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JUMPING THE GATES: USING BETA-OVERLAY STRATEGIES TO HEDGE LIQUIDITY CONSTRAINTS

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In response to the current financial crisis, a number of hedge funds have implemented "gates" on their funds that restrict withdrawals when the sum of redemption requests exceeds a certain percentage of the fund's total assets. To reduce the investor's risk exposures during these periods, we propose a futures overlay strategy designed to hedge out or control the common factor exposures of gated assets. By taking countervailing positions in stock, bond, currency, and commodity exposures, an investor can greatly reduce the systematic risks of their gated assets while still enjoying the benefits of manager-specific alpha. Such overlay strategies can also be used to reposition the betas of an investor's entire portfolio, effectively rebalancing asset-class exposures without having to trade the less liquid underlying assets during periods of market dislocation. To illustrate the costs and benefits of such overlays, we simulate the impact of a simple beta-hedging strategy applied to long/short equity hedge funds in the TASS database.

1 Introduction

The current financial crisis has created enormous stress in the hedge-fund industry, with wholesale liquidations at firesale prices causing hedge-fund managers to impose gates on investor redemptions. In many cases, such measures may well be justified because the unwinding of illiquid positions under duress can lead to extreme losses for both exiting and remaining investors. By instituting a gate, managers can unwind positions in a more orderly manner, preserving value for all investors. However, if unwinding positions in a more orderly manner takes months or, in some extreme cases, years, gated investors may be forced to bear certain risks that are no longer appropriate or "suitable".

In this paper, we argue that such circumstances can be remedied to some degree by implementing a futures overlay strategy in which the most significant factor risks of an investor's gated assets are hedged out or shaped to satisfy specific constraints from the moment an investor submits his

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redemption notice to the day when his assets are fully paid out. In this way, a manager can engage in the orderly liquidation of an investor's stake while, at the same time, a futures overlay can be implemented by either the manager or the investor to ensure that during a lengthy liquidation process, the investor's net exposures are more in line with original investment objectives.

The factors that we propose to use for hedging purposes are the most liquid exchange-traded futures contracts on stock indexes, bonds, interest rates, and commodities, and currency forward contracts on the major currencies. The reason for using liquid futures is simple: introducing illiquidity into the hedging vehicle would contradict the primary objective of the overlay, which is to reduce the main risk exposures of an illiquid hedge-fund investment. The reason for using currency forwards is the fact that they are even more liquid than currency futures, although the popularity of the latter has grown significantly in recent years and may eventually surpass the liquidity of forwards.

Of course, any hedging program that uses liquid instruments to hedge an illiquid portfolio will exhibit a certain degree of tracking error arising from at least two distinct sources: common but illiquid factors, and manager-specific factors. The former is unavoidable, and the latter may actually be desirable (if the investor still believes the manager has unique alpha) or at least tolerable (if the investor is well diversified across multiple managers). The key issue in determining the efficacy of the hedging program is the relative contribution of the hedgeable factors to the overall risk of the investor's assets, i.e., the R^2 of the risk model or "hedging equation". If the R^2 is close to 100%, then a hedging overlay strategy can neutralize nearly all of the risk of the investment; if the R^2 is close to 0%, the hedging strategy is nearly useless.

In between 0% and 100%—which is where most risk models fall—an investor can reduce part of the overall risk of his investment. We argue that this part—the risks due to the most liquid factor exposures—should be the highest priority for an investor to hedge for several reasons. First, it is the easiest set of risks to hedge by definition since there exist liquid futures contracts with which to implement the hedge. Second, it is likely to account for a significant amount of risk (otherwise, the corresponding futures would not be as liquid as they are), especially during periods of market dislocation when gates are triggered. Third, if the investor's assets are gated for an extended period of time, the risk profile of those assets can change significantly as market conditions change, and the investor may not be equipped to monitor those changes continuously during this period. Fourth, the typical investor is likely to have significant exposure to these same common-factor risks in other parts of his portfolio, particularly among traditional investment vehicles, hence a hedging program can enhance diversification. And finally, the investor selected the manager presumably because of the manager's unique sources of alpha, not the manager's betas, hence neutralizing those betas should have little impact on the manager's value-added.

Apart from hedging, futures overlay strategies can also be used to re-position an investor's overall portfolio to address broader liquidity constraints. For example, a pension fund that has experienced a significant market decline in its equity investments will be underweight stocks and overweight bonds, implying a significant rebalancing need to return to its strategic asset allocation. However, bond-market illiquidity may make such a rebalancing unusually costly. A futures overlay strategy that is long stock-index futures and short bond futures can alleviate this temporary imbalance, and as liquidity is restored to the bond markets, the overlay can be gradually reduced until it is no longer needed. Moreover, the liquidity, credit quality, and built-in leverage of exchange-traded futures allows such beta re-positioning overlays to be implemented cheaply, safely, and with relatively small amounts of capital.[1] In such applications, some betas are temporarily enhanced and others reduced, with the overall objective of maintaining a level set of exposures through changing market conditions.

In Section 2, we begin with a review of the basic definition of a linear risk model for alternative investments, and then describe the basic mechanics of beta-hedging and beta-repositioning in Section 3. We apply this framework in Section 4 to the universe of long/short equity hedge funds in the TASS database, and summarize the performance of the overlay during the past few years. In Section 5, we propose a dynamic implementation of beta overlay strategies in which overlays are applied selectively over time as a function of market conditions. By hedging only during periods of clear market dislocation, the overall performance drag of beta overlays can be reduced significantly at the expense of more frequent trading. We conclude in Section 6.

2 Linear risk models

The first step in constructing a beta-hedging overlay strategy, and perhaps the most important step, is to determine the relationship between the portfolio to be hedged and the hedging factors. Most hedging programs begin with a linear relationship, although more sophisticated programs can be constructed using nonlinear relationships at greater expense and complexity. In this paper, we shall focus only on linear hedging programs.

Denote by R_{it} the return of hedge fund i at date t, and let R_{it} satisfy the following linear relationship:

$$R_{it} = \alpha_i + \beta_{i1}\text{RiskFactor}_{1t} + \cdots + \beta_{iK}\text{RiskFactor}_{Kt} + \epsilon_{it} \qquad (1)$$

where RiskFactor_{kt} denotes the date-t return of risk factor k. This linear risk model may seem familiar to students of modern financial analysis, which is based on linear multi-factor models such as the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT). However, (1) differs in a few important ways.

First, the relationship we hypothesize is primarily a statistical one that is unfettered by any particular economic theory or philosophy. In particular, we place no restrictions on α_i, whereas the CAPM and APT assume that managers add no value above and beyond the risk premia associated with the risk factors.[2]

Second, we define "risk factor" differently from the usual academic context—our notion of a risk factor is an economic variable that satisfies three criteria:

1. **Definability.** It is a well-defined and measurable economic variable, i.e., there is a commonly accepted definition of the concept and an explicit way to measure it accurately.
2. **Commonality.** The variable is statistically and significantly related to a broad set of hedge funds or other investment vehicles.
3. **Tradability.** There exist liquid futures or forward contracts that capture the full economic effects of the variable.

The motivation for the first two conditions is obvious, but the third requires some explanation. Although economists have derived a number of linear factor models with a variety of factors, not all of them are based on marketable securities.[3] However, from a practical perspective, if one cannot trade the factor, there is no actionable consequence that can be derived from the risk model since it is impossible to alter the exposure to that factor. Therefore, our definition of a "risk factor" requires tradability so that any exposure identified in (1) can be actively managed.

Some examples and counter-examples may help to clarify these criteria. Examples of economic variables that satisfy our definition of risk factors are: the

S&P 500, Japanese 10-year government bonds, the US dollar index, oil, and gold. In each case, the variable: (1) has a precise definition that is widely agreed upon and which can be measured accurately; (2) is clearly related to many hedge funds and traditional investments; and (3) can be traded via futures or forward contracts.

On the other hand, the following are counter-examples that are not risk factors according to our criteria: fear, greed, illiquidity, and animal spirits. Despite the fact that all of these factors are quite relevant for hedge funds, none of them has a widely accepted definition that yields measurable quantities, nor can any of them be easily traded. Therefore, while such factors may have substantial economic justification, for the purposes of hedging beta exposures, we do not consider them as risk factors.

