) = C(898.47/1000) + (C + 15M)(847.62/1000)$$

which gives $C = \$822,810$. The CF's of the transactions are:

Transaction	CF today	CF in year 1	CF in year 2	CF in year 3
Buy 1-year zero	-14,151,000	15,000,000	0	0
Sell 2-year zero	739,270		-822,810	0
Sell 3-year zero	13,411,730	0	0	-15,822,810
Net	0	15,000,000	-822,810	-15,822,820

The forward interest rate for the 2-year forward loan is

$$f = 822,810/15,000,000 = 5.4854\%$$

which is locked in now.

37. (a) The current price of the bond is

$$P_0 = \frac{\$60}{1.065} + \frac{\$60}{1.065^2} + \frac{\$1060}{1.065^3} = \$986.76$$

You can sell it in one year for

$$P_1 = \frac{\$60}{1.065} + \frac{\$1060}{1.065^2} = \$990.90$$

But there is also the \$60 of coupon. So the total return is

$$RoR = (\$990.90 + \$60)/\$986.76 - 1 = 6.5\%$$

(b) The bond now has the same coupon rate as the yield to maturity so it is trading at par. So the new price $P_1 = \$1,000$ and

$$RoR = (\$1,000 + \$60)/\$986.76 - 1 = 7.4\%$$

38. (a) $D = \frac{\sum i=1^{20} \frac{i}{1.04^i}}{\sum i=1^{20} \frac{1}{1.04^i}} = 9.21$ years

(b) Need to match the duration and also the value of investment today should be equal to the total liabilities. So have the following two equations:

$V_5 * 5 + V_{20} * 20 = D * (V_5 + V_{20})$

$V_5 + V_{20} = \$13.59M$ Annuity formula

Solving gives $V_5 = \$9.78M$ and $V_{20} = \$3.81M$

$P_5 = V_5 * (1.04)^5 = \$11.9M$

$P_{20} = V_{20} * (1.04)^{20} = \$8.36M$

(c) No. Interest rates increase, so bigger fraction of present value given early. So duration decreases. So more in 5 year strips and less in 20 year strips.

39. (a) $P = \frac{\$8}{1.05} + \frac{\$8}{1.05^2} + \cdots + \frac{\$108}{1.05^{10}} = \$123.16$

$D = \frac{\$8}{1.05}\frac{1}{\$123.16} + \frac{\$8}{1.05^2}\frac{2}{\$123.16} + \cdots + \frac{\$108}{1.05^{10}}\frac{10}{\$123.16} = 7.54$

(b) Using pricing equation, the price will change to $P_{new} = \$122.28$.

Recall the approximation that

$$\Delta P/P \approx -MDy + CX \times (\Delta y)^2$$

$MD = D/(1+y) = 7.54/1.05 = 7.18$

Usign the first two terms only we have

$$\Delta P = -P \times MD \times \Delta y = -\$123.16 \times 7.18 \times 10 \times 10^{-5} = 0.88 \rightarrow P_{new} = \$122.28$$

(c) Using pricing equation: $P_{new} = \$107.02$.

Using duration approximation: $P_{new} = \$105.47$.

(d) Addign the convexity term the new prices are

$P_{new} = \$122.28$, and $P_{new} = \$107.13$

So not much has changed in the first case (part b) but the approximation is not much better for part c. In short, convexity is more important issue for large interest rate movements and hence one has to be hedged in terms of convexity in order to be immune to large interest rate movements.

40. (a) 4% per year

(b) No. The spot rates at $t = 5$ are not known at $t = 0$.

41. (a) $10000 \times (136\frac{3}{32}\%) = 13609.38$

(b) $437.50 in Feb. 2007, Aug. 2007, Feb. 2008, ..., Aug. 2020
$10,437.50 in Aug. 2020.

(c) $43,750 in Feb. 2007
$-9,956,250 in Aug. 2007
$206,250 in Feb. 2008
$10,206,250 in Aug. 2008.

42. (a) The 1, 2, 3, 4, 6 and 10-year spot interest rates are 5.08%, 4.93%, 4.91%, 4.80%, 4.87% and 5.05%, respectively.

    (b) $f_{1,2} = 1.0493^2/1.0508 - 1 = 0.0478$.
$f_{3,4} = 1.0480^4/1.0491^3 - 1 = 0.0447$.

    (c) $PV = 437.50/(1.0480^4) = 362.69$.

43. $P = Annuity(218.75, (1.0488)^0.5, 12) + 10000/(1.0488)^6 = \$9,769.56$.