Based on (1), we have the following characterization of the fund's expected return and variance:

$$\begin{aligned} \mathrm{E}[R_{it}] &= \alpha_i + \beta_{i1}\mathrm{E}[\mathrm{RiskFactor}_{1t}] + \cdots \\ &\quad + \beta_{iK}\mathrm{E}[\mathrm{RiskFactor}_{Kt}] \end{aligned} \tag{2}$$

$$\begin{aligned} \mathrm{Var}[R_{it}] &= \beta_{i1}^2\mathrm{Var}[\mathrm{RiskFactor}_{1t}] + \cdots \\ &\quad + \beta_{iK}^2\mathrm{Var}[\mathrm{RiskFactor}_{Kt}] \\ &\quad + \mathrm{Covariances} + \mathrm{Var}[\epsilon_{it}] \end{aligned} \tag{3}$$

where "Covariances" is the sum of all pairwise covariances between $\mathrm{RiskFactor}_{pt}$ and $\mathrm{RiskFactor}_{qt}$ weighted by the product of their respective beta coefficients $\beta_{ip}\beta_{iq}$.

This characterization implies that there are two distinct sources of a hedge fund's expected return: beta exposures β_{ik} multiplied by the risk premia associated with those exposures $\mathrm{E}[\mathrm{RiskFactor}_{kt}]$, and manager-specific alpha α_i. By "manager-specific", we do not mean to imply that a hedge fund's unique source of alpha is without risk—we are simply distinguishing this source of expected return from those that have clearly identifiable risk factors associated with them. In particular, it may well be the case that α_i arises from factors other than the K risk factors identified in (1), and a more-refined version—one that better reflects the particular investment style of a given manager—may yield a better-performing risk model.

From (3) we see that a hedge fund's variance has three distinct sources: the variances of the risk factors multiplied by the squared beta coefficients, the variance of the residual ϵ_{it}—which may be related to the specific economic sources of α_i—and the weighted covariances among the factors. This decomposition highlights the fact that a hedge fund can have several sources of risk, each of which should yield some risk premium, otherwise investors would not be willing to bear such risk. By taking on exposures to multiple risk factors, a hedge fund can generate attractive expected returns from the investor's perspective (see, for example, Lo, 2001).

Litterman (2005) calls such risk exposures "exotic betas" and argues that "[t]he adjective 'exotic' distinguishes it from market beta, the only beta which deserves to get paid a risk premium". We disagree—there are several well-established economic models that illustrate the possibility of multiple sources of systematic risk, each of which commands a positive risk premium, e.g., Merton (1973) and Ross (1976). We believe that hedge funds are practical illustrations of these multi-factor models of expected returns, and on average, have net long exposures to such risk factors. For example, long/short equity managers are typically net long the S&P 500, hence they benefit to some degree from the normally positive equity risk premium. Equity market-neutral managers are typically long volatility, CTAs are typically long commodities, and global macro managers are typically long bonds. Therefore, hedging away the beta exposures of these managers will, on average, require short positions in risk factors that normally yield positive expected returns.

Accordingly, by hedging away certain risk factors, an investor will be forgoing the normally positive risk premia associated with such factors. Therefore, one or more of the following conditions must hold for a rational investor to implement a beta-hedging overlay strategy:

(A1) The investor believes that the expected returns of the risk factors to be hedged are temporarily negative during the hedging period.
(A2) The investor believes that the risk reduction from hedging the risk factors is worth the price of forgoing the normally positive expected returns of the risk factors to be hedged during the hedging period.
(A3) The investor already has significant exposure to the risk factors to be hedged, and therefore does not wish to have any additional exposure.
(A4) The risk factors to be hedged are incidental to the expected return of the manager, but they contribute more than proportionally to the manager's volatility.

Of course, any successful hedging program also requires the following condition:

(B) The risk factors in the linear risk model (1) account for a significant fraction of the variability in the manager's returns.

If (B) is not satisfied, there is no point to hedging since the risk factors are unable to capture much of the manager's risks. In such cases, implementing a beta-hedging overlay can actually *increase* the overall risk to the investor while simultaneously reducing the expected return because of transactions costs and forgone potential risk premia. One measure of a risk model's effectiveness is its R^2, which is simply the estimated fraction of the total variance

$$R^2 \equiv \frac{\text{Var}[\sum_k \beta_{ik}\text{RiskFactor}_{kt}]}{\text{Var}[R_{it}]}. \qquad (4)$$

For the majority of hedge funds in the TASS database, the R^2's range from 25% to 75% for a three-factor risk model, and where a hedge fund falls in this range depends on several characteristics: the hedge fund's investment style, the set of risk factors, and the time period. A very rough guideline for the minimum R^2 needed to implement an effective hedging-overlay strategy is 25%—any value lower than this threshold raises the possibility that the hedge will do more harm than good.

3 Beta overlays

Assuming that one or more of conditions (A1)–(A4) and (B) hold, we now proceed to construct a beta-hedging overlay program or "beta-blocker" for the investor's stake in manager i.[4] Denote by R_{ht} the return of a hedging portfolio consisting of futures or forward contracts corresponding to the K risk factors in (1). To hedge out all of the factor risks of manager i, we simply take countervailing positions in each of the factors, so that the sum of R_{ht} and R_{it} contains no factor exposures:

$$\begin{aligned} R_{ht} + R_{it} &= \alpha_i + \epsilon_{it} \\ R_{ht} &= -\sum_{k=1}^{K} \beta_{ik}\text{RiskFactor}_{kt} \end{aligned} \qquad (5)$$

and this is always achievable given our definition of risk factors. Appendix A.1 provides a more detailed discussion of the mechanics of this process for a given dollar investment in manager i and specific notional values for futures contracts corresponding to the K risk factors.

With this beta-blocker in place, the risk reduction of the post-hedge portfolio can be quantified as:

$$\frac{\text{Var}[R_{ht} + R_{it}]}{\text{Var}[R_{it}]} = 1 - R^2 \qquad (6)$$

where R^2 is defined in (4). Therefore, the percentage reduction in volatility δ due to the beta-blocker is

Table 1 Percentage volatility reduction from linear beta-blockers for various levels of R^2.

R^2	δ
5%	3%
10%	5%
20%	11%
30%	16%
40%	23%
50%	29%
60%	37%
70%	45%
80%	55%
90%	68%
95%	78%

simply:

$$\delta \equiv 100 \times (1 - \sqrt{1 - R^2}) \qquad (7)$$

which is tabulated in Table 1 for various levels of R^2. These figures show that for a hedge fund with an R^2 of 50%, a beta-blocker will reduce the volatility by about 29%.

Of course, the values in Table 1 and the beta-blocker (5) are all based on *estimates*, not the true theoretical values of β_{ik} and R^2 which are unobservable. Therefore, the realized performance of the beta-blocker may differ from the estimated performance. Moreover, as the parameters β_{ik} and R^2 change—which they are likely to do for the typical hedge fund—performance differences may arise. These and other practicalities create "tracking error" in the beta-blocker, and a number of techniques can be employed to mitigate its effects, including time-varying-parameter regression, regime-switching models, and robust estimation. Appendix A.2 contains a more detailed discussion of tracking error.

Given the generality of the beta-blocking framework (5) that we have proposed, it is clear that this approach can be applied to any collection of managers, both alternative and traditional, and can be used to hedge only a portion of the beta exposures if desired. The only pre-requisite is condition (B): we must be able to construct a risk model for *each manager* that adequately captures that manager's risk exposures. Unlike the case of traditional assets where a single risk model, e.g., the MSCI/BARRA Gobal Equity Model or the Northfield Global Risk Model, can cover the risk profiles of an entire class of managers, alternative assets are considerably more heterogeneous. However, constructing individual risk models for each alternatives manager allows the investor to integrate his traditional and alternatives portfolios in a relatively seamless manner. We argue that such an integration is not only desirable, but also indispensable in determining the overall risk/reward profile of an investor's portfolio.[5]

In some cases, an investor may be less interested in neutralizing certain betas than in gaining exposures to them in a cost-effective manner. In these cases, similar overlay strategies can be used to "reposition" the investor's betas, reducing those that the investor is not willing to bet on, and accentuating those that the investor is. Since the mechanics are so similar to those of the beta-blocker, we relegate the details of such beta-repositioning strategies to Appendix A.1.

4 Hedging Long/Short Equity managers

To illustrate the empirical relevance of the beta-blocker program, we apply this hedge to the universe of Long/Short Equity funds from the Lipper TASS hedge fund database during the period from January 2000 through October 2008. We find that, on average, the beta-blocker program reduces the volatility, maximum draw-downs, and autocorrelations, and increases the Sharpe ratios of the funds, with a modest reduction in average monthly return. We also find that the beta-blocker program is more effective for funds with higher average regression R^2 values, as suggested by (7) in Section 3.