44. (a) Short $\frac{10}{1.0508} = \$9.5166$ million worth of one-year strip.
Long \$9.5166 million worth of two-year strip.
Cash flow at $t = 1$: 0.
Cash flow at $t = 2$: $9.5166 \times (1.0493)^2 = \$10.478$ million.

    (b) Short $\frac{10}{1.0508} = \$9.5166$ million worth of one-year strip.
Long \$9.5166 million worth of six-year strip.
Cash flow at $t = 1$: 0.
Cash flow at $t = 5$: $9.5166 \times (1.0487)^6 = \$12.659$ million.

45. (a) 4%

    (b) $200 \times (1.04)^3 = 224.97$.

46. Not available but we have many other pension questions

47. Bond underwriting

If the underwriter purchases the bonds from the corporate client, then it assumes the full risk of being unable to resell the bonds at the stipulated offering price. In other words, the underwriter bears the risk of interest rate movement between the time of purchase and the time of resale. For long maturity bonds, it is generally true that its duration is also long. Thus, bonds with long maturities are more exposed to interest rate movement risk. Therefore, the underwriter demands a larger spread (higher underwriting fees) between the purchase price and stipulated offering price.

48. (a) Use the formula that

$$D = \sum_{t=1}^{T} t \times w_t \quad \text{where } w_t = \frac{CF_t/(1+y)^t}{BondPrice}$$

Durations are 1.97, 4.61, 8.35, 14.13 for A through D, respectively.

    (b) You can either do this question by aggregating the cashflows as each time and using an approach identical to what we did in part a. Here is an alternative.

To calculate the duration of a portfolio, it is easier to calculate its modified duration first. Recall that modified duration if $MD = D/(1 + y)$ and that $\Delta P/P = -MD \times \Delta y$

Is is not have to show that the modified duration of a portfolio is simply the weighted sum of the modified durations of the individual bonds where the weights are proportional to the value of the bond to the total value.

In this case, the modified durations A through D are 1.85, 4.35, 7.88, and 13.33, respectively. For the portfolio we have:

$$MD_{portfolio} = \frac{10}{60} \times 1.85 + \frac{20}{60} \times 4.35 + \frac{20}{60} \times 7.88 + \frac{10}{60} \times 13.33 = 6.6$$

Hence the duration is

$$D_{portfolio} = 6.6 \times 1.06 = 7$$

(c)
$$\Delta P/P = -MD \times \Delta y = 7 \times 20bps = 140bps = 0.14\%$$

49. (a) PV bond = PV semiannual payments + PV principal payment

$= 300 \times \frac{1}{0.05/2} \times (1 - \frac{1}{(1+0.05/2)^{20}}) + \frac{10,000}{1.025^{20}} = \$10,779.45$

Note: if 5% is an effective annual rate, the semiannual rate is $1.05 = (1 + r/2)^2$ so $r/2 = 2.47\%$

Then, PV bond $= 300 \times \frac{1}{0.0247} \times (1 - \frac{1}{(1+0.0247)^{20}}) + \frac{10,000}{1.0247^{20}} = \$10,828.57$

(b) To calculate the IRR sovle this following equation

$0 = -11240 + \frac{300}{(1+IRR/2)^1} + \frac{300}{(1+IRR/2)^2} + \cdots + \frac{300}{(1+IRR/2)^{20}}$

So: $IRR/2 = 2.23\%$

50. (a) $D = \frac{200*0.5+300*1+500*5-900*1}{100} = 20$

It means that the equity has the same sensibility to interest rates as a 20 year strip

(b) $dP/P = -D/(1 + y) * dy = -20/1.03 * 0.01 = -0.1941$. Given the \$100 value, $dP = -\$19.41M$

So the new price is \$80.58M. Because of convexity, the change will be smaller.

51. Not available but there are several other pension related question dealing with duration matching.

52. (a) A: $y = \frac{105}{100.96} - 1 = 0.040016$.

B: y solves $\frac{6.5}{(1+y)^1} + \frac{6.5}{(1+y)^2} + \frac{106.5}{(1+y)^3} = 106.29$; $y = 0.042238$.

C: y solves $\frac{2}{(1+y)^1} + \frac{2}{(1+y)^2} + \frac{102}{(1+y)^3} = 93.84$; $y = 0.042294$.

(b) $r_{0,1} = \frac{105}{100.96} = 1.0400$.

$r_{0,2}$: consider the portfolio of $+1$ B, $-\frac{106.5}{102} = -1.0441$ C and $\frac{2\times1.0441-6.5\times1}{105} = -0.042017$ A. The portfolio pays 0 at time $t = 1, 3$ and $6.5\times1-2\times1.0441 = 4.4118$ at time $t = 2$. The portfolio costs $100.96 \times (-0.042017) + 106.29 \times 1 - 93.84 \times 1.0441 = 4.0680$. Then, $r_{0,2} = \left[\frac{4.4118}{4.0680}\right]^{1/2} = 1.041398$.