For this analysis, we selected all Long/Short Equity funds from the Lipper TASS "Live" database (as of December 1, 2008) that report: (1) returns on a monthly basis and value their assets under management in US dollars; (2) assets under management of at least 500MM USD at some point during 2008; and (3) monthly returns for every month between January 2004 and October 2008. There are 47 funds that meet all of these criteria.

To estimate the risk models of Section 2, we use the following 15 factors:

- 6 equity factors: S&P 500 Futures, S&P/TSE 60 Futures, FTSE 100 Futures, DAX Futures, CAC 40 Futures and TOPIX Futures
- 5 10-year government bond factors: US 10-Year Futures, Canadian 10-year Futures, Euro-Bund Futures, Long Gilt Futures, and Japanese 10-Year Futures
- 4 foreign exchange factors: EUR Forwards, CAD Forwards, GBP Forwards, JPY For wards

To estimate the betas of each fund, we use a 24-month trailing window and a statistical factor-selection algorithm to select 2 of the 15 factors and estimate the regression coefficients using ordinary least-squares regression on these two factors. In implementing the overlay strategy for month t, we employ the betas estimated using data through month $t-2$, and scale the magnitude of the hedge based on the net-asset-value at the end of month $t-2$.[6] Our simulations do not include transaction costs; however, since the overlay positions change only on a monthly basis and involve only highly liquid instruments, the associated costs are likely to be negligible.

Figure 1 reports the distribution of average regression α's and average R^2's among the risk models for the 47 funds in our sample. The median R^2 is 0.49 and the distribution of average α has mean 0.83% and median 0.76%, suggesting that the selected risk factors do, in fact, account for a significant fraction of volatility, even in the presence of substantial manager-specific alpha. Indeed, the mean and median fund returns are 0.94% and 0.77% per month, respectively. Figure 5 shows the distribution of annual returns, standard deviation, Sharpe ratio,[7] and maximum drawdown for all funds (in blue) and all funds with the beta-blocker overlay (in red).

To develop some intuition for the amount of monthly turnover generated by a beta-hedging program based on these estimates, note that there are two potential reasons for changing positions each month: (i) a change in the selected factors due to

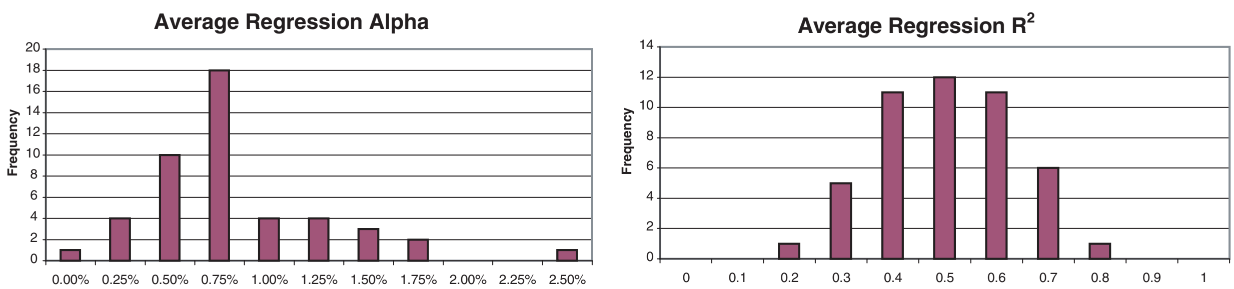


Figure 1 Distribution of average regression alphas and R^2's for 24-month rolling-window regressions of the returns of 47 Long/Short Equity funds in the TASS "Live" database from January 2000 to October 2008 using two factors per regression chosen statistically from a universe of 15 factors.

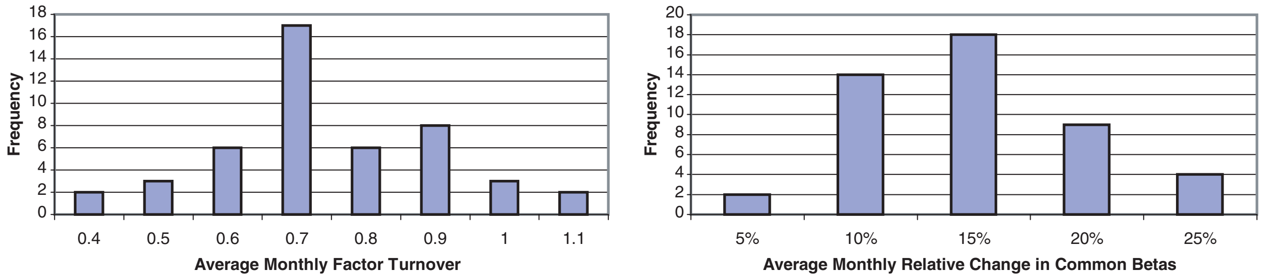


Figure 2 The distribution of (i) monthly factor turnover and (ii) monthly changes in betas for the 47 Long/Short Equity funds in the TASS "Live" database from January 2000 to October 2008.

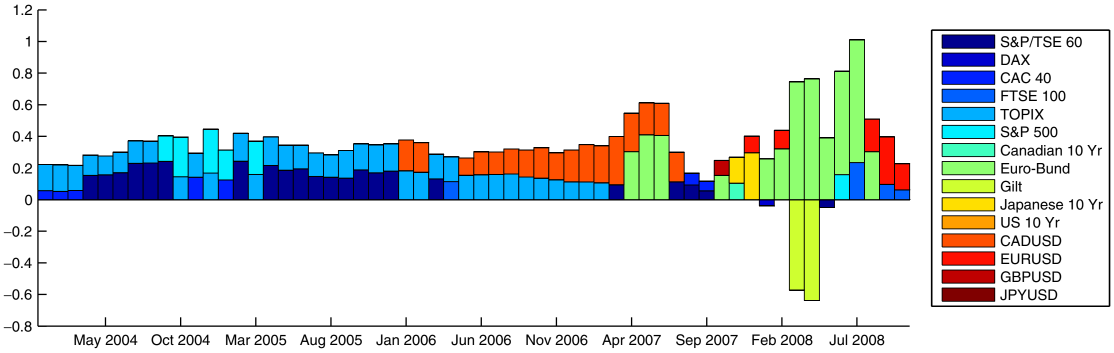
our factor-rotation algorithm; and (ii) a change in the estimated betas for those factors that persist from the previous month. The left half of Figure 2 shows the distribution of the average number of factors that change on a monthly basis; since we are selecting 2 factors each month, a value of 2 would represent a complete monthly turnover. To put these value into perspective, Figures 3a and 3b show the evolution of the betas of the funds with the maximum turnover (1.14) and median turnover (0.727) respectively. The right half of Figure 2 shows the distribution of the average percentage change in the betas of factors that are selected in consecutive months.[8]

Figure 3c shows the average betas across the 47 Long/Short Equity funds in the TASS "Live" database. These are the betas that would be estimated for a portfolio consisting of an equal-weighted allocation to each of the 47 funds. The monthly performance of such a portfolio, together with the monthly performance of the overlay is illustrated in Figure 4.

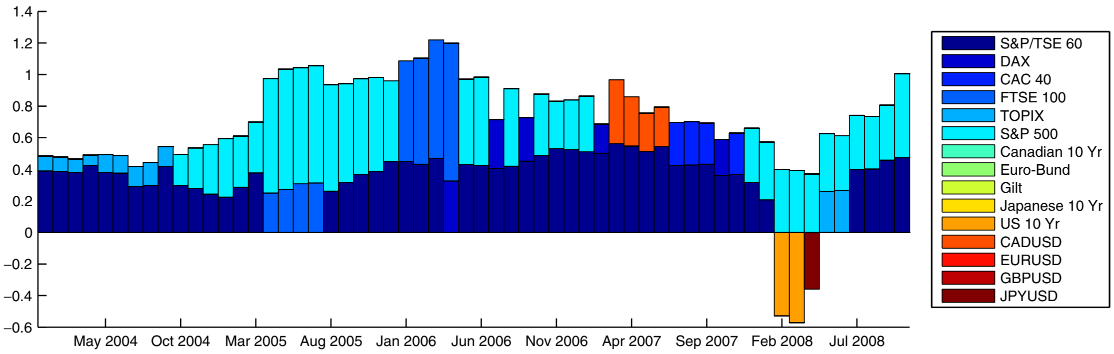
The left panel of Table 2 summarizes the impact of the beta-blocker overlay on the 47 funds. On an average, the overlay reduces the annual return by 0.61% while reducing volatility by 11.6% and reducing maximum drawdowns by 20.4%. Equation (7) suggests that the degree of volatility reduction improves as the regression R^2 increases, and we confirm this empirically in the right panel of Table 2 where we summarize the impact of the beta-blocker overlay for the subset of 21 funds with average R^2 greater than 0.5. We also note that the correlation between the percentage of volatility reduction and the empirical values of $1 - \sqrt{1 - R^2}$ is 75%, again demonstrating the relevance of the analysis of Section 3.