$r_{0,3} = \left[\frac{106.5}{93.84-6.5/(1+r_{0,1})-6.5/(1+r_{0,2})^2}\right]^{1/3} - 1 = 0.042323$.

$f_{0,1} = 0.040016$.

$f_{1,2} = 1.041398^2/1.040016 - 1 = 0.042781$.

$f_{2,3} = 1.042323^2/1.041398^2 - 1 = 0.044175$.

53. (a) Zero-coupon bond: $y = 4.5\%$.

Coupon bond: $P = \frac{5}{1.035} + \frac{5}{1.04^2} + \frac{105}{1.045^3} = 101.46$; y solves $\frac{5}{1+y} + \frac{5}{(1+y)^2} + \frac{105}{(1+y)^3} = 101.46$; $y = 0.044675$.

(b) Zero-coupon bond: $P_0 = \frac{100}{1.045^3} = 87.63$; $P_1 = \frac{100}{1.04^2} = 92.46$; realized return $= \frac{92.46}{87.63} - 1 = 0.055072$.

Coupon bond: $P_1 = \frac{5}{1.035} + \frac{105}{1.04^2} = 101.91$; realized return $= \frac{101.91+5}{101.46} - 1 = 0.053659$.

54.

(1)	(2)	(3)	(4)	(5)
Yrs	Pmt	PV of Pmt	Wt of Pmt	(1) × (4)
1	\$1 M	\$0.9091 M	0.2744	0.2744
2	\$2 M	\$1.6529 M	0.4989	0.9978
3	\$1 M	\$0.7513 M	0.2267	0.6801
Total		\$3.3133 M	1.0000	1.9523

Duration is 1.9523 years.

55. Not available but there is another very similar question ID:020205

56. (Assuming semiannual coupon payment: YTM per six months is $(1.06)^0.5 = 1.02956$.)

(a) $P = Annuity(2.5, 0.02956, 20) + 100/1.06^10 = \$93.18$.

(b) $D = \frac{1\frac{2.5}{1.02956} + 2\frac{2.5}{1.02956^2} + ... + 20\frac{2.5}{1.02956}}{\frac{2.5}{1.02956} + \frac{2.5}{1.02956^2} + ... + \frac{2.5}{1.02956}} = 15.8067$ periods (of 6 months).

$MD = \frac{15.8067}{1.02956} = 15.3529$.

(c) $\Delta P = 93.18(1 - 15.3529 \times (1.07^{0.5} - 1.06^{0.5})) = \$86.25$.

(d) $P = Annuity(2.5, (1.07)^{0.5}, 20) + 100/1.07^{10} = \$86.56$.

57. (a) The net cash flow in May-June, 2010 will be $-(1.04)^{0.5} + 0.055/2 = -\$0.99230$ billion.

(b) The transaction's NPV is 1-1=0.

58. (a) There is a one-to-one mapping between a bond's price and YTM (y). y is the solution to $P = Annuity(4.4375, y, 20) + 100/(1 + y)^{20}$.

(b) $Annuity(4.4375, 0.02315, 20) + 100/(1.02315)^20 = 133.67$.

59. **(a)** iv; **(b)** iv; **(c)** iv.

60. (a) (i) Increased. $r \approx R - i$.

(b) (i) A. Bond A has a smaller coupon payment, so its value depends more heavily on the principal repayment, which occurs far into the future. Therefore, it is more sensitive to change in the yield.

(c) (i) $y > 6\%$. Since the bond is sold at a discount, $y > c$.

(d) (i) Profit. $f_{1,2} = 1.06^2/1.05 - 1 = 0.07010 > 0.06$. The price has gone up.

61. (a) Zero-coupon bond: $P = \frac{100}{1.055^3} = 94.79; D = 3; MD = \frac{3}{1.055} = 2.844$.

Coupon bond: $P = \frac{5}{1.055} + \frac{5}{1.055^2} + \frac{105}{1.055^3} = 98.65; MD = \frac{1}{1.055}\left[\frac{1\frac{5}{1.055} + 2\frac{5}{1.055^2} + 3\frac{105}{1.055^3}}{94.79}\right] = 2.7094$.

(b) Zero-coupon bond: $\Delta P = -2.844 \times 0.001 = -0.0028436$. Actual % change $= \frac{100/1.056^3 - 94.79}{94.79} = -0.0028382$.

Coupon bond: $\Delta P = -2.7094 \times 0.001 = -0.0027094$. New price $= \$98.38$; actual % change $= \frac{98.38 - 98.65}{98.65} = -0.0027043$.

(c) Zero-coupon bond: $\Delta P = -2.844 \times 0.02 = -0.056872$. Actual % change $= \frac{100/1.075^3 - 94.79}{94.79} = -0.054782$.

Coupon bond: $\Delta P = -2.7094 \times 0.02 = -0.054187$. New price $= \$93.50$; actual % change $= \frac{93.50 - 98.65}{98.65} = -0.052228$.

(d) Portfolio: $MD = 2.844 \times 0.4 + 2.7094 \times 0.6 = 2.7631$.

62. (a) The present value of the annuities is \$1 M /0.1 = \$10 M.

The duration is $1.10/0.10 = 11$ years.

Let $x =$ weight of 5 year zeros, and $1 - x =$ weight of 20 year zeros. Then

$$11 = 5x + 20(1 - x)$$

and so $x = 0.60$ (in 5 year zeros), and $1 - x = 0.40$ (in 20 year zeros).

5 year zeros: $\$10M \times 0.60 = \$6M$ market value;

20 year zeros: $\$10$ M $\times 0.40 = \$4M$ market value.

(b) Face value of 5 year zeros: $\$6M \times (1.10)^5 = \$9.66M$; Face value of 20 year zeros: $\$4M \times (1.10)^{20} = \$26.91M$.

63. (a) Depends. If the bond is held to maturity and all coupon payments are reinvested at the YTM, then higher YTM means higher return, ceteris paribus.

(b) No. Forward rates are expectations of future spot rate; they are unrelated to the current spot rate.

(c) No. Since the market expects a rate cut, it is already incorporated into today's prices.

64. (a) PV of obligation:

$$PV = \sum_{t=1}^{2} \frac{\$10,000}{(1.08)^t} = \$17,832.65.$$

Duration of obligation:

(1)	(2)	(3)	(4)	(5)
Yr	Pmt	PV of Pmt	Wt of Pmt	$(1) \times (4)$
1	$\$10,000$	$\$9,259.26$	0.51923	0.51923
2	$\$10,000$	$\$8,573.39$	0.48077	0.96154
Total		$\$17,832.65$	1.0000	1.48077

Duration of obligation is 1.4808 years.

(b) Zero coupon bond with a duration of 1.4808 years would immunize the obligation. The present value of this bond must be $17,832.65, thus the face value (feature redemption value) must be:

$$\$17,832.65 \times (1.08)^{1.4808} = \$19,985.26.$$

(c) If interest rates increase to 9%, the value of the bond would be:

$$\frac{\$19,985.26}{(1.09)^{1.4808}} = \$17,590.92.$$

The tuition obligation would be:

$$PV = \sum_{t=1}^{2} \frac{\$10,000}{(1.09)^2} = \$17,591.11.$$

The net position changes by only $0.19,
If interest rates decline to 7%:
The value of the zero coupon bond would increase to:

$$\frac{\$19,985.26}{(1.07)^{1.4808}} = \$18,079.99.$$

The tuition obligation would increase to

$$PV = \sum_{t=1}^{2} \frac{\$10,000}{(1.07)^2} = \$18,080.18.$$

The net position changes by $0.19.
As interest rates change, so does the duration of the stream of tuition payments, thus the slight net differences.

65. (a) Real return is $1.05/1.02 - 1 = 0.029412$.

(b) Real return is $1.05/1.03 - 1 = 0.019417$.

(c) (i) The cash flows at time $t = 1, 2, 3, 4$ and 5 are $5.10, 5.20, 5.31, 5.41$ and 115.93, respectively.
(ii) The cash flows at time $t = 1, 2, 3, 4$ and 5 are $5.15, 5.30, 5.46, 5.63$ and 121.72, respectively.

3 Common Stock Solutions

1. (a) UNCERTAIN/FALSE

    We can increase the present value of a share of common stock with a new investment only if $ROE > r$, where r is a discount rate (capitalization rate)

    If a new investment results in $ROE < r$, the price of the stock will decline even though earnings could be higher. Many students missed this point.

2. (a) False: the market value of a share of stock equals the discounted value of the stream of future dividends per share. It "works" with earnings if and only if PVGO = 0, and the plowback ratio = 0.

3. (a) False. The discount rate on one asset is determined by its risk characteristics.

    (b) False. A growth company has investment opportunities with expected return higher than the required rate of return. However, earnings may not be growing. For example, the firm could be investing heavily, leading to lower current earnings and dividend.