Another common consequence of the beta-blocker overlay is a reduction in autocorrelations of returns. As shown in Lo (2001, 2002) and Getmansky, Lo and Makarov (2004), autocorrelation in hedge-fund returns is a proxy for illiquidity exposure, and Figure 6 implies that a number of long/short equity funds contain illiquid investments, e.g., the median first-order autocorrelation is 20.6%. Not surprisingly, Figure 6 shows that the combined returns of these funds and the overlay strategies have considerably lower autocorrelation and greater liquidity, with the distribution of autocorrelations shifted left by the beta-blocker and a median autocorrelation of 9.2%.

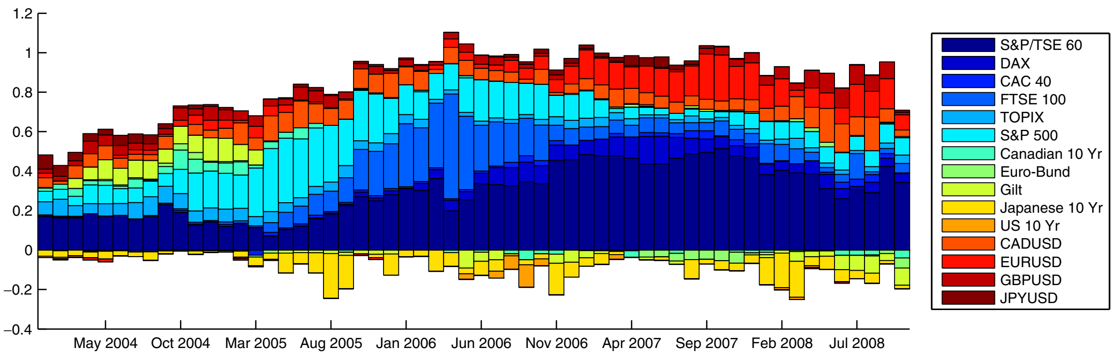
Table 3 reports the percentage change in autocorrelation resulting from the beta-blocker, as well as the percentage change in annual standard deviation (taking into account the first-order autocorrelation) and the annual Sharpe ratio (again, accounting for first-order autocorrelation; see Lo, 2002). As with Table 2, the beta-blocker overlay is, on average, more effective for the subset of funds with average R^2 in excess of 0.5.



(a)



(b)



(c)

Figure 3 The evolution of the estimated betas of (a) the Long/Short Equity fund from the TASS "Live" database with the largest factor turnover (1.14 factors per month), (b) the Long/Short Equity fund with the median factor turnover (0.727 factors per month), and (c) an equal-weighted portfolio of the 47 Long/Short Equity funds in our sample.

Table 3 shows that another advantage of the reduced autocorrelation is a reduction in longer-horizon (e.g., annual) volatility, even for fixed levels of monthly volatility, due to the fact that the annual variance is the sum of monthly variances plus all the pairwise covariances of the twelve individual monthly returns (which are directly related to the autocorrelations).

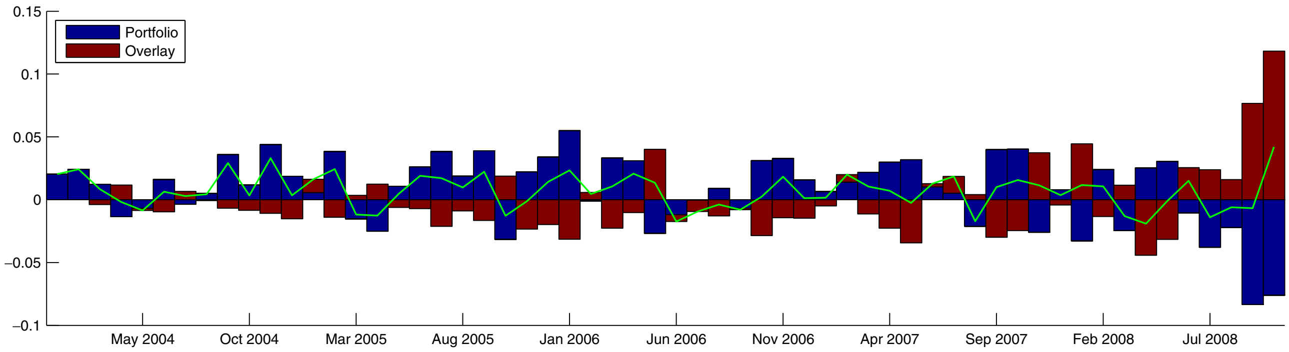


Figure 4 The monthly returns of an equal-weighted portfolio of the 47 Long/Short Equity fund from the TASS "Live" database (Blue), the beta-blocker monthly overlay (Red) and the resulting performance of the hedged portfolio (Green).

Table 2 Summary statistics on the impact of the beta-blocker overlay for 47 Long/Short Equity funds in the TASS "Live" database from January 2000 to October 2008. The left panel reports results for all 47 funds in our sample, and the right panel reports results for the subset of 21 funds whose average regression R^2 is greater than 0.5 (for a 24-month rolling-window regressions using two factors per regression chosen statistically from a universe of 15 factors).

	All 47 Funds				21 Funds with Average $R^2 > 0.5$			
	Change in annual return	% Change in volatility	% Change in Sharpe ratio	% Change in max draw-down	Change in annual return	% Change in volatility	% Change in Sharpe ratio	% Change in max draw-down
Mean	−0.61%	−11.6%	2.8%	−20.4%	0.38%	−25.1%	12.5%	−38.9%
25th Percentile	−1.86%	−26.7%	−15.3%	−48.0%	−0.93%	−36.0%	−7.2%	−61.8%
Median	−0.40%	−9.2%	3.1%	−32.7%	−0.36%	−31.4%	44.0%	−42.5%
75th Percentile	1.27%	3.0%	42.8%	−5.3%	1.76%	−16.1%	77.8%	−27.4%

As noted in Section 2, the beta-blocker overlay is not without cost—we see in Table 2 that the overlay reduces annual returns by an average of 0.61% over the entire sample period. Moreover, the cost will tend to be greater during "normal" and/or favorable market conditions. Indeed, since funds tend to have long exposures to various risk factors, the overlay will often be short risk factors that, during normal market conditions, offer positive risk premia. Thus, the criteria set forth in Section 2 must be taken into consideration in deciding what factors to hedge and to what degree.

To help put these considerations in perspective, we tabulate in Table 4 the annual returns, volatilities, and their ratios of the 15 risk factors used in this analysis. These results show that during the last few years (other than 2008), US and foreign equities have been the most costly factors to hedge, with annual returns that range from 1.4% to 44.9% during the 2003–2006 subperiod. Of course, equities are also the most volatile asset class, with annual volatilities during this same period that range from 9.7% to 31.3%. While such high levels of volatility seem to go hand-in-hand

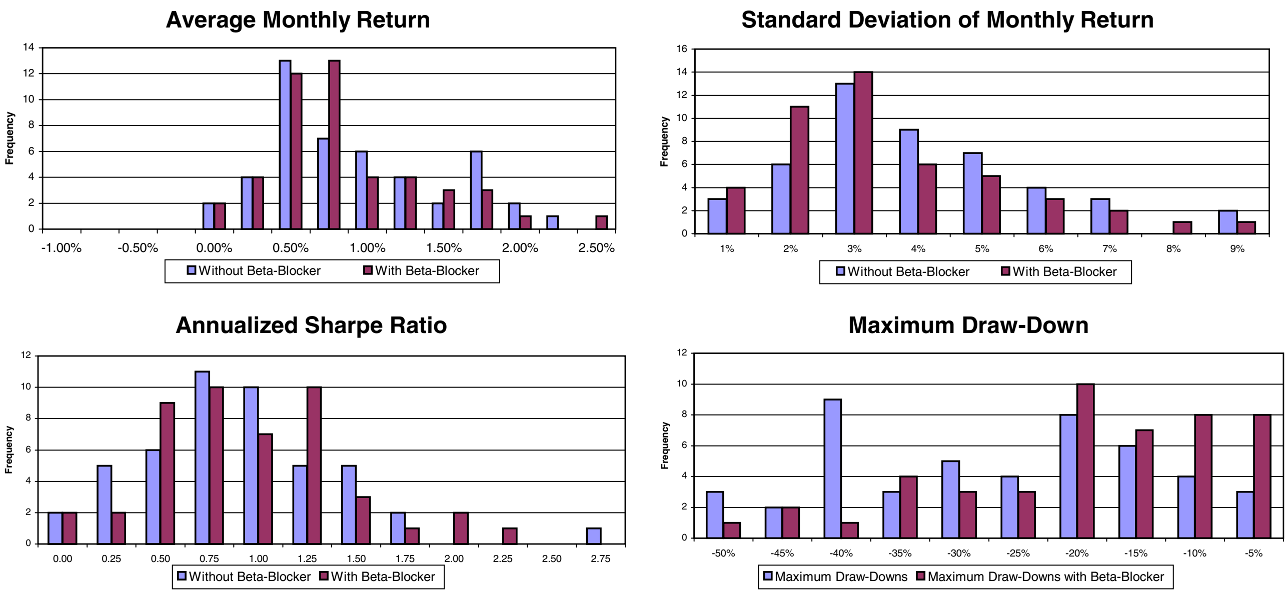


Figure 5 Distribution of return statistics for 47 Long/Short Equity funds in the TASS "Live" database from January 2000 to October 2008 with and without beta-blocker overlays constructed from 24-month rolling-window regressions using two factors per regression chosen statistically from a universe of 15 factors.

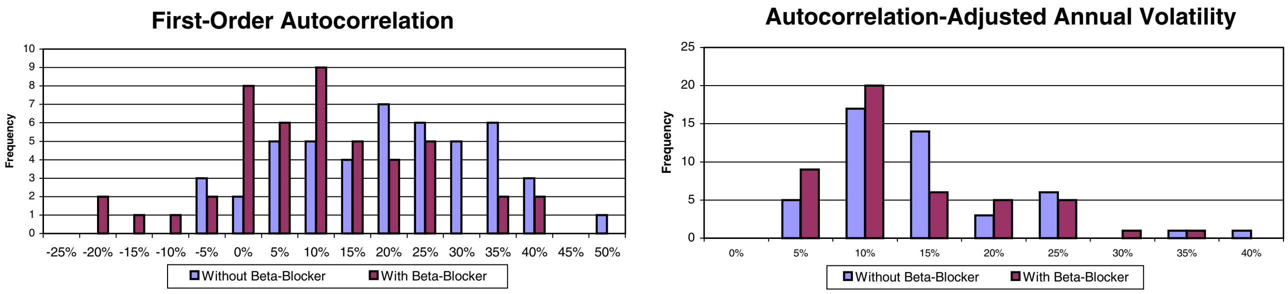


Figure 6 Distributions of autocorrelations and autocorrelation-adjusted annual volatility of 47 Long/Short Equity funds in the TASS "Live" database from January 2000 to October 2008 constructed from 24-month rolling-window regressions using two factors chosen statistically from a universe of 15 factors.

with large risk premia, during periods of market dislocation such as 2000–2002 and 2008, equities can also yield double-digit losses. Therefore, hedging out equity-beta exposure may or may not cause a performance drag, depending on market conditions, but it will definitely reduce portfolio risk.

The potential performance gaps between periods of calm and periods of dislocation are highlighted in Table 5, which summarizes the impact of the beta-blocker overlay during a period of relative calm (2005–2006) and a more turbulent period (2007–2008). While the goal of reducing volatility is achieved on average during both periods, it is clear that the costs of hedging are much greater on average during 2005–2006, underscoring the importance of the hedging conditions set out in Section 2, and opening the door for the possibility of hedging selectively as a function of market conditions. We turn to this possibility in the next section.

Table 3 The impact of the beta-blocker overlay on first-order autocorrelations, autocorrelation-adjusted annual volatility and autocorrelation-adjusted annual Sharpe ratios for 47 Long/Short Equity funds in the TASS "Live" database from January 2000 to October 2008. The left panel reports results for all 47 funds in our sample, and the right panel reports results for the subset of 21 funds whose average regression R^2 is greater than 0.5 (for a 24-month rolling-window regressions using two factors per regression chosen statistically from a universe of 15 factors).

	All 47 funds			21 Funds with average $R^2 > 0.5$		
	% Change in first-order autocorrelation	% Change in AC-adjusted volatility	% Change in AC-adjusted Sharpe ratio	% Change in first-order autocorrelation	% Change in AC-adjusted volatility	% Change in AC-adjusted Sharpe ratio
Mean	−52.6%	−14.5%	6.8%	−88.3%	−27.5%	16.4%
25th Percentile	−108.7%	−33.5%	−15.7%	−126.9%	−40.6%	−9.4%
Median	−37.2%	−14.2%	12.2%	−44.6%	−35.9%	50.2%
75th Percentile	−16.1%	4.7%	49.5%	−23.2%	−15.6%	86.0%

5 Dynamic implementations of beta overlays

For those investors who have committed to a long-term hedging program, it may be possible to avoid some of the performance drag of a full beta-blocker overlay by formalizing conditions under which the overlay should be fully engaged and when it should be inactive. For example, one might choose to hedge portfolio betas—either completely or partially—only during periods where the volatility of the beta exposures exceeds a certain threshold, or based on an external condition, e.g., only when the VIX Index exceeds 50, or as part of other risk-management protocols. Hedging only during those periods when the portfolio is deemed to be at "high risk" and forgoing the overlay during other periods may seem like market-timing, but in fact is closer to volatility-timing, a considerably less daunting challenge. In fact, there is mounting evidence that volatility is both time-varying and persistent, and most investors do respond dynamically to sharp changes in risk, which is consistent with a dynamic implementation of beta overlay strategies.

To maximize the effectiveness of such dynamic implementations, such beta overlay strategies should be updated on a *daily* basis, despite the fact that the betas are updated only monthly. While the monthly estimated betas must (by definition) remain static until new return data is available—presumably at least a month later—selective hedging can occur on a daily basis as the volatilities of the hedging factors change from day to day.

To illustrate the flexibility of daily hedging using the beta-blocker framework, we implement dynamic hedging overlays for the 47 Long/Short Equity funds from the "Live" TASS database from January 2000 through October 2008 using the following simple algorithm: Whenever the trailing 2-week (daily) volatility of the beta exposures exceeds the trailing 2-year (daily) volatility estimate of the beta exposures, we hedge the betas to bring them back in line with the 2-year volatility estimate. Thus if the 2-week volatility of the betas is 3.0% and the 2-year volatility of the betas is 2.0%, then we put on a partial hedge equal to one-third of the beta exposure to bring the short-term volatility to the same level as the long-term volatility.

Table 6 reports the results of a daily simulation (not including transaction costs) of the hedge described

Table 4 Annual returns, volatilities, and return/volatility ratios of the 15 hedging factors.