4. False. The goal of a manager should always be to maximize the market value of the firm. The firm's risk will be reflected in its market value and shareholders can always unload any undesirable risk in the market. (Caveat: It could be true if shareholders can't sell or have blocked access to financial markets like in private firms).

5. False. DDM use future dividend for discounting, so it works even with firms that have not paid dividends in the past.

6. (a) Growth stocks are stock of companies that have access to growth opportunities, where investment opportunities earn expected returns higher than the required rate of return on capital, or when PVGO > 0. A stock with growing dividends may not be a growth stock. A growth stock may be a stock with DPS growing slower than the required rate of return.

    Comment: to get a full credit, its not enough just to mention that growth stock dont usually have rapidly growing dividends. Many students said that growth stocks have DPS growing slower than the required rate of return. This is not necessarily true.

7. (a) False: To be a growth stock, its PVGO must be positive. It can have a positive PVGO, and yet not have made a profit since its IPO.
    OR
    Depends: If its PVGO is also non-positive, it is not a growth stock.

8. (a) False. A growth company has ROE greater than r, its cost of capital.

    (b) False. Higher volatility are not necessarily associated with higher returns as part of the volatility may be due to idiosyncratic risks.

9. False. The price of a stock equals the present value of all expected future dividends per share, discounted at the appropriate rate.

10. No. P = E/r + PVGO. So a higher P/E ratio may just mean higher PVGO.

11. (a) $g = ROE \times b = 12\% \times 0.5 = 6\%$. It is the same for both earnings and dividends.

(b) $P = \frac{D_1}{r-g} == \frac{BVPS \times ROE \times p}{r-g} = \frac{\$0.6}{12\%-6\%} = \$10$

(c) We have assumed that the first payment of dividend happens at the end of the current year.

(d) $P = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \cdots$. Note that $D_t = BVPS_t \times ROE_t \times p$, where p is the payout ratio. If all the quantities are restated in real terms, we will have a higher discount rate in the denominator of the DCF. But the BVPS will also be higher by the same factor so they will cancel out and the price won't change. We have assumed that the inflation will affect the assets and the discount factor in the same way here.

(e) No change. The price will remain as \$10/share as the ROE of the firm is the same as its cost of capital, i.e. it does not have access to any growth opportunities so the PVGO=0.

12. Price of Qs stock = \$2.2/9.5\% = \$23.16

Price of Rs stock = \$1.4/(9.5\%-3\%) = \$21.54

Price of Ss stock = $1/(1 + 9.5\%) + 1.5/(1 + 9.5\%)^2 + 2/(1 + 9.5\%)^3 + 2.5/9.5\%/(1 + 9.5\%)^3 = \$23.73$

13. PV_0 of the first 5 years $= \frac{1}{1.08} + \frac{1.1}{1.08^2} + \frac{1.1^2}{1.08^3} + \frac{1.1^3}{1.08^4} + \frac{1.1^4}{1.08^5} = 4.8043$.
PV_0 of all dividends after the first 5 years $= \frac{1}{1.08^5} \frac{1.1^5 \times 1.02}{0.08-0.02} = 16.9395$.
$P_0 = 4.8043 + 16.9395 = 21.7438$.

14. (a) $p_0 = \frac{D_1}{r-g} = \frac{\$5 \cdot 0.3}{0.1-0.08} = \$75$.

(b) Yes, it has growth opportunities, and b > 0. $PVGO = p_0 - \frac{E_1}{r} = \$75 - \frac{\$5}{0.1} = \$25$.

15. (a) g solves $\frac{5 \times 0.4 \times (1+g)}{0.10-g} = 80$; $g = 0.073171$.

(b) PVGO $= 80 - \frac{5}{0.1} = 30$.

16. The growth rate of dividend after year 4 is $g_2 = 0.10 \cdot (1 - 0.6) = 0.04$.

The following tables give answers to part (a) (b) and (c). We see that today's price in part (a) is the same as that in part (c), but higher than that in part (b). The reason is that the ROE after year 4 is the same as the cost of capital (both at 0.10). Therefore, after year 4, it does not matter what the payout ratio is, and the price will remains the same. However, since from year 1 to year 3, the ROE is higher than cost of capital, a increase in payout ratio will reduce the growth rate of earnings (and dividends), therefore, the price in part (b) is lower.

Year	BVPS0	EPS	DivPS	BVPS1	Price	P/E
0	-	-	-	100.000	113.698	-
1	100.000	15.000	4.500	110.500	120.568	7.274
2	110.500	16.575	4.972	122.103	127.653	6.970
3	122.103	18.315	5.495	134.923	134.923	10.000
4	134.923	13.492	8.095	140.320	140.320	10.000

Price and P/E

Payout Ration for the first three years is 0.3000, and is 0.600 after year 4.

Year	BVPS0	EPS	DivPS	BVPS1	Price	P/E
0	-	-	-	100.000	113.147	-
1	100.000	15.000	9.000	106.000	115.461	7.262
2	106.000	15.900	9.540	112.360	117.467	6.970
3	112.360	16.854	10.112	119.102	119.102	10.000
4	119.102	11.910	7.146	123.866	123.866	10.000

Price and P/E

Payout Ration for the first three years is 0.6000, and is 0.600 after year 4.

Year	BVPS0	EPS	DivPS	BVPS1	Price	P/E
0	-	-	-	100.000	113.698	-
1	100.000	15.000	4.500	110.500	120.568	7.274
2	110.500	16.575	4.972	122.103	127.653	6.970
3	122.103	18.315	5.495	134.923	134.923	10.000
4	134.923	13.492	12.143	136.272	136.272	10.000

Price and P/E

Payout Ration for the first three years is 0.3000, and is 0.900 after year 4.

17. $P_0 = D_1/(r - g)$

where,

P_0 = current price

D_1 = expected dividend next period

r = cost of equity or market capitalization rate

g = expected growth rate in dividends

In this example, we have $20/(0.12 - 0.10) = 1000$

A 23% drop in P_0 will cause it to drop from 1000 to 770. This can be caused by a 23% drop in the estimate of D_1 (from 20 to 15.4), or it can be caused by a 5% increase in the estimate of the cost of equity (from 12% to 12.6%), or it can be caused by a 6% decrease in the estimate of the expected growth rate in dividends (from 10% to 9.4%).

Though the first possibility is not a slight change, the last two possibilities are. (5 points)

18. (a) $P = \frac{D_1}{r-g} = \frac{\$5}{10\% - 6\%} = \$125$

(b) $P = \frac{EPS_1}{r} + PVGO = \frac{\$8}{10\%} + PVGO \rightarrow PVGO = \$45$

19. First, calculate the nominal growth rate:

$1 + g = (1.04)(1.02)$ so $g = .0608$

$PV = \frac{4.5}{0.095 - 0.0608} = \$131.58$

20. The table below summerizes the calculations:

Year	1	2	3	4	5
BB BVPS	$35.00	$39.20	$43.90	$49.17	$55.07
Investment	$ 4.20	$ 4.70	$ 5.27	$ 5.90	$ 2.20
EB BVPS	$39.20	$43.90	$49.17	$55.07	$57.28
EPS	$ 5.60	$ 6.27	$ 7.02	$ 7.87	$ 5.51
DPS	$ 1.40	$ 1.57	$ 1.76	$ 1.97	$ 3.30
PV(DPS)	$ 1.28	$ 1.31	$ 1.34	$ 1.37	
PV(TV)	$41.79				
Total share price	$47.08				
ROE	16%	16%	16%	16%	10%
g	12%	12%	12%	12%	4%
Payout Ratio	25%	25%	25%	25%	60%
Plowback Ratio	75%	75%	75%	75%	40%
Cost of Capital	9.50%				
PVGO	$(11.87)				

If we calculate PVGO as $Price - EPS/r$, we obtain a negative PVGO. How come that PVGO is negative while ROE > cost of capital at any time? Because the ROE on current assets decreases over time, which means that, by taking current ROE, we overestimate the value of assets in place. So, calculate an average ROE from now to the infinite $= 10\%*35 = 3.5$ Value of assets in place $= 3.5/0.095 = \$36.84$ "real" PVGO $= \$47.08 - \$36.84 = \$10.24$

21. The table below summerizes the calculations:

Year	1	2	3	4	5

	1	2	3	4	5
BB BVPS	$35.00	$39.20	$43.90	$49.17	$55.07
Investment	$ 4.20	$ 4.70	$ 5.27	$ 5.90	$ 2.20
EB BVPS	$39.20	$43.90	$49.17	$55.07	$57.28
EPS	$ 5.60	$ 6.27	$ 7.02	$ 7.87	$ 5.51
DPS	$ 3.36	$ 3.76	$ 4.21	$ 4.72	$ 3.30
Net cash needed	$ 1.96	$ 2.20	$ 2.46	$ 2.75	$ -
PV(DPS)	$ 3.07	$ 3.14	$ 3.21	$ 3.28	
PV(TV)	$41.79				
Total share price	$54.49				

ROE	16%	16%	16%	16%	10%
G	12%	12%	12%	12%	4%
payout ratio	60%	60%	60%	60%	60%
plowback ratio	75%	75%	75%	75%	40%
cost of capital	9.5%				

PVGO = $(4.46)

At end of year 1, the company needs $1.96 per share of additional cash, or $9,800,000. At the new price of $54.49 per share, this represents 179,850 shares to be issued at t=1. The only effect on the ex-dividend price per share today comes from the change in the payout policy, which increases the price per share from $47.08 to $54.49. Dividend policy does not create value. What creates value is that the company gets money at 9.5% and invests it at 10%.