Year	S&P/ TSE 60 futures	DAX futures	CAC 40 futures	FTSE 100 futures	TOPIX futures	S&P 500 futures	Canadian 10 yr futures	Euro-bund futures	Long gilt futures	Japanese 10 yr futures	US 10 yr futures	CAD forwards	EUR forwards	GBP forwards	JPY forwards
Annual returns															
2000	3.4%	−10.5%	−3.9%	−14.9%	−24.3%	−15.1%	6.2%	5.3%	4.8%	4.4%	9.9%	−4.3%	−9.1%	−7.9%	−15.8%
2001	−17.2%	−28.5%	−25.2%	−18.2%	−19.7%	−15.9%	1.7%	−0.6%	−1.9%	4.6%	3.4%	−5.7%	−5.2%	−1.5%	−16.4%
2002	−15.7%	−47.0%	−37.1%	−25.6%	−16.7%	−23.6%	8.1%	8.4%	6.9%	5.0%	15.2%	1.7%	19.7%	13.1%	8.8%
2003	22.2%	32.5%	15.3%	13.5%	27.2%	28.7%	3.4%	2.4%	−1.7%	−0.8%	2.3%	24.0%	21.0%	13.6%	9.0%
2004	11.5%	4.4%	7.1%	5.9%	11.3%	9.2%	5.8%	7.9%	2.4%	3.2%	4.0%	8.8%	8.5%	10.3%	3.2%
2005	23.3%	24.2%	24.1%	15.2%	44.9%	1.4%	3.5%	4.2%	2.8%	0.6%	−0.7%	2.8%	−13.6%	−9.0%	−15.9%
2006	15.1%	18.5%	17.3%	9.1%	3.3%	10.1%	0.2%	−3.8%	−4.8%	0.2%	−1.7%	−1.3%	9.1%	13.4%	−5.7%
2007	6.9%	17.4%	0.2%	1.5%	−12.4%	−0.1%	1.3%	−2.2%	0.3%	3.4%	6.6%	16.7%	9.2%	1.8%	1.6%
2008	−28.9%	−43.5%	−41.5%	−33.6%	−43.7%	−39.6%	6.6%	7.8%	4.3%	1.8%	12.4%	−19.9%	−11.7%	−20.7%	14.4%
Annual Volatilities															
2000	28.7%	23.6%	23.8%	20.1%	23.8%	22.8%	5.4%	4.7%	5.7%	3.5%	5.5%	5.3%	12.1%	8.8%	10.6%
2001	20.7%	28.9%	26.9%	20.9%	26.8%	21.8%	6.9%	4.9%	5.2%	3.4%	7.3%	5.7%	11.4%	8.3%	10.1%
2002	18.2%	37.6%	34.6%	27.6%	24.2%	26.4%	6.1%	5.1%	4.8%	2.9%	7.0%	6.2%	9.0%	6.8%	10.3%
2003	11.2%	31.3%	25.5%	19.4%	22.3%	16.4%	6.0%	6.2%	5.7%	4.7%	7.7%	9.0%	10.0%	8.0%	8.4%
2004	12.3%	15.7%	13.8%	10.0%	17.5%	11.0%	5.7%	4.1%	4.3%	3.7%	6.3%	8.8%	10.7%	10.3%	10.1%
2005	11.3%	11.8%	10.8%	8.5%	13.7%	10.1%	4.2%	3.8%	4.1%	2.9%	4.5%	7.9%	9.0%	8.3%	8.7%
2006	13.8%	15.3%	14.6%	12.4%	18.9%	9.7%	3.9%	3.8%	3.7%	3.6%	3.8%	7.2%	7.4%	7.8%	8.2%
2007	15.4%	14.9%	16.9%	16.9%	20.6%	15.6%	4.7%	4.2%	4.4%	3.4%	5.2%	9.3%	6.1%	7.0%	9.2%
2008	40.4%	39.0%	39.5%	37.3%	50.2%	41.6%	7.0%	6.8%	7.5%	6.0%	9.7%	15.9%	13.0%	13.1%	14.6%
Ratios of Annual Returns to Annual Volatilities															
2000	0.12	−0.44	−0.16	−0.74	−1.02	−0.66	1.15	1.12	0.84	1.25	1.80	−0.82	−0.75	−0.90	−1.48
2001	−0.83	−0.99	−0.94	−0.87	−0.74	−0.73	0.24	−0.12	−0.37	1.35	0.46	−1.01	−0.45	−0.18	−1.62
2002	−0.86	−1.25	−1.07	−0.93	−0.69	−0.90	1.33	1.66	1.44	1.72	2.17	0.27	2.19	1.92	0.85
2003	1.99	1.04	0.60	0.70	1.22	1.75	0.57	0.39	−0.29	−0.17	0.30	2.67	2.09	1.70	1.08
2004	0.94	0.28	0.52	0.59	0.64	0.84	1.02	1.95	0.57	0.85	0.64	1.01	0.80	1.00	0.32
2005	2.05	2.05	2.23	1.78	3.28	0.14	0.82	1.10	0.68	0.21	−0.15	0.35	−1.51	−1.07	−1.83
2006	1.09	1.21	1.19	0.73	0.18	1.04	0.06	−0.99	−1.28	0.07	−0.43	−0.18	1.22	1.72	−0.69
2007	0.45	1.17	0.01	0.09	−0.60	0.00	0.27	−0.52	0.07	1.01	1.26	1.79	1.51	0.26	0.17
2008	−0.72	−1.12	−1.05	−0.90	−0.87	−0.95	0.94	1.14	0.57	0.30	1.27	−1.25	−0.91	−1.58	0.99

Note: 2008 returns do not include December.

Table 5 Summary statistics on the impact of the beta-blocker overlay for 47 Long/Short Equity funds in the TASS "Live" database from January 2005 through December 2006 (in the left panel) and from January 2007 through October 2008 (in the right panel).

	2005–2006				2007–2008			
	Change in annual return	% Change in volatility	% Change in Sharpe ratio	% Change in max draw-down	Change in annual return	% Change in volatility	% Change in Sharpe ratio	% Change in max draw-down
Mean	−8.87%	−21.5%	−47.2%	8.3%	8.63%	−14.8%	−87.0%	−27.7%
25th Percentile	−11.32%	−31.3%	−73.0%	−22.4%	3.26%	−42.9%	−157.6%	−60.5%
Median	−8.58%	−22.8%	−43.4%	3.5%	8.27%	−16.2%	−46.4%	−45.7%
75th Percentile	−4.81%	−11.3%	−17.2%	20.5%	14.63%	8.2%	16.4%	−20.7%

Table 6 Summary statistics on the impact of a daily hedging overlay (seeking to neutralize excess beta exposure during periods when the 2-week trailing volatility of beta exposure exceeds the 2-year trailing volatility of beta exposures) for 47 Long/Short Equity funds in the TASS "Live" database from January 2000 to October 2008. The left panel reports results for the period from January 2005 through December 2006 and the the right panel reports results for the period from January 2007 through October 2008.

	2005–2006				2007–2008			
	Change in annual return	% Change in volatility	% Change in Sharpe ratio	% Change in max draw-down	Change in annual return	% Change in volatility	% Change in Sharpe ratio	% Change in max draw-down
Mean	−0.45%	−2.5%	−0.8%	−3.7%	4.54%	−8.1%	−8.0%	−16.6%
25th Percentile	−0.77%	−4.3%	−4.0%	−12.8%	1.85%	−18.5%	−56.0%	−23.2%
Median	−0.34%	−2.2%	−0.34%	−1.1%	3.61%	−9.3%	−8.7%	−17.9%
75th Percentile	−0.10%	−0.3%	4.3%	2.6%	5.81%	0.2%	18.3%	−8.9%

above. Comparing the results with those from Table 5, we note that the overlay costs much less during the calm period from 2005–2006 (0.45% per year on average vs. 8.87%), while still providing a significant positive average annual return of 4.54% during the more turbulent period of 2007–2008. Not surprisingly, the return and volatility-reduction of the dynamic hedge is not as great as the full beta-blocker overlay—indeed, the hedge is only active part of the time and when it is active it is only hedging a portion of the beta exposure. However, such a hedge is just one example of a dynamic hedging program, and the appropriate trade-offs of performance drag and volatility-reduction for a given portfolio will vary depending on the investor's objectives and risk preferences.

6 Conclusion

The current credit crisis has upended the investment processes of many investors and managers, and the ubiquity of "unwind risk" has been used to justify a number of extraordinary measures including the raising of gates, the creation of illiquidity side-pockets, and complete suspensions of all withdrawals. Originally motivated by the desire to protect a fund's remaining investors from the panic unwinding of illiquid assets during periods of market dislocation, gates are now being used by some managers of even relatively liquid assets such as exchange-traded equities to retain investment capital, not necessarily to protect investor wealth.

In this paper, we have argued that investors need not stand idly by during these periods, but can manage their risk exposures pro-actively by using beta-blockers and beta-repositioning strategies to adjust their portfolios in the face of liquidity constraints. Although such strategies cannot generate liquidity from gated assets, the built-in leverage of exchange-traded futures allows investors great flexibility for reshaping their portfolio exposures in a capital-efficient manner. Moreover, because most broker/dealers accept securities as collateral for futures positions, even if an investor cannot liquidate his assets, he can often pledge them as collateral to support a futures overlay program. Although illiquid assets will undoubtedly suffer significant "haircuts" in terms of their collateral value, the magnitude of the typical institutional investor's portfolio is likely to be many multiples greater than the margin needed to support a futures overlay for that portfolio.