Once the company announces this measure, its market cap goes up to 5,000,000*54.49. Once it announces it will issue shares at the fair price to cover the additional cash needs, the new price per share becomes 5,000,000*(54.49+1.96)/(5,000,000+179,850)= $54.49!

The reason why this issue does not impact the price today is that is is done at the fair market value of the share, and every dollar invested shows up either as growth or as dividend.

22. (a) $P = \frac{5}{1.12} + \frac{5 \times 1.1}{1.12^2} + \frac{5 \times 1.1^2}{1.12^3} + \frac{1}{1.12^3} \frac{5 \times 1.1^2 \times 1.05}{0.12 - 0.05} = 77.75.$

(b) $\text{PVGO} = 77.75 - \frac{5/0.3}{0.12} = -61.14.$

(c) $P/E = \frac{77.75}{5/0.3} = 4.6650.$

(d) $P_1 = \frac{5 \times 1.1}{1.12} + \frac{5 \times 1.1^2}{1.12^2} + \frac{1}{1.12^2} \frac{5 \times 1.1^2 \times 1.05}{0.12 - 0.05} = 82.08.$
$E_2 = 5.5/0.3 = 18.33.$
$P/E_1 = \frac{82.08}{18.33} = 4.4770.$

(e) 10%: (a) 109.09 (b) -57.58 (c) 6.5455 (d) 6.2727
14%: (a) 60.34 (b) -58.70 (c) 3.62060 (d) 3.4795

23. (a) We compute the market value for the two division separately. For division A, the cash flow is risk-free, so we need to use the treasury rate to discount. The PV of the division A is

$$PV_A = \frac{\$2m}{0.05 - 0.02} = \$66.667m. \tag{2}$$

The PV of division B is the difference between the currently market value of the whole firm and the PV of division A:

$$PV_B = PV - PV_A = \$87m - \$66.667 = \$20.3333m. \tag{3}$$

The cost of capital for division B is

$$r_B = \frac{E_B}{PV_B} + g_B = \frac{\$2m}{\$20.333m} + 0.04 = 0.138. \tag{4}$$

(b) The NPV of the new project is

$$NPV = -\$5m + \frac{0.8}{r_B - g_B} = -\$5m + \frac{0.8}{0.138 - 0.04} = \$3.133m. \tag{5}$$

So the new market value of the firm is $\$87m + \$3.133m = \$90.133m$.

24. (a) Cost of capital is $4 + 1.2 \times 8 = 13.6\%$.

(b) $P = 5/0.136 = \$36.76$; hence P/E $= 7.35$.

(c)
$$\beta = \rho \frac{\sigma_i}{\sigma_m} = \frac{0.1 \times 0.5}{0.2} = 0.25.$$

Hence the required rate of return on the new project is $4 + 0.25 \times 8 = 6\%$.

(d) Present value of the growth opportunity is $-3 + 0.8/0.06 = \$10.33$ as of year 1. Thus price will be

$$P = 36.76 + \frac{10.33}{1.06} = \$46.51$$

and P/E $= 9.30$.

25. Not available. We have at least two other questions very similar to this.

26. The analyst uses a constant growth one-stage DCF model: $P_0 = DIV_1/(r - g)$

But the growth rate is actually going to decline over the next years. The analyst should use a two-stage DCF analysis to value the company; assuming constant growth rate leads to overestimating the cost of capital. For example, if we assume that growth rate stays at 16% next year, but falls at 10% starting year 2, then the right discount rate is r such as $\$62 = 0.5/(1 + r) + 0.55/(r - 0.1)/(1 + r)$ which leads to $r = 10.81\%$

27. (a) The company should issue 200,000 shares.

(b) The company's value with the new project is $50 + \frac{1.2}{0.12-0.03} = \$63.3333$ million. The share price is then $\frac{63.3333}{1+0.2} = \$52.78$.

(c) The NPV of the new project is $\frac{1.2}{0.12-0.03} - 10 = \$3.3333$ million. In anticipation to this project, the stock price moves to \$53.33 even before the new equity issue. The company needs to issue $\frac{10,000,000}{53.33} = 187,500$ shares, and the stock price will not change after the issue.

28. It is not a fair offer. If you have market prices you should use them. NPV is just a tool to estimate market prices when they do not exist or market anomalies occur.

Value per share from your aunt = share price-dividend foregone = $22-1.4/1.08 = \$20.7$

Major mistakes: a. Not use of the market price, b. Not consider the dividend, c. Not discount the dividend

29. (a) $d_1 = EPS(1-b) = 5(1-0.4) = 3$; $g = ROE \times b = 12(0.4)\% = 4.8\%$. So $P = d_1/(r-g) = 3/(0.09-0.048) = 71.43$.
So $P/E = 71.43/5 = 14.29$

(b) P/E will increase, because ROE ¿ r here, that is, investors are getting more than what they want

(c) Plowback. Do not give dividends

30. (a) Dividend next year = Earnings * (1 - Plowback) = 0.5
Dividend growth rate = 12%*plowback = 6%

(b) Pre-announcement stock price = price under no growth = $\frac{1}{10\%} = 10$
Post-announcement stock price: Using DDM,

$$P = \frac{D_1}{r-g} = \frac{.5}{10\% - 12\% * .5} = 12.5$$

(c) Pre-announcement PE = 10, post-announcement PE = 12.5
Since ROE is higher than the cost of capital, the company is a growth company.

31. (a) Using the Gordon Growth Model, we can solve for the value of g.

$$P = \frac{D_1}{r-g}$$
$$20 = \frac{0.5}{0.1-g}$$
$$g = 0.1 - \frac{0.5}{20} = 7.50\%$$

(b) $g = ROE \times b$ where b is the plowback ratio $b = \frac{0.75}{1.25} = 0.6$, thus

$$ROE = \frac{g}{b} = \frac{0.075}{0.6} = 12.5\%$$

(c) MW Co. is a growth company since ROE on its new investments (12.5%) is higher than the cost of capital (10%), and b > 0.

32. From the original valuation, we can find r = EPS/P = 12.5%
Now if the earnings grow forever at 2.5%, then the new value is
EPS1/(r-g) = 2.5*(1+2.5%)/(12.5%-2.5%) = 25.625 million
So the value of the company will increase by 5.625 million
*Here we assumed the company always payout all the earnings as dividend.

33. (a) g = ROE*b = 12%. Therefore, price = D1/(r-g) = 4/(12.5%-12%) = 800. PVGO = 800 -$20/12.5% = 640

(b) As ROE < r, the company should stop expanding. Then its value will be $160 ($20 per year forever).

34. No. EPS is only growing at 12.7% for the next three years, not forever. The expected rate of return can only increase by less than that amount. In addition, there may be a cost to the rapid growth (e.g. part of the current earnings may be retained), so the rate of return is lowered further.

35. We will calculate the price of the stock a year from today. At that point, the firm can either not take the project or if it does, the project will reduce the current EPS by $3 but as a result the EPS moving forward will be higher by $3 \times 20\% = \$0.6$.

In the first case, the price is the PV of the perpetuity of $5 discounted at 10%. So $P = \$55$ (note that the first earning is at time zero, but this is more of less a timing assumption and other assumption as long as clearly markes would be valid as well).

In the second case, the price is $2 plus the PV of a perpetuity of $5.6 starting in 1 year discounted at 10%. So price will be $P = \$58$

36. We assume that dividends are paid continuously and discounting is also done continuously. We will divide each year into N parts each with length of $1/N$. Assume dividend is paid at the end of each little time section. Fix a time horizon T and let's calculate the PV:

$$PV(0,T) = \sum_{i=0}^{TN} \left[\left(\frac{0.6}{N} \left(1 + \frac{0.06}{N} \right)^i \right) \left(1 + \frac{0.12}{N} \right)^{-i} \right]$$

$$= \sum_{i=0}^{TN} \left[\left(\frac{0.6}{N} \left(1 + \frac{0.06}{N} \right)^{N\frac{i}{N}} \right) \left(1 + \frac{0.12}{N} \right)^{N\frac{-i}{N}} \right]$$

Now, taking the limit at $N \to \infty$ and usign the trick that $e^{rt} = \lim_{n \to \infty} (1 + r/n)^{nt}$, we

get:

$$
\begin{aligned}
PV(0,T) &= \int_0^T 0.6e^{0.06t}e^{-0.12t}dt \\
&= \int_0^T 0.6e^{-0.06t}dt \\
&= \frac{0.6}{-0.06}(e^{-.06T} - 1)
\end{aligned}
$$

Now as $T \to \infty$ the PV become equal to \$10. So the combination of continously paying dividend, continous compounding and contimous growth cancel each other out and we have the same PV.