Although we have focused on only two uses of overlay strategies in this paper, there are clearly many other applications of our framework, including hedge-fund beta replication (Hasanhodzic and Lo, 2006, 2007), global tactical asset allocation, transition management for alternatives (Chafkin and Lo, 2008), and dynamic risk management (Lo, 2001, 2008). Moreover, we have used linear factor models to highlight the potential value of beta hedging even with relatively simple risk models, but more sophisticated models that incorporate time-varying volatilities and nonlinear relations among the factors may yield even better performance. As investors become more familiar with the risks of alternatives, we expect all of these applications to grow in importance and sophistication.

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A Appendix

This appendix contains the technical details to the beta-blocker and beta-repositioning strategies described in Section 3. Section A.1 outlines the analytical framework and Section A. 2 contains an analysis of tracking error.

A.1 Beta-blocker and beta-repositioning strategies

Consider a portfolio consisting of Strategy A plus a futures overlay program involving K types of futures contracts, where N_{kt} contracts of contract k are held at date t, $k = 1, \ldots, K$. Denote by PL_t the profit/loss of the portfolio at date t which is given by:

$$\mathrm{PL}_t = V_{t-1}R_{at} + \sum_{k=1}^{K} N_{kt-1}m_k(F_{kt} - F_{kt-1}), \tag{A.1}$$

where F_{kt} denotes the date-t futures price of type k. Consider a linear regression of R_{at} on the K futures-contract returns:

$$\begin{aligned} R_{at} &= \alpha + \sum_{k=1}^{K} \beta_k R_{kt} + \epsilon_t, \\ R^2 &= 1 - \mathrm{Var}[\epsilon_t]/\mathrm{Var}[R_{at}] \end{aligned} \tag{A.2}$$

where $\mathrm{E}[\epsilon_t|\{R_{kt}\}] = 0$ by construction, and assume that $\{\epsilon_t\}$ is white noise (which should be tested). Then (A.1) can be re-expressed as:

$$\mathrm{PL}_t = V_{t-1}\alpha + \sum_{k=1}^{K} (V_{t-1}\beta_k + N_{kt-1}F_{kt-1}m_k) \times R_{kt} + V_{t-1}\epsilon_t \tag{A.3a}$$

$$R_{pt} = \frac{\mathrm{PL}_t}{V_{t-1}} = \alpha + \sum_{k=1}^{K} \gamma_{kt-1}R_{kt} + \epsilon_t \tag{A.3b}$$

$$\gamma_{kt-1} \equiv \beta_k + \frac{N_{kt-1}F_{kt-1}m_k}{V_{t-1}}. \tag{A.3c}$$

To minimize Strategy A's exposure to movements in the K futures prices, the number of futures contracts $\{N_{kt}\}$ held in the portfolio should be adjusted so as to minimize the net exposures $\{\gamma_{kt}\}$. Specifically,

$$\gamma^*_{kt} = \beta_k + \frac{N^*_{kt}F_{kt}m_k}{V_t} = 0 \tag{A.4a}$$

$$\Rightarrow N^*_{kt} = -\frac{V_t\beta_k}{F_{kt}m_k}. \tag{A.4b}$$

With such a futures overlay in place, the return of the portfolio becomes[9]:

$$\mathrm{PL}^*_t = V_{t-1}(\alpha + \epsilon_t) \tag{A.5a}$$

$$R^*_{pt} = \frac{\mathrm{PL}^*_t}{V_{t-1}} = \alpha + \epsilon_t \tag{A.5b}$$

$$\frac{\mathrm{Var}[R^*_{pt}]}{\mathrm{Var}[R_{at}]} = 1 - R^2 \tag{A.5c}$$

so the reduction in the volatility of the portfolio due to the futures overlay is simply $1 - \sqrt{1 - R^2}$ where R^2 is the coefficient of multiple determination of the linear projection (A.2).

Like beta-blockers, beta-repositioning overlays make use of futures and forward contracts to alter the betas of an investor's portfolio. But in contrast to beta-blockers, repositioning overlays are meant to *generate* beta exposures, not neutralize them. Denote by R_{pt} the date-t return of an investor's entire portfolio, and consider the risk model (1) applied to this portfolio:

$$\begin{aligned} R_{pt} = \alpha_p &+ \beta_{p1}\mathrm{RiskFactor}_{1t} + \cdots \\ &+ \beta_{pK}\mathrm{RiskFactor}_{Kt} + \epsilon_{Pt}. \end{aligned} \tag{A.6}$$

In this context, β_{pk} is the beta exposure of an investor's entire portfolio to risk factor k, which is of course a weighted average of the betas of each manager to factor k, weighted by the fraction of assets allocated to that manager. Denote by R^*_t the target portfolio of the investor, which is determined by the investor's strategic asset-allocation, and apply the same risk factors from (A.6) to this portfolio to obtain the target betas $\{\beta^*_k\}$:

$$\begin{aligned} R^*_t = \alpha^* &+ \beta^*_1\mathrm{RiskFactor}_{1t} + \cdots \\ &+ \beta^*_K\mathrm{RiskFactor}_{Kt} + \epsilon^*_t. \end{aligned} \tag{A.7}$$

Then the beta-repositioning portfolio return R_{ht} is given by the difference between (A.7) and (A.6):

$$\begin{aligned} R_{ht} &\equiv R_t^* - R_{pt} \\ &= \alpha_h + (\beta_1^* - \beta_{p1})\text{RiskFactor}_{1t} + \cdots \\ &\quad + (\beta_K^* - \beta_{pK})\text{RiskFactor}_{Kt} + \epsilon_{ht} \\ &= \alpha_h + \beta_{h1}\text{RiskFactor}_{1t} + \cdots \\ &\quad + \beta_{hK}\text{RiskFactor}_{Kt} + \epsilon_{ht} \end{aligned} \tag{A.8}$$

where $\beta_{hk} \equiv \beta_k^* - \beta_{pk}$.

The beta-repositioning portfolio return Rh_t can be achieved in the identical manner to the beta-blocker using futures and forward contracts.

For purposes of repositioning a portfolio's aggregate exposures, it is often easier to formulate the hedging objective in terms of target portfolio weights or notional exposures rather than the target betas of (A.7). To that end, consider a portfolio with weights $\boldsymbol{\omega}_t = [\omega_{1t} \cdots \omega_{nt}]'$ and suppose the target weights (perhaps from an investor's strategic asset-allocation process) is given by $\boldsymbol{\omega}_t^*$. Then the notional exposures $\mathbf{X}_t \equiv [X_{1t} \cdots X_{nt}]'$ required to restore the portfolio $\boldsymbol{\omega}_t$ to its desired weights $\boldsymbol{\omega}_t^*$ is given by:

$$X_{it} \equiv V_t(\omega_{it}^* - \omega_{it}) \tag{A.9}$$

where V_t is the total assets in the portfolio. If E_i is the notional exposure of a futures contract for asset i, then the number of such contracts N_{it} required in a repositioning overlay is given by:

$$N_{it} \equiv \text{round}(X_{it}/E_i) \tag{A.10}$$

where round($\cdot$) is the function that rounds its argument to the nearest integer.

Table A.1 presents an illustrative example of a \$1B passive portfolio that is initially 60% invested in the S&P 500 Index and 40% invested in the Lehman US Aggregate Index at the start of 2008. By the end of November 2008, price movements alone have changed the asset allocation to 52.6% in the Lehman Index and 47.4% in the S&P 500. Table A.1 shows that using S&P 500 and Lehman Index futures that trade on the Chicago Mercantile Exchange, it is a simple matter to construct a beta-repositioning strategy. Morever, the last column of Table A.1 shows how capital efficient such an overlay strategy is, with a maximum margin requirement of approximately \$15MM to reposition a \$1B portfolio.[10]

As with beta-blockers, beta-repositioning overlay strategies will be only as effective as the underlying risk models allow. However, because we are now applying the risk model (1) to an investor's entire portfolio containing both traditional and alternative assets, the R^2 is likely to be considerably higher. Moreover, the objective is no longer risk reduction, but rather changing the factor exposures of the portfolio, and the ability to achieve this latter objective does not depend on the risk model's R^2. In addition, the natural leverage incorporated into exchange-traded futures contracts, the standardization of those contracts, the existence of a clearing corporation that intermediates all transactions, and the fact that futures are marked-to-market daily make beta-repositioning strategies ideal for institutional investors.

A.2 Tracking error

If the optimal hedging strategy $\{N_{kt}^*\}$ is not implemented continuously—either because of transactions costs or other implementation frictions—tracking errors will arise due to the fact that the optimal net exposures $\{\gamma_{kt}^*\}$ will fluctuate as futures prices, capital, and betas fluctuate. To quantify the impact of such fluctuations, suppose that a futures overlay implemented on date t_0 and left unchanged through date $t > t_0$. Denote by $\tilde{\gamma}_{kt}$ the resulting net exposure on date t, which is obviously not optimal in the sense of (A.4):

$$\tilde{\gamma}_{kt} = \beta_k + \frac{N_{kt_0}^* F_{kt} m_k}{V_{t-1}} = \left(1 - \frac{F_{kt}/F_{kt_0}}{V_{t-1}/V_{t_0}}\right)\beta_k. \tag{A.11}$$

Table A.1 Sample beta-repositioning overlay for a $1B portfolio initially invested 40% in the Lehman US Aggregate Index and 60% in the S&P 500 on December 31, 2007, where the overlay consists of Lehman Aggregate and S&P 500 futures with notional exposures set to maintain a 40/60 asset allocation for the overall portfolio.

Date	Lehman US aggregate index total return	S&P 500 index return*	Lehman US aggregate index level	S&P 500 index level	Bond AUM ($MM)	Stock AUM ($MM)	Bond wgt	Stock wgt	Target notional Bond exposure ($MM)	Target notional Stock exposure ($MM)	Target number of Lehman contracts	Target number of S&P 500 contracts	Bond margin ($MM)	Stock margin ($MM)	Max total margin ($MM)
Initial			1,281.70	1468.36	$400.00	$600.00	40.0%	60.0%	$ —	$ —	0	0			
200801	1.68%	−6.12%	1,304.91	1378.55	$406.72	$563.30	41.9%	58.1%	$(18.71)	$18.71	−143	54	$0.2	$1.7	$1.9
200802	0.14%	−3.48%	1,306.86	1330.63	$407.28	$543.72	42.8%	57.2%	$(26.88)	$26.88	−206	81	$0.3	$2.5	$2.8
200803	0.34%	−0.60%	1,311.66	1322.70	$408.67	$540.48	43.1%	56.9%	$(29.01)	$29.01	−221	88	$0.4	$2.7	$3.1
200804	−0.21%	4.75%	1,308.71	1385.59	$407.82	$566.18	41.9%	58.1%	$(18.22)	$18.22	−139	53	$0.2	$1.6	$1.9
200805	−0.73%	1.07%	1,298.38	1400.38	$404.83	$572.22	41.4%	58.6%	$(14.01)	$14.01	−108	40	$0.2	$1.2	$1.4
200806	−0.08%	−8.60%	1,297.25	1280.00	$404.50	$523.03	43.6%	56.4%	$(33.49)	$33.49	−258	105	$0.4	$3.2	$3.7
200807	−0.08%	−0.99%	1,296.11	1267.38	$404.17	$517.88	43.8%	56.2%	$(35.35)	$35.35	−273	112	$0.4	$3.5	$3.9
200808	0.95%	1.22%	1,309.36	1282.83	$408.01	$524.19	43.8%	56.2%	$(35.13)	$35.13	−268	110	$0.4	$3.4	$3.8
200809	−1.34%	−9.08%	1,290.43	1164.74	$402.53	$476.60	45.8%	54.2%	$(50.88)	$50.88	−394	175	$0.6	$5.4	$6.1
200810	−2.36%	−16.94%	1,257.61	968.75	$393.03	$395.85	49.8%	50.2%	$(77.48)	$77.48	−616	320	$1.0	$9.9	$10.9
200811	3.25%	−7.48%	1,301.80	896.24	$405.82	$366.22	52.6%	47.4%	$(97.00)	$97.00	−745	433	$1.2	$13.4	$14.6

*S&P 500 Index Return does not include dividends.

This shows that the magnitude of the net exposure is inversely related to the absolute value of the ratio of the growth of the futures price to the growth of capital between t_0 and t. The larger the difference in growth between the k-th futures price and capital, the larger the absolute value of the net exposure $\tilde{\gamma}_{kt}$.

Another implication of (A.11) is that the sign of the net exposure is opposite to the relative growth of the futures price and capital, i.e., if the futures price grows faster than the capital, the net exposure $\tilde{\gamma}_{kt}$ to the futures price F_{kt} will be of the opposite sign of β_k, and if the capital grows faster than the futures price, the net exposure will be of the same sign as β_k.

Two special cases are worth noting. When capital is being injected into the strategy, V_t will be increasing much faster than F_{kt}, in which case $\tilde{\gamma}_{kt}$ is likely to be of the same sign as β_k. Therefore, it may be necessary to increase the frequency of rebalancings for those futures contracts k for which β_k is largest in absolute value.

The second special case involves the steady state in which the level of capital is fixed over time, say at V, hence $V_t/V_{t_0} = 1$. In this case:

$$\tilde{\gamma}_{kt} = \left(1 - \frac{F_{kt}}{F_{kt_0}}\right)\beta_k = -R_k(t_0, t)\beta_k \tag{A.12}$$

where $R_k(t_0, t)$ is the compounded net return of the k-th futures contract between t_0 and t. For most index futures contracts, the expected return over any finite interval is positive (because of the risk premium implicit in the index), hence the net exposure $\tilde{\gamma}_{kt}$ will tend to be of the opposite sign of β_k. Moreover, assuming that the one-period futures return R_{kt} is independently and identically distributed with mean μ_k and variance σ_k^2, we have:

$$\begin{aligned} \mathrm{E}_{t_0}[\tilde{\gamma}_{kt}] &= -\,\mathrm{E}_{t_0}[R_k(t_0, t)]\beta_k \\ &= (1 - (1 + \mu_k)^{t-t_0-1})\beta_k \end{aligned} \tag{A.13a}$$

$$\begin{aligned} \mathrm{Var}_{t_0}[\tilde{\gamma}_{kt}] &= \mathrm{Var}_{t_0}[R_k(t_0, t)]\beta_k^2 \\ &= [(\sigma_k^2 - (1 + \mu_k)^2)^{t-t_0-1} \\ &\quad - (1 + \mu_k)^{2(t-t_0-1)}]\beta_k^2. \end{aligned} \tag{A.13b}$$

These expressions can be used to select the most important betas to hedge, as well as to quantify the remaining exposures of the hedged portfolio.

Notes

1 In fact, some futures brokers will accept securities as collateral, albeit with some "haircut", but this should pose little concern for the pension fund since a large fraction of their assets are intended to be buy-and-hold.

2 This hypothesis, which seems to hold for most mutual funds but has been soundly rejected for hedge funds, implies that $\alpha_i = (1 - \sum_k \beta_{ik})R_f$ where R_f is the return on the riskless asset.

3 For example, Breeden's (1979) consumption-based CAPM (CCAPM) relates the expected return of an asset to its beta with respect to aggregate consumption, which currently has no tradable market instrument associated with it.

4 We have borrowed the term "beta-blocker" from the pharmaceutical industry where it refers to a class of drugs used to treat hypertension and heart-attack patients by blocking so-called "beta receptors" in the heart and kidneys. Given recent market conditions, blocking financial betas may yield similar salutary effects.

5 See Lo (2008, Chapter 8) for a more detailed discussion of this integrated investment framework.

6 This two-month lag reflects the fact that fund returns for month t are not typically available in time to implement the hedge for month $t+1$ because of reporting delays. Of course, if the returns are available before the end of month $t+1$, then the hedge can and should be implemented earlier.

7 All Sharpe ratios reported in this paper are computed with respect to a 0% riskfree rate.

8 For a given factor, this percentage change is defined to be $|\beta_t - \beta_{t-1}|/|\beta_{t-1}|$.

9 Note that for practical purposes, N_{kt}^* must be rounded to an integer hence the equalities (A.5a)–(A.5c) hold only approximately, subject to rounding errors in $\{N_{kt}^*\}$.

10 These values are computed under the assumption that the initial margin requirements for the CME Lehman Index and S&P 500 futures contracts are currently $1,620 and

$30,938, respectively, per contract, which are "speculative" margins (our hedging overlay strategy may be eligible for the lower "hedging" margin levels). Also, these figures are initial margin requirements; maintenance margin levels may be lower. The maximum total margin is simply the sum of these two margin requirements, i.e., no cross-netting is assumed.

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